Nuclear Direct Reactions to Continuum 4

– How to get Nuclear Structure Information –

Munetake ICHIMURA (RNC)

VII. Response Function

1. Response Function and Polarization Propagator
2. Mean Field Approximation
3. Tamm-Dancoff Approximation
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VIII. Inclusive Breakup Reactions

1. Breakup Processes
2. Formalism
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VII. Response Function

Here I sketch how to calculate the response functions (Recall **V. DWBA**)

1. Polarization Propagator

1.1. One-body Density Operator

For unified expression, we write

$$\rho_F(\boldsymbol{r}) = \sum_k F_k \delta(\boldsymbol{r} - \boldsymbol{r}_k)$$

where

$$F_k = \sigma_{a,k}^{(\alpha)}, \quad (\alpha = 0, 1,\ a = 0, x, y, z)$$

1.2 Polarization Propagator

Introduce

- **Polarization propagators**

$$\Pi_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega)$$
$$\equiv \langle \Phi_A|\, \rho_F^{\dagger}(\boldsymbol{r})\, \frac{1}{\omega - H_{\mathrm{A}} + \mathrm{i}\,\delta}\, \rho_{F'}(\boldsymbol{r}')|\Phi_A\rangle$$

H_A : Internal Hamiltonian of the nucleus A.

- **Response Functions**

We can write

$$R_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = -\frac{1}{\pi}\mathrm{Im}\;\Pi_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega)$$

2. Mean Field Approximation

2.1 Hamiltonian

This is the 0-th order approximation.

Approximate H_{A} by

Mean Field Hamiltonian, H_0

$$H_A \longrightarrow H_0 = \sum_k \hat{h}_k - T_{\mathrm{c.m.}}$$

$\hat{h}_k$: **Single-particle Hamiltonian**

for the k-th nucleon in A

$$\hat{h}_k = T_k + U_k^{\mathrm{m.f}}$$

$U_k^{\mathrm{m.f}}$: Mean field (Hartree-Fock field)

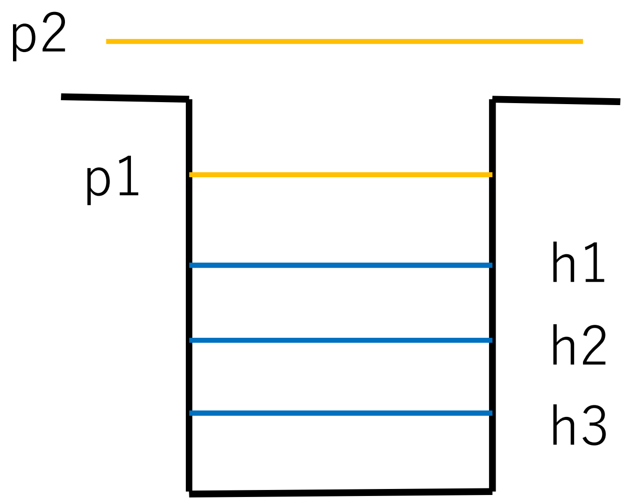
- Single particle states

  $|h\rangle$: occupied single particle state

  $|p\rangle$: unoccupied single particle state

They obey

$$\hat{h}|h\rangle = \epsilon_h|h\rangle, \quad \hat{h}|p\rangle = \epsilon_p|p\rangle,$$



2.2. Free Polarization Propagator

The polarization propagator in the mean field approximation is called

Free polarization propagator

$$\Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega)$$
$$= \langle\Phi^{(0)}_A|\rho^{\dagger}_F(\boldsymbol{r})\frac{1}{\omega-(H_0-\mathcal{E}^{(0)}_0)+\mathrm{i}\,\delta}\rho_{F'}(\boldsymbol{r}')|\Phi^{(0)}_A\rangle$$

$\Phi^{(0)}_A$: Ground state of A

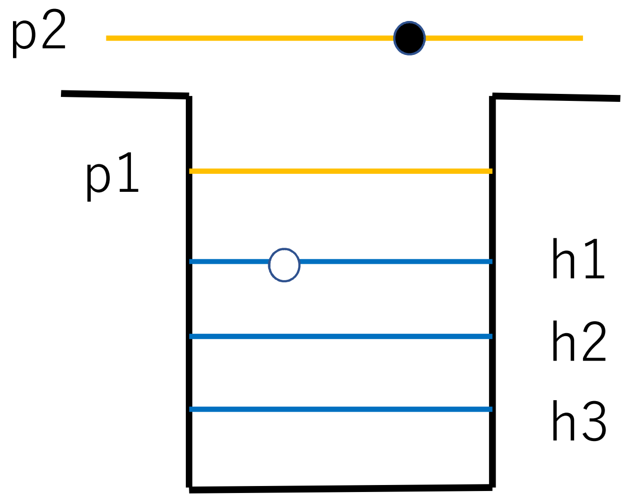
in the mean field approximation

$$H_0\Phi^{(0)}_A=\mathcal{E}^{(0)}_0\Phi^{(0)}_A$$

Note the operation of the density operator

$$\rho_{F'}(\boldsymbol{r'})|\Phi_A^{(0)}\rangle = \sum_{h,p} |h^{-1}p\rangle\langle h^{-1}p|\rho_{F'}(\boldsymbol{r'})|\Phi_A^{(0)}\rangle$$

the sum of 1-particle-1-hole states.



We can write

$$\Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \sum_{p,h}\langle\Phi^{(0)}_A|\rho^\dagger_F(\boldsymbol{r})|h^{-1}p\rangle \times \frac{1}{\omega-(\epsilon_p-\epsilon_h)+\mathrm{i}\,\delta} \times \langle h^{-1}p|\rho_{F'}(\boldsymbol{r}')|\Phi^{(0)}_A\rangle = \langle\Phi^{(0)}_A|\rho^\dagger_F(\boldsymbol{r})G_{ph}(\omega)\rho_{F'}(\boldsymbol{r}')|\Phi^{(0)}_A\rangle$$

Here we introduced

- **ph Green's function**

$$G_{ph}(\omega) = \sum_{h,p}|h^{-1}p\rangle\frac{1}{\omega-(\epsilon_p-\epsilon_h)+\mathrm{i}\,\delta}\langle h^{-1}p|$$

How to cope with the infinite sum $\sum_{p\in\mathrm{unocc}}$?
p runs continuously !

cf. $\sum_h$ run only finite number of states, and thus can be handled.

Further manipulation

$$G_{ph}(\omega) = \sum_h |h^{-1}\rangle \; g(\omega + \epsilon_h)\langle h^{-1}|$$

with

$$\begin{aligned}
g(\epsilon) &= \sum_{p\in\mathrm{unocc}} |p\rangle \frac{1}{\epsilon - \epsilon_p + \mathrm{i}\,\delta} \langle p| \\
&= \sum_{p\in\mathrm{full}} |p\rangle \frac{1}{\epsilon - \epsilon_p + \mathrm{i}\,\delta} \langle p| \\
&- \sum_h |h\rangle \frac{1}{\epsilon - \epsilon_p + \mathrm{i}\,\delta} \langle h| \\
&= g_{\mathrm{sp}}(\epsilon) - \sum_h |h\rangle \frac{1}{\epsilon - \epsilon_h + \mathrm{i}\,\delta} \langle h|
\end{aligned}$$

where

$$\begin{aligned}
g_{\mathrm{sp}}(\epsilon) &= \sum_{p\in\mathrm{full}} |p\rangle \frac{1}{\epsilon - \epsilon_p + \mathrm{i}\,\delta} \langle p| \\
&= \frac{1}{\epsilon - \hat{h} + \mathrm{i}\,\delta}
\end{aligned}$$

- The single particle Green's function

$g_{\rm sp}(\epsilon)$ in $\boldsymbol{r}$ representation

$$g_{\rm sp}(\boldsymbol{r},\boldsymbol{r}';\epsilon) = \langle \boldsymbol{r}|g_{\rm sp}(\epsilon)|\boldsymbol{r}'\rangle$$
$$= \langle \boldsymbol{r}|\frac{1}{\epsilon - \hat{h} + \mathrm{i}\,\delta}|\boldsymbol{r}'\rangle$$

is known to be calculable.

