Polarizations and spin observables

- What is polarization
- Induced Polarization P_y and analyzing power A_y
- Parity conservation
- Spin transfers D_{ij}
- Spin-parity dependences on D_{ij} at 0 degrees
- Spin measurements
- Experimental evidence for usefulness of D_{ij}
- Homework

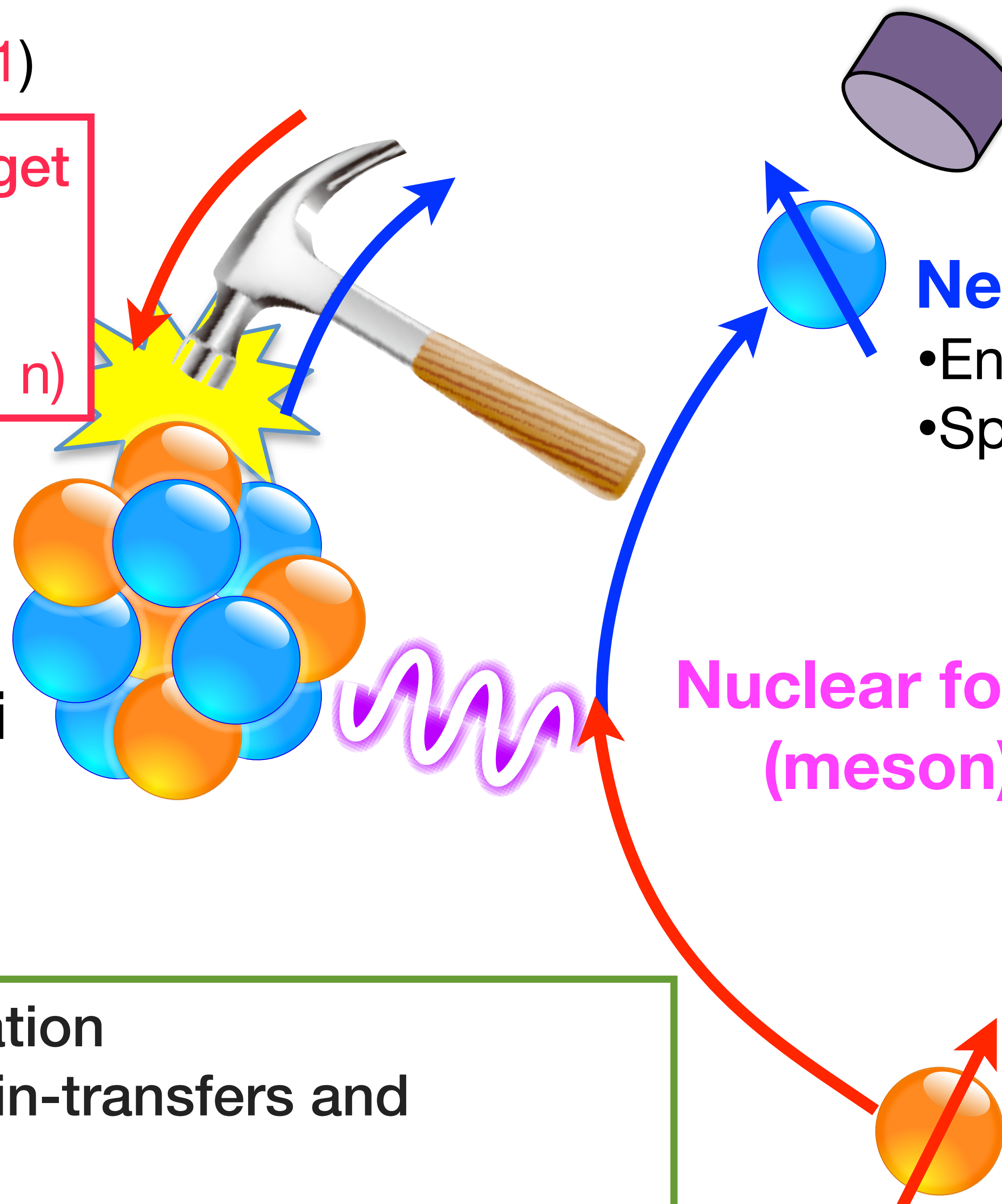
Fermi and Gamow-Teller excitations by (p,n)

Excite Fermi and Gamow-Teller transitions by (p,n)

- Fermi ($\Delta S=0$)
- Gamow-Teller ($\Delta S=1$)

Spin-transfer to the target
↕
Spin-transfer in (p,n)
(spin-transfer b/w p and n)

Ref.
Lecture by Ichimura-san
"Spin Observables"



✤ What is spin/polarization
✤ Relation between spin-transfers and target excitations
✤ What is the best energy for $\Delta S=1$ excitations

What is polarizations?

What is polarization

In quantum mechanics, polarization is often treated with a density operator

Simple explanations would be useful for understanding the essence of spin physics with nuclear reactions

Nucleon (proton and neutron) is a particle with spin 1/2

- Two magnetic substates (m=±1/2) along a quantization axis
  - *often called as up-spin (m=+1/2) and down-spin (m=-1/2) states.*
- An assembly of particles (incident beam, scattered particles, etc.)

→ can be described by the population $\tilde{\boldsymbol{p}}(\boldsymbol{m})$ for each m state.