- Calculation of $g_{\rm sp}(\boldsymbol{r},\boldsymbol{r}';\epsilon)$

  (Ignore spins)

Angular momentum representation

$$g_{\rm sp}(\boldsymbol{r},\boldsymbol{r}';\epsilon) = \sum_{lm} Y_{lm}(\Omega_r)\frac{g_l(r,r';\epsilon)}{rr'}Y_{lm}^{\dagger}(\Omega_{r'})$$

The radial parts

$$g_l(r, r'; \epsilon) = \frac{2m_N}{W(f_l, h_l)} f_l(r_<; \epsilon) h_l(r_>; \epsilon)$$

where $r_< = \min(r, r'),\ r_> = \max(r, r')$,

$f_l(r; \epsilon)$ and $h_l(r; \epsilon)$:
regular and singular solutions of the equation

$$\left[-\frac{1}{2m_N}\frac{d^2}{dr^2} + \frac{1}{2m_N}\frac{l(l+1)}{r^2} + U^{\mathrm{m.f}}(r)\right] u_l(r; \epsilon)$$
$$= \epsilon\ u_l(r; \epsilon)$$

$W(f, h)$: Wronskian

$$W(f, h) = \begin{vmatrix} f & h \\ f' & h' \end{vmatrix}$$

Thus

$$g(\boldsymbol{r},\boldsymbol{r}';\epsilon) = \langle \boldsymbol{r}|g(\epsilon)|\boldsymbol{r}'\rangle$$

$$= g_{\rm sp}(\boldsymbol{r},\boldsymbol{r}';\epsilon) - \sum_h \phi_h(\boldsymbol{r}) \frac{1}{\epsilon - \epsilon_h + \mathrm{i}\,\delta} \phi_h^*(\boldsymbol{r}')$$

is calculable

$\phi_h(\boldsymbol{r})$: Bound state wave function of the state $|h\rangle$

Now we can calculate

- **Free Polarization Propagator**

$$\Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega)$$

$$= \sum_h \langle \Phi^{(0)}_A|\rho^\dagger_F(\boldsymbol{r})|h\rangle g(\boldsymbol{r},\boldsymbol{r}';\omega+\epsilon_h)\langle h|\rho_{F'}(\boldsymbol{r}')|\Phi^{(0)}_A\rangle$$

and get

- **Free Response Function**

$$R^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = -\frac{1}{\pi}\mathrm{Im}\;\Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega)$$

[Comment]

In actual calculations, various refinements should be taken into account.

- Spins, Isospins
- Δ isobar,
- Complex mean field
  (representing particle spreading width)
- Energy-dependent mean field
- (radial dependent) effective mass

$$m_N \longrightarrow m^*(r)$$

- Perey factor
- Spreading widths of holes
- Orthogonality condition
- etc.

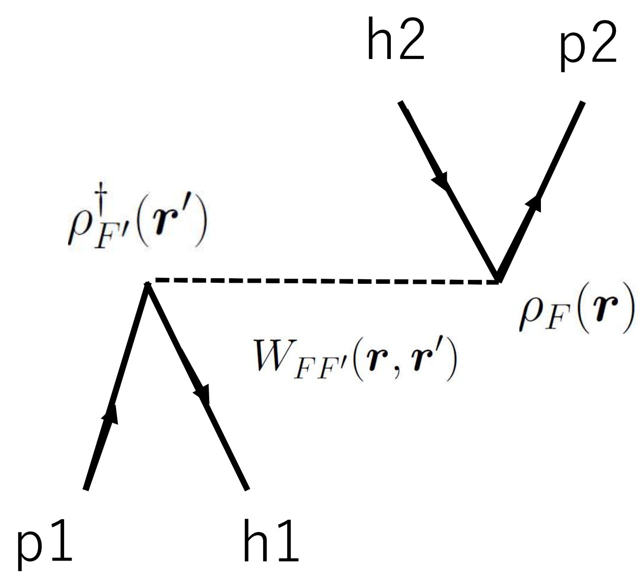
For details, see Manual of the program RESPQ in http://www.nishina.riken.jp/researcher/archive/program_e.html

3. Tamm-Dancoff Approximation

(Usually abbreviated **TDA**)

Consider the nuclear correlations induced by the ph interaction V_{ph}

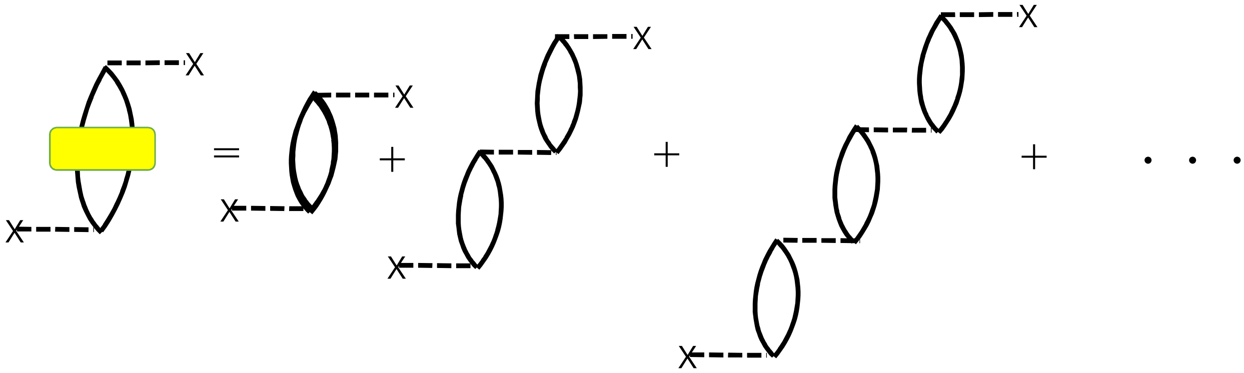
● ph interaction V_{ph}



$$V_{ph} = \sum_{FF'} \int d^3\boldsymbol{r} d^3\boldsymbol{r}' \rho_F(\boldsymbol{r}) W_{FF'}(\boldsymbol{r},\boldsymbol{r}') \rho^{\dagger}_{F'}(\boldsymbol{r}')$$

● Polarization propagators in TDA

Take account of the correlation



The polarization propagator
with this correlation is given
by the solution of the equation

$$\begin{aligned}\Pi^{\rm TDA}_{FF'}(\boldsymbol{r},\boldsymbol{r}') = &\Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}') \\ &+ \sum_{F''F'''}\int d^3\boldsymbol{r}''d^3\boldsymbol{r}'''\Pi^{(0)}_{FF''}(\boldsymbol{r},\boldsymbol{r}'') \\ &\times W_{F''F'''}(\boldsymbol{r}'',\boldsymbol{r}''')\Pi^{\rm TDA}_{F'''F'}(\boldsymbol{r}''',\boldsymbol{r}')\end{aligned}$$

4. Random Phase Approximation – Ring approximation

(Commonly abbreviated as **RPA**)

More elaborated approximation.

Generalize

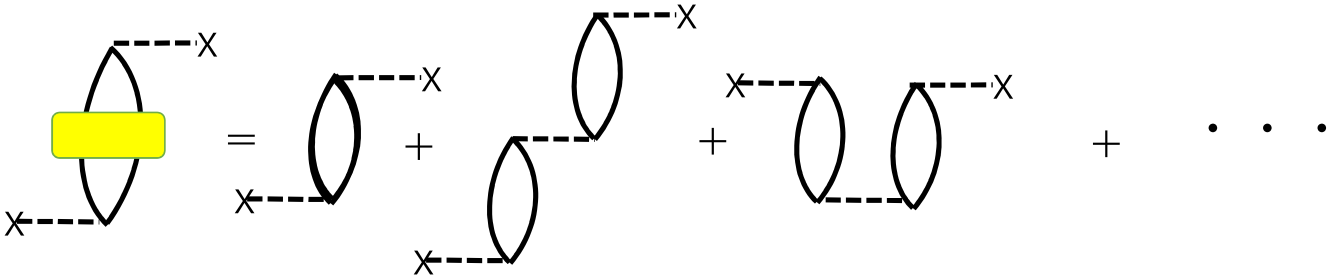
Free polarization propagator $\Pi^{(0)}_{FF'}$ as

$$
\begin{aligned}
&\Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) \\
&= \langle\Phi_0|\rho^{\dagger}_{F}(\boldsymbol{r})\frac{1}{\omega-(H_0-\mathcal{E}_0)+\mathrm{i}\,\delta}\rho_{F'}(\boldsymbol{r}')|\Phi_0\rangle \\
&+ \langle\Phi_0|\rho_{F'}(\boldsymbol{r}')\frac{1}{-\omega-(H_0-\mathcal{E}_0)+\mathrm{i}\,\delta}\rho^{\dagger}_{F}(\boldsymbol{r})|\Phi_0\rangle
\end{aligned}
$$

● **Polarization propagators in RPA**

Given by solving the RPA equation

$$\Pi^{\mathrm{RPA}}_{FF'}(\boldsymbol{r},\boldsymbol{r}') = \Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}') + \sum_{F''F'''}\int d^3\boldsymbol{r}'' d^3\boldsymbol{r}''' \Pi^{(0)}_{FF''}(\boldsymbol{r},\boldsymbol{r}'') \times W_{F''F'''}(\boldsymbol{r}'',\boldsymbol{r}''')\Pi^{\mathrm{RPA}}_{F'''F'}(\boldsymbol{r}''',\boldsymbol{r}')$$



Once this approximation had been called
New Tamm-Dancoff Approximation

5. Fermi Gas Model

Recall PWBA formula

$$\frac{d^2\sigma}{d\omega^* d\Omega} = K \frac{\sqrt{s}}{m_A} \, |\tilde{V}(\boldsymbol{q}^*)|^2 \; R_\rho(\omega, \boldsymbol{q}^*)$$

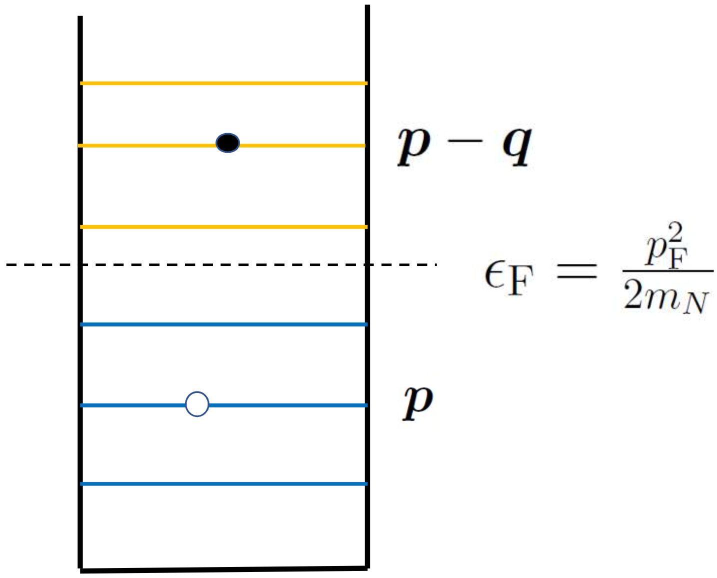
$$R_\rho(\omega, \boldsymbol{q})$$
$$= -\frac{1}{\pi}\mathrm{Im}\langle\Phi_A|\tilde{\rho}^\dagger(\boldsymbol{q})\frac{1}{\omega - H_A + \mathrm{i}\eta}\tilde{\rho}(\boldsymbol{q})|\Phi_A\rangle$$

$$\tilde{\rho}((\boldsymbol{p}) = \sum_{k=1}^{A} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}_k}$$

Let us calculate the free response function $R_\rho(\omega, \boldsymbol{q})$ in a simple model.

Fermi gas model provides the analytic form, from which we can learn some characteristic of the response functions.

- Fermi gas model



$$\Pi^{(0)}(\boldsymbol{q},\omega) = \sum_{p,h} \langle \Phi_A^{(0)} | \tilde{\rho}^\dagger(\boldsymbol{q}) | h^{-1}p \rangle$$

$$\times \frac{1}{\omega - (\epsilon_p - \epsilon_h) + \mathrm{i}\,\delta} \langle h^{-1}p | \tilde{\rho}((\boldsymbol{q}) | \Phi_A^{(0)} \rangle$$

$$= \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\theta(p_F - p)\theta(|\boldsymbol{p}-\boldsymbol{q}| - p_F)}{\omega - \left(\frac{(\boldsymbol{p}-\boldsymbol{q})^2}{2m_N} - \frac{p^2}{2m_N}\right) + \mathrm{i}\delta}$$

$$= \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\theta(p_F - p)\theta(|\boldsymbol{p}-\boldsymbol{q}| - p_F)}{\omega - \left(\frac{q^2}{2m_N} - \frac{\boldsymbol{q}\cdot\boldsymbol{p}}{m_N}\right) + \mathrm{i}\delta}$$

$$|h\rangle = |\boldsymbol{p}\rangle, \qquad |p\rangle = |\boldsymbol{p}-\boldsymbol{q}\rangle$$

The free response function

$$R^{(0)}(\boldsymbol{q},\omega) = -\frac{1}{\pi}\mathrm{Im}\Pi^{(0)}(\boldsymbol{q},\omega)$$

$\Pi^{(0)}(\boldsymbol{q},\omega)$ can analytically be calculated.
It is known as the Lindhart function.

A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-particle Systems*, McGraw-Hill, Inc. (1971)

[Just for fun]

Analytical form of $R^{(0)}(\boldsymbol{q},\omega)$

p_{F} : Fermi momentum
$\epsilon_{\mathrm{F}} = \frac{p_{\mathrm{F}}^2}{2m_N}$: Fermi energy

Set

$$x = \frac{q}{2p_{\mathrm{F}}}, \quad y = \frac{\omega}{\epsilon_{\mathrm{F}}}$$

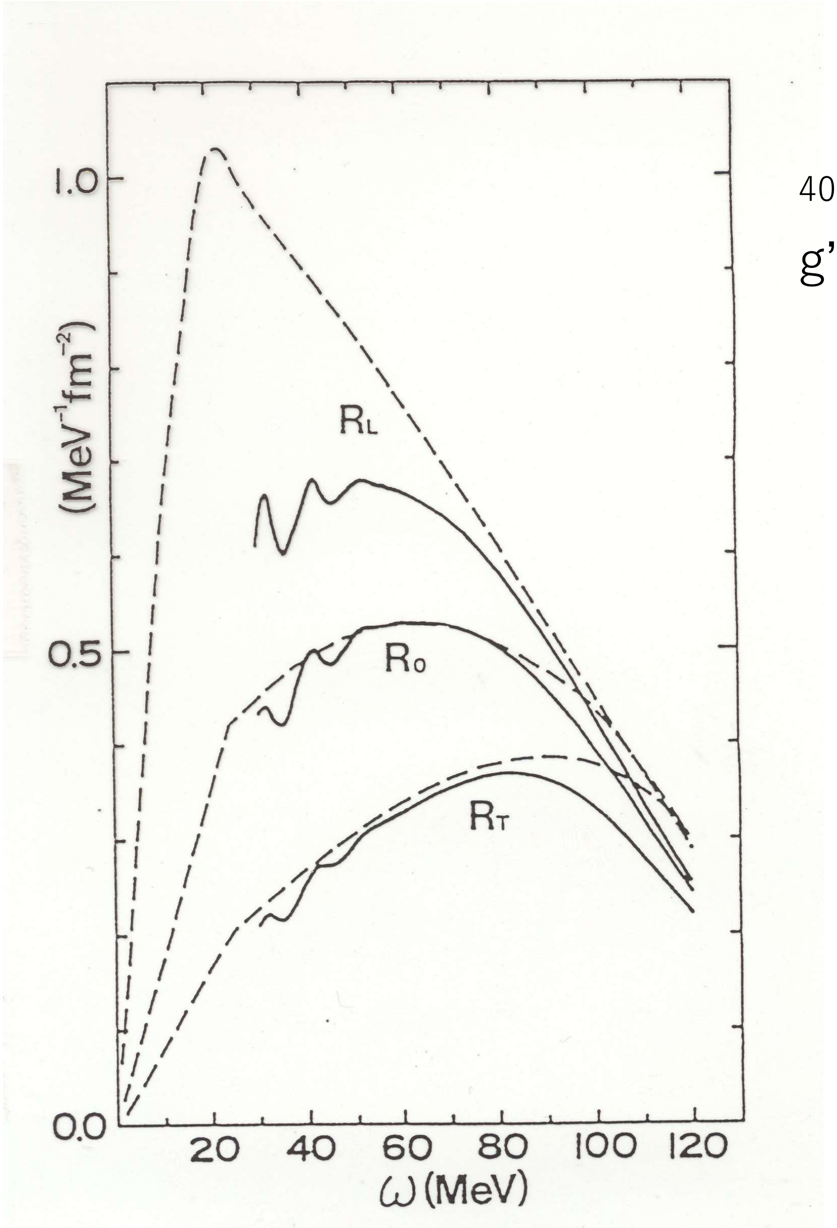
For $0 \leq x \leq 1$

$$R^{(0)}(\boldsymbol{q},\omega) = \frac{m_N p_{\mathrm{F}}}{(2\pi)^2} \begin{cases} \frac{y}{4x} & \text{for } \frac{y}{4x} < 1 - x \\ \frac{1-(x-\frac{y}{4x})^2}{4x} & \text{for } 1 - x < \frac{y}{4x} < 1 + x \end{cases}$$

For $x > 1$

$$R^{(0)}(\boldsymbol{q},\omega) = \frac{m_N p_{\mathrm{F}}}{(2\pi)^2} \frac{1-(x-\frac{y}{4x})^2}{4x} \quad \text{for } x - 1 < \frac{y}{4x} < 1 + x$$

● What spectrum $R^{(0)}(\boldsymbol{q}, \omega)$ has ?



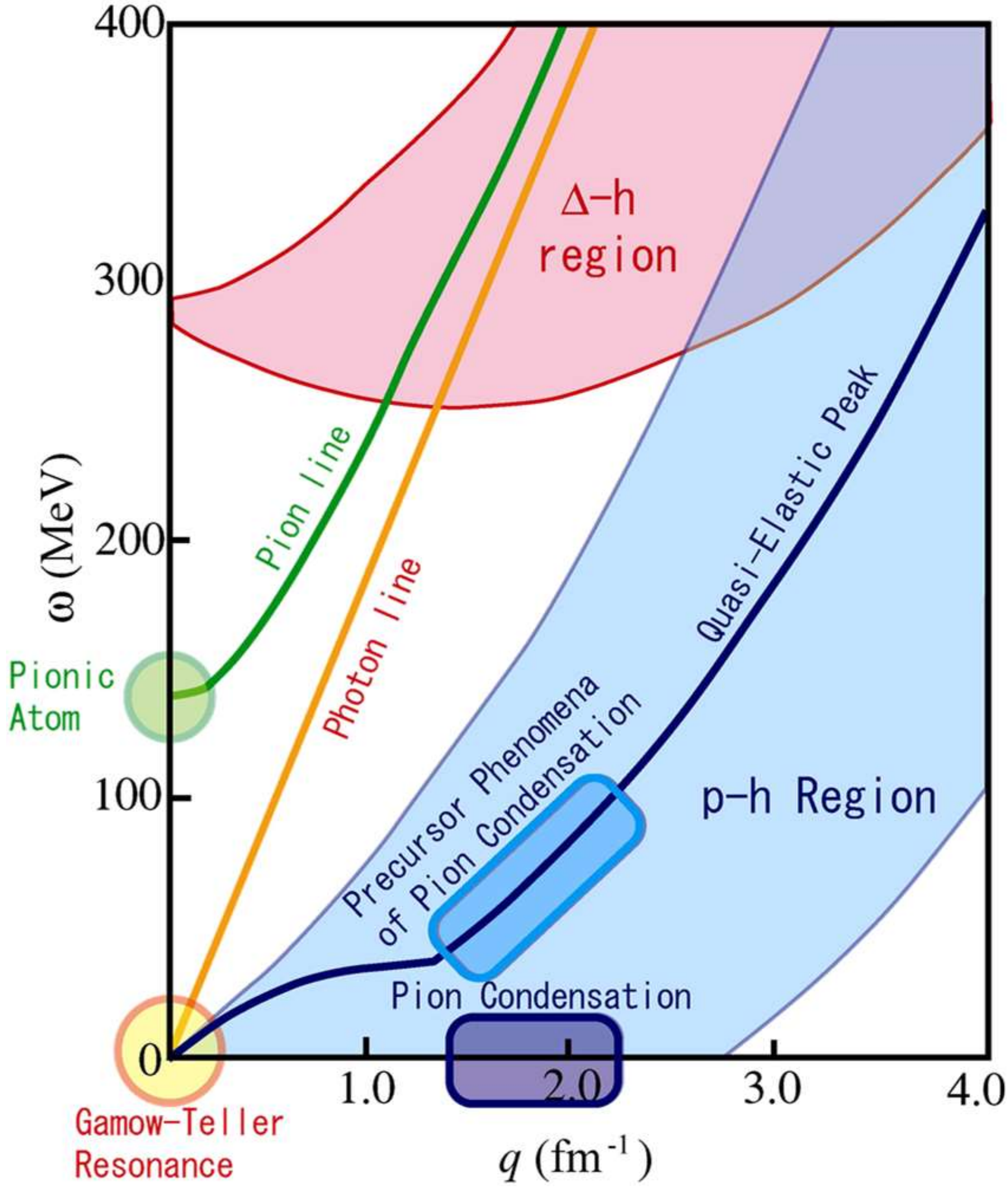
^{40}Ca q=1.75 fm^{-1}

$g'_{NN}=g'_{N\Delta}= g'_{\Delta\Delta} = 0.6$

M. Ichimura et al,
PR 39 (1989) 1446

Peak at $\omega_{\text{peak}} = \dfrac{q^2}{2m_N}$

- Whereis $R^{(0)}(\boldsymbol{q}, \omega)$ finite ?



● What region can we study ?

A reaction can reach very limited region.

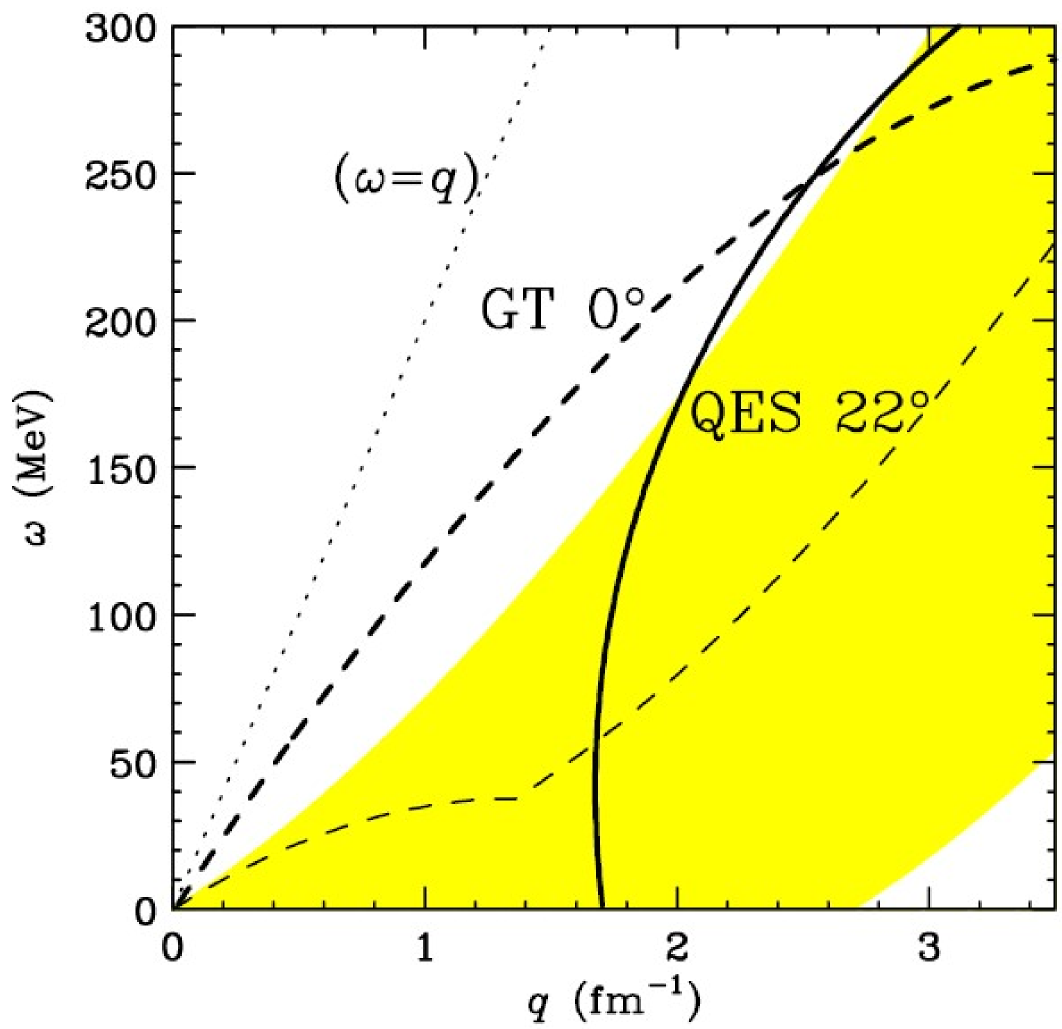


図 1 ^{90}Zr$(p,n), T_p = 300$ MeV, $\theta = 0$deg, ^{12}C$(p,n), T_p = 350$ MeV, $\theta = 22$deg

M. Ichimura, H. Sakai and T. Wakasa, Prog. Part. Mucl. Phys. **56**, 446 (2006)

6. Relation to familiar quantities

Relation between $R(q,\omega)$ and familiar quantities.

(1) GT strength

$$B_{\mathrm{GT}^{\pm}}(\omega) = \sum_X |\langle\Phi_X|\sum_k t_k^{\pm}\boldsymbol{\sigma}_k|\Phi_A\rangle|^2\delta(\omega-\omega_X)$$

$$R_{\mathrm{GT}^{\pm}}(q,\omega) = \sum_X |\langle\Phi_X|\sum_k t_k^{\pm}\boldsymbol{\sigma}_k \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_i}|\Phi_A\rangle|^2\delta(\omega-\omega_X)$$

Thus

$$B_{\mathrm{GT}^{\pm}}(\omega) = R_{\mathrm{GT}^{\pm}}(q=0,\omega)$$

(2) Fermi transition strength

$$B_{\mathrm{F}^{\pm}}(\omega) = \sum_X |\langle\Phi_X|\sum_k t_k^{\pm}|\Phi_A\rangle|^2\delta(\omega-\omega_X)$$

$$R_{\mathrm{F}^{\pm}}(q,\omega) = \sum_X |\langle\Phi_X|\sum_k t_k^{\pm}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_i}|\Phi_A\rangle|^2\delta(\omega-\omega_X)$$

$$B_{\mathrm{F}^{\pm}}(\omega) = R_{\mathrm{F}^{\pm}}(q=0,\omega)$$

(3) E1 transition strength (> GDR)

For the case $J_A = 0$

$$B_{\mathrm{E1}}(\omega) = \sum_{X \in 1^-} |\langle \Phi_X | \sum_k t_{z,k} \boldsymbol{r}_k | \Phi_A \rangle|^2 \delta(\omega - \omega_X)$$

Response Function to the 1^- states

$$\begin{aligned}
& R_{\mathrm{IV1-}}(q,\omega) \\
&= \sum_{X \in 1^-} |\langle \Phi_X | \sum_k t_{z,k} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_i} | \Phi_A \rangle|^2 \delta(\omega - \omega_X) \\
&= \sum_{X \in 1^-} |\langle \Phi_X | \sum_k t_{z,k} \left(1 - \mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_i + O(q^2)\right) | \Phi_A \rangle|^2 \\
& \times \delta(\omega - \omega_X) \\
&= q^2 \sum_{X \in 1^-} |\langle \Phi_X | \sum_k t_{z,k} \boldsymbol{r}_i | \Phi_A \rangle|^2 \delta(\omega - \omega_X) + O(q^4)
\end{aligned}$$

Thus

$$B_{\mathrm{E1}}(\omega) = \lim_{q \to 0} \frac{1}{q^2} R_{\mathrm{IV1^-}}(q,\omega)$$

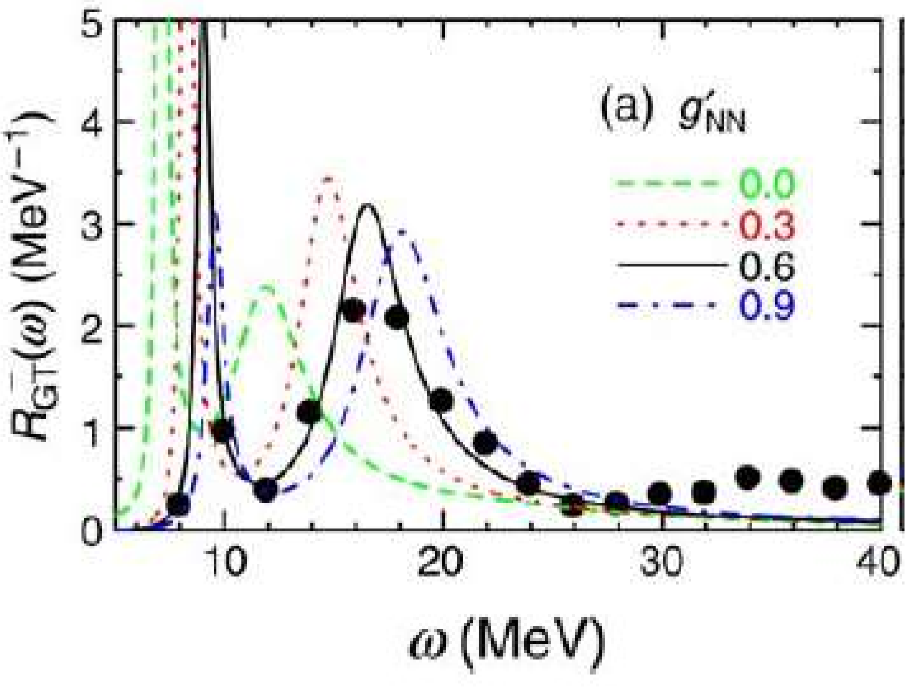
7. Discussion

7.1 Comments on the Fermi gas model

(1) Fermi gas model is heuristic,
but not necessarily realistic.

(2) It may reasonably work for large q region

(3) But for small q region, it is useless and even misleading.



Spectrum at $q = 0$, ^{90}Zr to ^{90}Nb

(4) If you want to have $R(q, \omega)$,

First calculate $R(\boldsymbol{r}, \boldsymbol{r}'; \omega)$,

by the methods described in subsec. 2-4.

Then take its Fourier transform

$$R(q, \omega) = \tilde{R}(\boldsymbol{q}, \boldsymbol{q}; \omega)$$

7.2 Comments on calculation of $R(\boldsymbol{r}, \boldsymbol{r}'; \omega)$

(1) Choices of the mean field is crucial.

(2) Choice of effective *ph* interaction is crucial.

(3) Calculations are carried out in the angular momentum representation.
Namely, calculate $R^J_{SL,S'L'}(r, r')$

(4) Taking suitable linear combinations of $R^J_{SL,S'L'}(r, r')$, we can calculate
the response functions we want,
such as R_{S}, R_{L}, R_{T}, etc.

(5) How to include nuclear correlations beyond TDA or RPA in the framework of the present formalism is a longstanding subject

There are lots of matters to be discussed about response functions.
But they are out of scope in this lecture.

For details about comments (3) and (4), see Manual of the program RESPQ in http://www.nishina.riken.jp/researcher/archive/program_e.html

7.3 Comments on PWIA

(1) Factorized form

$$\frac{d^2\sigma}{d\omega^* d\Omega} = K\ |V_i(q)|^2 R(q,\omega)$$

is very attractive nature to extract nuclear information $R_i(q,\omega)$

(2) This doesn't hold in DWIA or more elaborate reaction theories.

(3) PWIA is heuristic, but not realistic in general.

(4) It may work for some cases, if one allows to use normalization factor as

$$\frac{d^2\sigma}{d\omega^* d\Omega} = N_{\rm eff}\ \left[K\ |V_i(q)|^2 R(q,\omega)\right]$$

$N_{\rm eff}$: Effective nucleon number

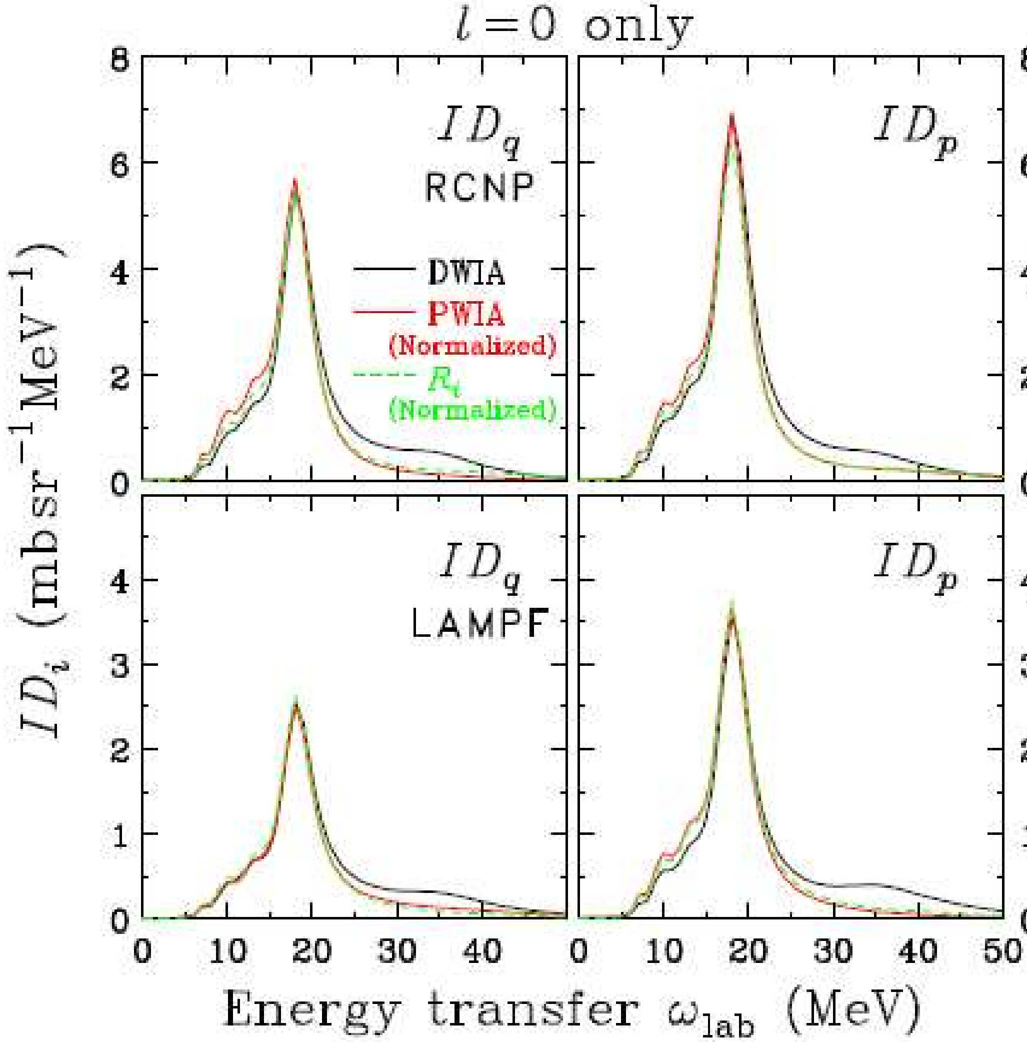


図 2 ^{208}Pb(p,n) at 296 MeV. T. Wakasa, Private communication

Looks OK, but to extract $R_i(q,\omega)$ we need to know $N_{\rm eff}$ from other independent data or by theoretical calculation.

● Taddeucci's Prescription

A kind od the N_{eff} method.

Applied to GT transitions, etc., very often.

Set a semi-empirical ansatz

$$\frac{d^2\sigma(q,\omega)}{d\omega d\Omega} = \hat{\sigma} F(q,\omega) R(q=0,\omega)$$

$\hat{\sigma}$: unit cross section

$F(q,\omega)$: Normalized angular distribution

$$F(q=0,\omega) = 1$$

e.g. $\quad R(q=0,\omega) = R_{\text{F}}(\omega)$ or $R_{\text{GT}(\omega)}$

In N_{eff} method,

$$\hat{\sigma} = N_{\text{eff}} K(q=0) |V(q=0)|^2$$

- Calculate $F(q,\omega)$ by DWBA
  with simple nuclear structure model.
  * ω dependence is not care for .

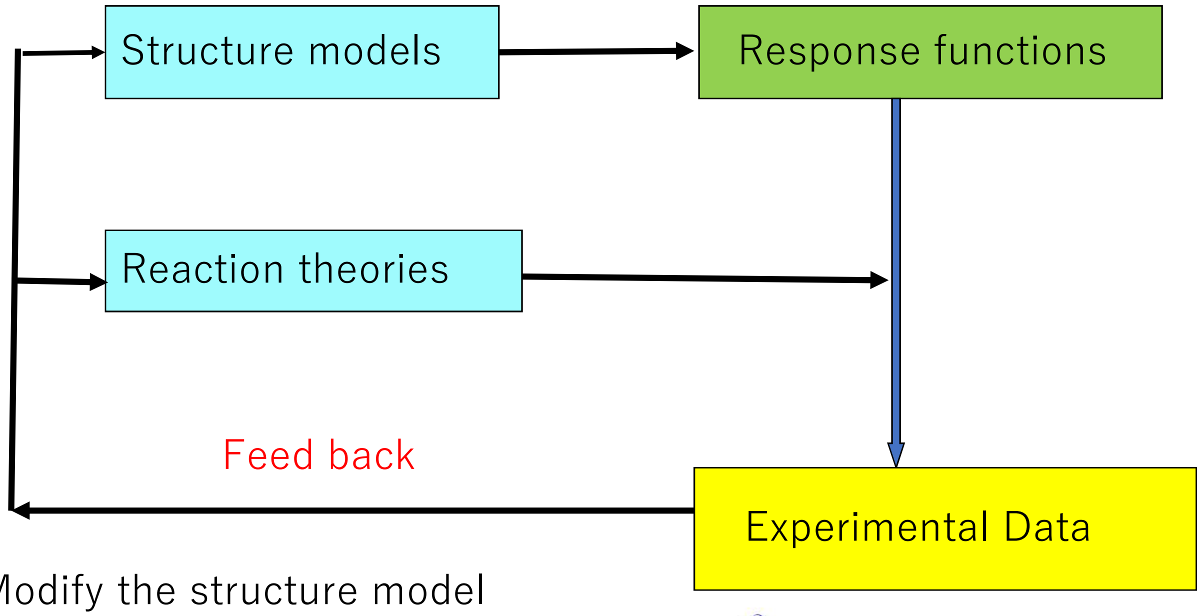
- From observed database of
  $$\frac{d^2\sigma(q,\omega)}{d\omega d\Omega}, \text{ and } R(q=0,\omega)$$
  Evaluate $\hat{\sigma}$.

- Apply the formula to the newly observed
  data, and obtain $R(q=0,\omega)$.

Careful calibration is needed !

7.4 For more general cases

My opinion is



Modify the structure model
(parameters in the model, etc.)

$\frac{d^2\sigma}{d\omega^* d\Omega}$ ID_0 ID_n ID_q ID_p

Refine the reaction theory

VIII. Inclusive Breakup Reactions

1. Breakup Processes

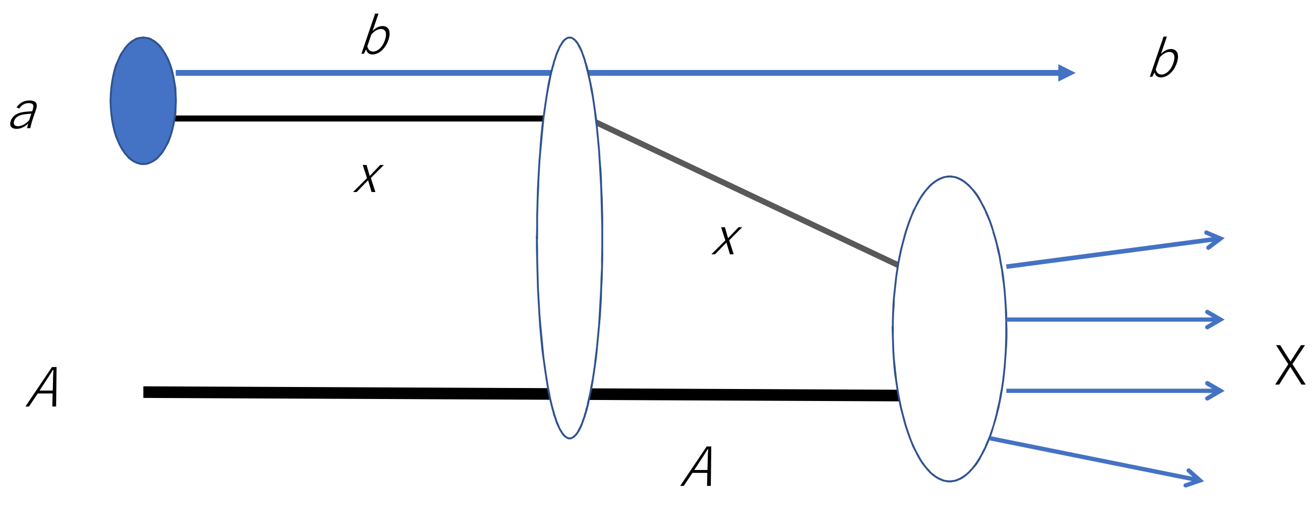
Consider the inclusive breakup reactions

$$a + A \longrightarrow b + \text{anything}$$

$$a = b + x$$

Assume

b and x : structureless



The process is decomposed

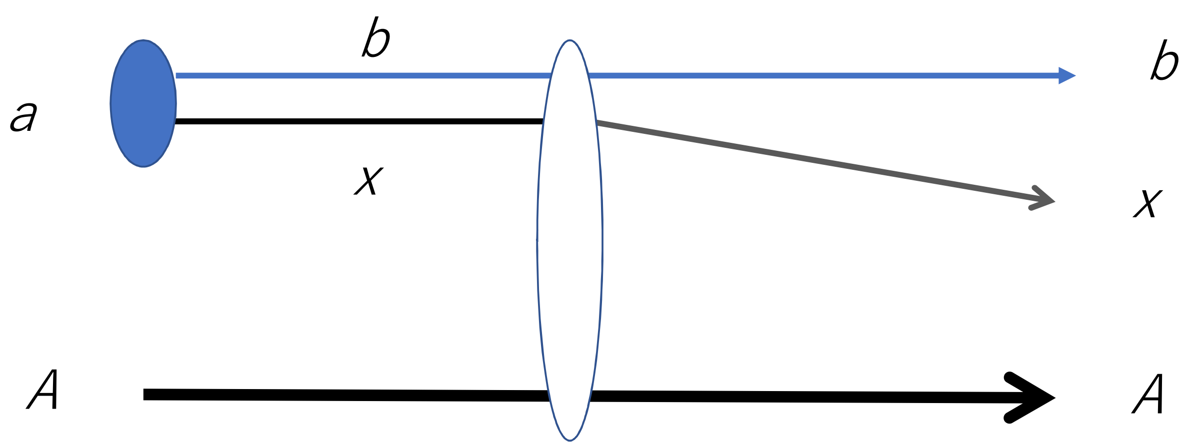
(1) Elastic breakup

(2) Inelastic breakup

(3) Transfer reaction

(4) Breakup fusion (Incomplete fusion)

- Elastic breakup



We will consider the decomposition

Elastic Breakup + Non-elastic Breakup

$$\frac{d^2\sigma^{\text{inc}}}{dE_b d\Omega_b} = \frac{d^2\sigma^{\text{EBU}}}{dE_b d\Omega_b} + \frac{d^2\sigma^{\text{NEB}}}{dE_b d\Omega_b}$$

2. Formalism

- Hamiltonian

$$
\begin{aligned}
H &= T_b + T_x + H_A + V_{xb} + V_{xA} + V_{bA} \\
&= (T_b + U_{bA}) + (T_x + V_{xA}) + H_A \\
&+ (V_{xb} + V_{bA} - U_{bA}) \\
&= (T_a + U_{aA}) + (T_{bx} + V_{bx}) + H_A \\
&+ (V_{xA} + V_{bA} - U_{aA})
\end{aligned}
$$

- Wave functions

$$H_A\Phi_A = E_A\Phi_A$$

$$H_X\Phi_X = (T_x + V_{xA} + H_A)\Phi_X = E_X\Phi_X$$

$$(T_{bx} + V_{bx})\phi_a = \epsilon_a\phi_a$$

- Distorted waves

$$(T_a + U_{aA})\chi_a^{(+)} = E_a\chi_a^{(+)}$$

$$(T_b + U_{bA})\chi_b^{(-)} = E_b\chi_b^{(-)}$$

- Total energy of the initial state

$$E_i = E_A + \epsilon_a + E_a$$

- DWBA

$$\begin{aligned} T_{fi} &= \langle \Phi_X \chi_b^{(-)} | V_{xb} + V_{bA} - U_{bA} | \Phi_A \phi_a \chi_a^{(+)} \rangle \\ &= \langle \Phi_X \chi_b^{(-)} | V^{\text{post}} | \Phi_A \phi_a \chi_a^{(+)} \rangle \end{aligned}$$

- Inclusive cross section

$$\frac{d^2\sigma^{\text{inc}}}{dE_b d\Omega_b} = K \sum_X \left| \langle \Phi_X \chi_b^{(-)} | V^{\text{post}} | \Phi_A \phi_a \chi_a^{(+)} \rangle \right|^2 \times \delta(E_i - E_b - E_X)$$

Using the completeness, we get

$$\frac{d^2\sigma^{\text{inc}}}{dE_b d\Omega_b} = K \, \langle \Phi_A \phi_a \chi_a^{(+)} | V^{\text{post},\dagger} | \chi_b^{(-)} \rangle \times \delta(E_i - E_b - H_X) \langle \chi_b^{(-)} | V^{\text{post}} | \Phi_A \phi_a \chi_a^{(+)} \rangle$$

Assuming the excitation of A by $V^{\rm post}$ is very small, we can write

$$\langle\chi_b^{(-)}|V^{\rm post}|\Phi_A\phi_a\chi_a^{(+)}\rangle$$
$$= |\Phi_A\rangle\langle\chi_b^{(-)}\Phi_A|V^{\rm post}|\Phi_A\phi_a\chi_a^{(+)}\rangle$$

Then we get

$$\frac{d^2\sigma^{\rm inc}}{dE_b d\Omega_b}$$
$$= K\ \langle\Phi_A\phi_a\chi_a^{(+)}|V^{\rm post,\dagger}|\Phi_A\chi_b^{(-)}\rangle$$
$$\times\ \langle\Phi_A|\delta(E_i - E_b - (T_x + H_A + V_{xA}))|\Phi_A\rangle$$
$$\times\ \langle\chi_b^{(-)}\Phi_A|V^{\rm post}|\Phi_A\phi_a\chi_a^{(+)}\rangle$$
$$= K\ \langle\Phi_A\phi_a\chi_a^{(+)}|V^{\rm post,\dagger}|\Phi_A\chi_b^{(-)}\rangle$$
$$\times\ \langle\Phi_A|\delta(\omega - T_x - V_{xA})|\Phi_A\rangle$$
$$\times\ \langle\chi_b^{(-)}\Phi_A|V^{\rm post}|\Phi_A\phi_a\chi_a^{(+)}\rangle$$

where

$$\omega = E_a + \epsilon_a - E_b$$

is the energy transfer

Introducing the Green's function of x

$$G_x(\omega) = \langle \Phi_A | \frac{1}{\omega - (T_x + V_{xA}) + \mathrm{i}\delta} | \Phi_A \rangle$$
$$= \frac{1}{\omega - T_x - U_x + \mathrm{i}\delta}$$

with Optical potential of x on A

$$U_x = V_x + \mathrm{i}W_x$$

All excitations of A are included through U.

- Inclusive breakup cross section

$$\frac{d^2\sigma^{\mathrm{inc}}}{dE_b d\Omega_b} = -\frac{K}{\pi}\mathrm{Im}\int d^3\boldsymbol{r}'_x \int d^3\boldsymbol{r}_x$$
$$\times\ S^\dagger(\boldsymbol{r}'_x) G_x(\boldsymbol{r}'_x, \boldsymbol{r}_x) S(\boldsymbol{r}_x)$$

where

$$S(\boldsymbol{r}_x) = \langle \boldsymbol{r}_x \chi_b^{(-)} \Phi_A | V^{\mathrm{post}} | \Phi_A \phi_a \chi_a^{(+)} \rangle$$

$$G_x(\boldsymbol{r}'_x, \boldsymbol{r}_x; \omega) = \langle \boldsymbol{r}'_x | G_x(\omega) | \boldsymbol{r}_x \rangle$$

[Comment]

About the relation

$$\langle \Phi_A | \frac{1}{\omega - (T_x + V_{xA}) + i\delta} | \Phi_A \rangle = \frac{1}{\omega - T_x - U_x + i\delta}$$

Note

$$\langle \Phi_A | \frac{1}{\omega - (T_x + V_{xA}) + i\delta} | \Phi_A \rangle$$
$$\neq \frac{1}{\langle \Phi_A | \omega - (T_x + V_{xA}) + i\delta | \Phi_A \rangle}$$

Set

$$P = |\Phi_A\rangle\langle\Phi_A|, \quad Q = 1 - P, \quad \omega^+ = \omega + i\delta$$

By short manupulation

$$P \frac{1}{\omega^+ - (T_x + V_{xA})} P$$
$$= \frac{P}{\omega^+ - T_x - PV_{xA}P - PV_{xA}Q \frac{1}{\omega^+ - T_x - QV_{xA}Q} QV_{xA}P}$$
$$= \frac{1}{\omega^+ - T_x - U_x}$$

[Excersize] When $AB = 1$, express PBP by PAP, PAQ, QAP, QAQ

3. Decomposition of elastic and non-elastic breakup

An identity of the Green's function

$$\mathrm{Im}G_x = (1 + G_x^\dagger U_x^\dagger)\mathrm{Im}\left[G_x^{(0)}\right](1 + U_x G_x) + G_x^\dagger W_x G_x$$

where

$$G_x^{(0)} = \frac{1}{\omega - T_x + \mathrm{i}\delta}$$

Use

$$\mathrm{Im}G_x^{(0)} = \sum_{\boldsymbol{k}} |\boldsymbol{k}\rangle \delta(\omega - \frac{k^2}{2m_x})\langle \boldsymbol{k}|$$

we get

$$(1 + G_x^\dagger U_x^\dagger)\mathrm{Im}\left[G_x^{(0)}\right](1 + U_x G_x) = \sum_{\boldsymbol{k}} |\chi_{\boldsymbol{k}}^{(-)}\rangle \delta\left(\omega - \frac{k^2}{2m_x}\right)\langle \chi_{\boldsymbol{k}}^{(-)}|$$

A. Kasano and M. Ichimura, PL 115B, 81(1982)

Now the first term gives

Elastic Breakup Cross Section

$$\frac{d^2\sigma^{\rm EBU}}{dE_b d\Omega_b} = K \sum_{\boldsymbol{k}} |\langle \chi_{\boldsymbol{k}}^{(-)} \chi_b^{(-)} \Phi_A | V^{\rm post} | \Phi_A \phi_a \chi_a^{(+)} \rangle|^2$$
$$\times\, \delta(\omega - \frac{k^2}{2m_x})$$

Consequently the seond term gives

Non-elastic Breakup Cross Section

$$\frac{d^2\sigma^{\rm NEB}}{dE_b d\Omega_b} = -\frac{K}{\pi} \langle \psi_x | W_x | \psi_x \rangle$$

where

$$\psi_x(\boldsymbol{r}) = G_x \langle \chi_b^{(-)} \Phi_A | V^{\rm post} | \Phi_A \phi_a \chi_a^{(+)} \rangle$$
$$= \int G_x(\boldsymbol{r}, \boldsymbol{r}'; \omega) S(\boldsymbol{r}') d^3\boldsymbol{r}'$$

This formalism is called IAV model

M. Ichimura, N. Austern and C.M. Vincent, Phys. Rev. **C32**, 431(1985)

4. Applications

Jin Lei and A.M. Moro, PR **C92**, 044616(2015)

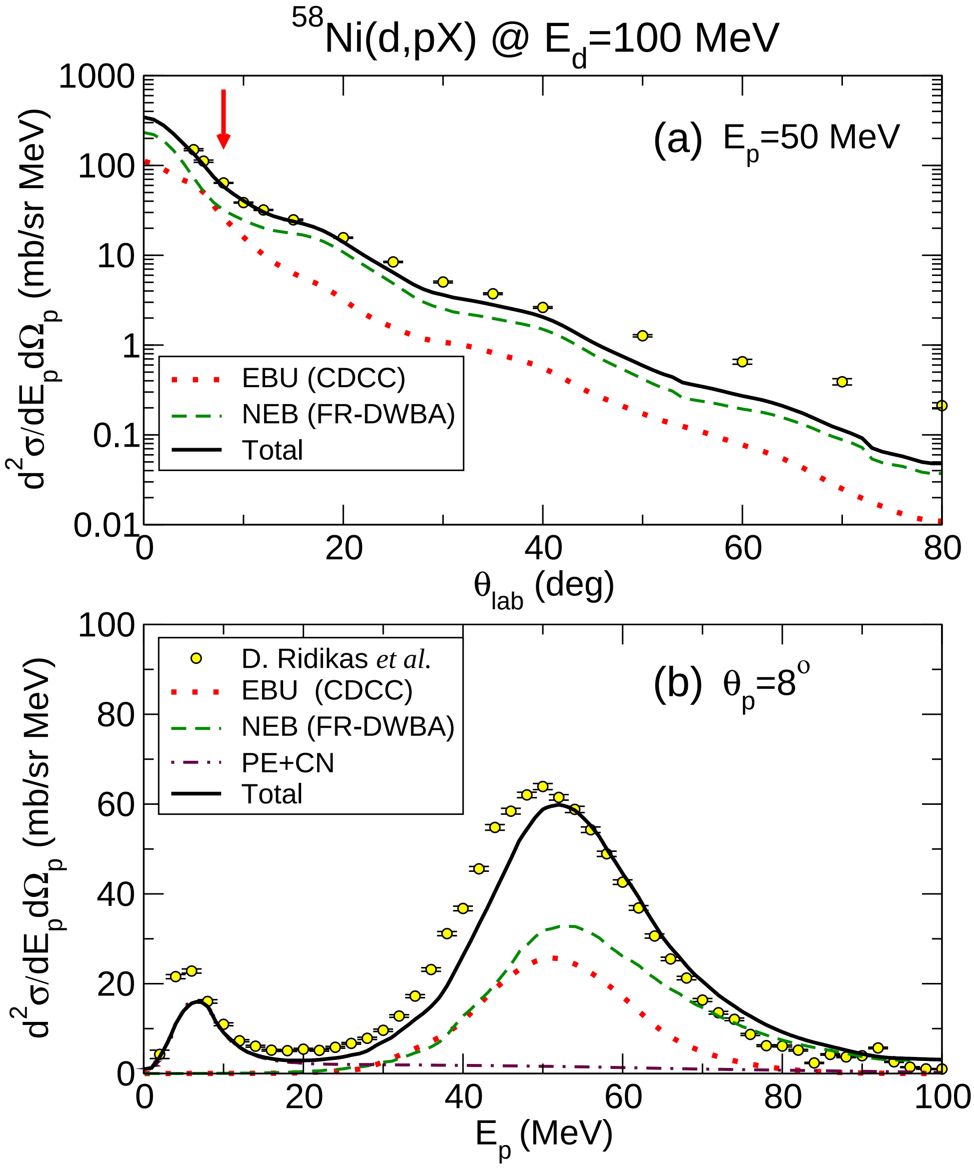


FIG. 4. (Color online) Double differential cross section of protons emitted in the ^{58}Ni(d,pX) reaction at $E_d = 100$ MeV in the laboratory frame. (a) Proton angular distribution for a fixed proton energy of $E_p = 50$ MeV. (b) Energy distribution for protons emitted at a laboratory angle of 8° (arrow in top figure). The meaning of the lines is the same as in Fig. 3, and are also indicated by the labels. Experimental data are from Ref. [44].

^{209}Bi (^{6}Li, αX)

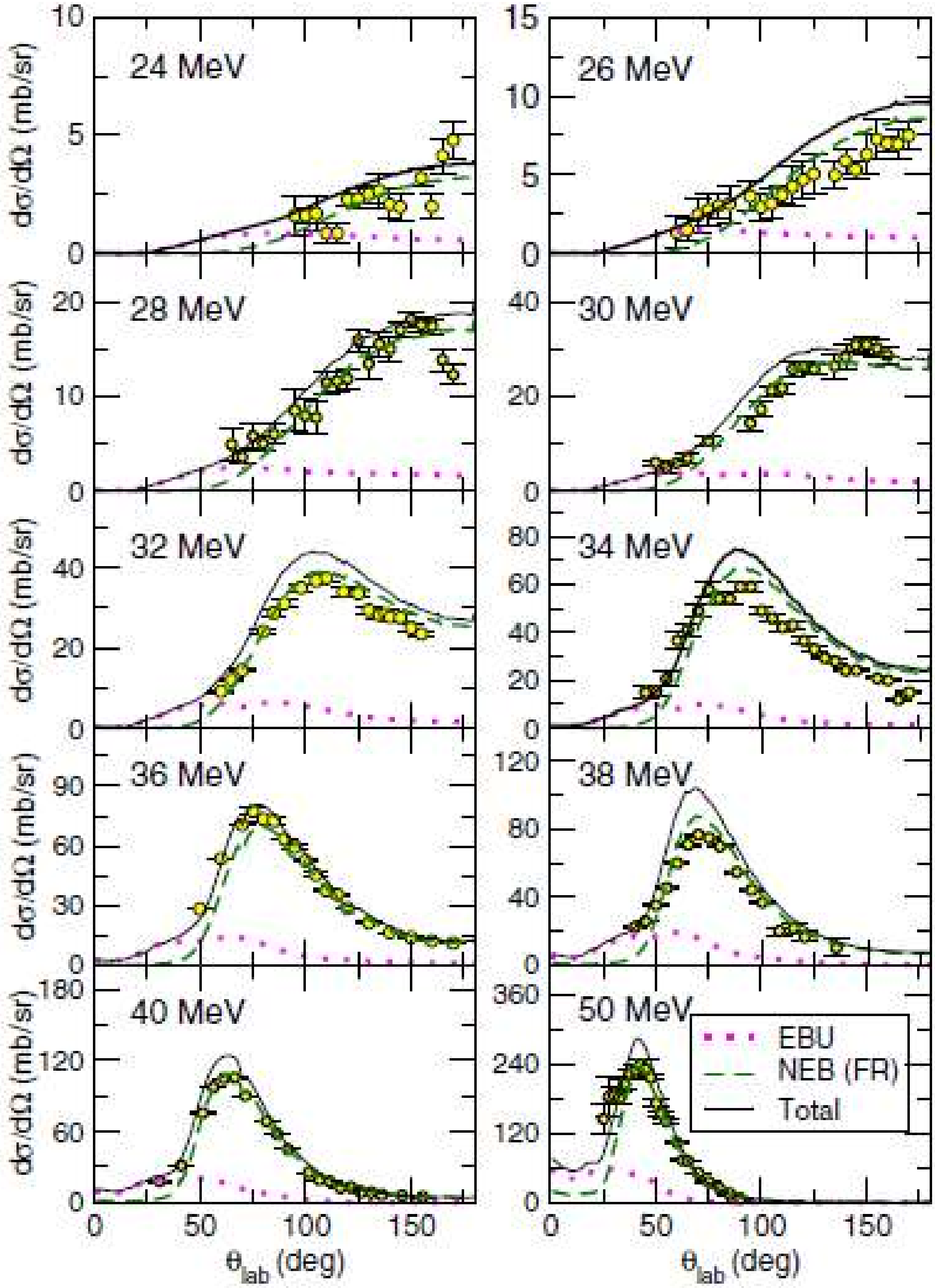


FIG. 6. (Color online) Angular distribution of α particles produced in the reaction ^{6}Li + ^{209}Bi at the incident energies indicated by the labels. The dotted, dashed, and solid lines correspond to the EBU (CDCC), NEB (FR-DWBA), and their sum, respectively. Experimental data are from Ref. [57].