Then "polarized" and "unpolarized" mean:

- polarized : $\tilde{\boldsymbol{p}}(1/2) \neq \tilde{\boldsymbol{p}}(-1/2)$
- unpolarized: $\tilde{\boldsymbol{p}}(1/2) = \tilde{\boldsymbol{p}}(-1/2)$

with the normalization of

$$\tilde{\boldsymbol{p}}(1/2) + \tilde{\boldsymbol{p}}(-1/2) = 1$$

What is polarization

Instead of using population parameters

The distribution of populations can be described in terms of "moments"

- These moments are called as "polarization"
- For nucleons, it is simple as explained in the followings
(deuteron with spin=1 is rather complicated)

The first moment (polarization p_y) with respect to y-axis is defined as

$$p_y \equiv \frac{1}{(1/2)} \sum_m m\tilde{p}_y(m) = \tilde{p}_y(1/2) - \tilde{p}_y(-1/2)$$

- p_y is bounded by $-1 \leqq p_y \leqq +1$

Both populations, $\tilde{p}_y(+1/2)$ and $\tilde{p}_y(-1/2)$, can be specified by p_z as follows:

$$\tilde{p}_y(+1/2) = \frac{1}{2}(1 + p_y)$$

$$\tilde{p}_y(-1/2) = \frac{1}{2}(1 - p_y)$$

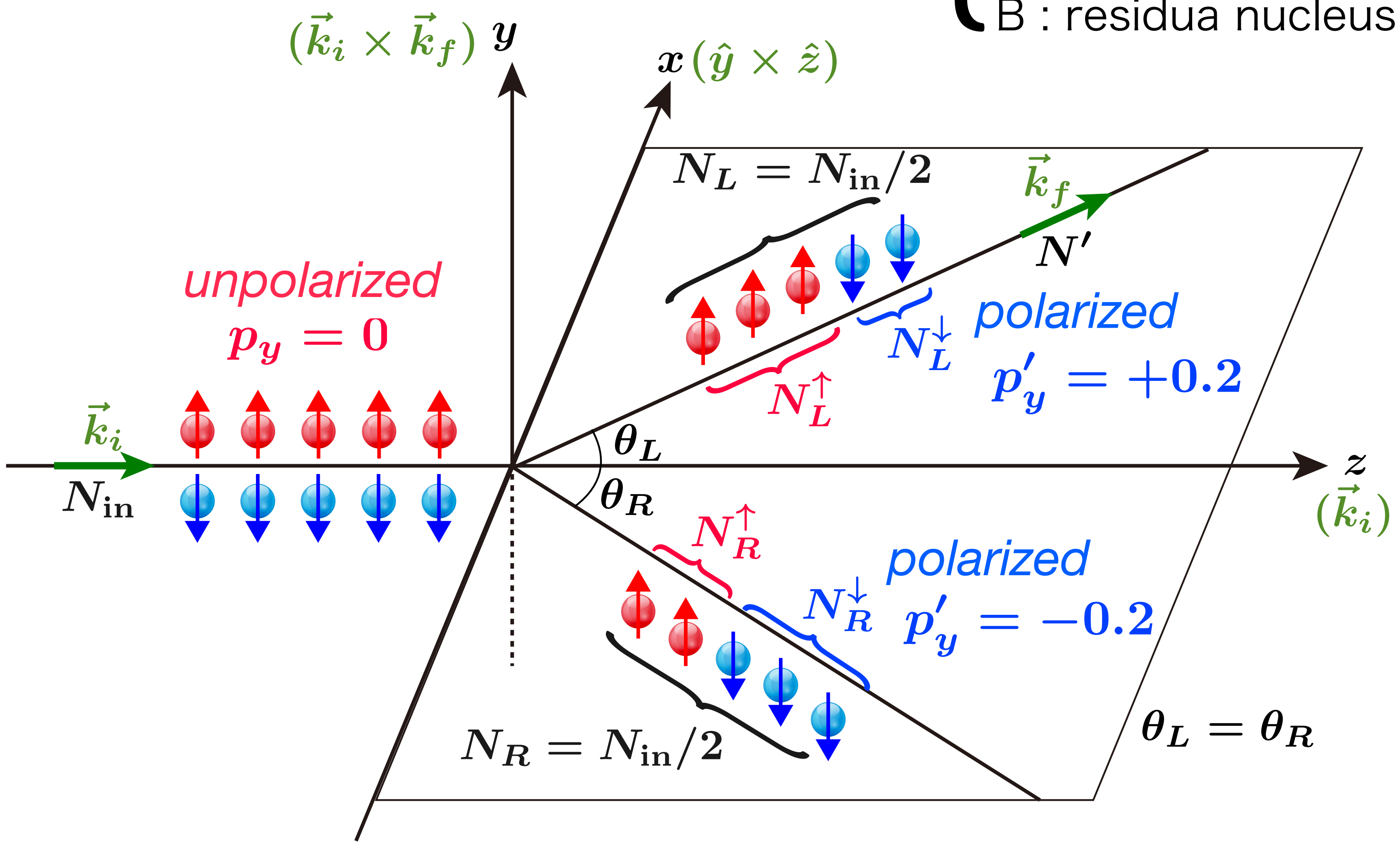
Induced polarization P_y and Analyzing power A_y

Induced polarization p_y

Consider nucleon-induced two-body scattering (reaction):

- N + A → N' + B   or   A(N,N')B

N : incident nucleon
A : target nucleus
N' : scattered nucleon
B : residua nucleus



$y\text{-axis} \parallel \vec{k}_i \times \vec{k}_f$

(normal to the reaction plane)

In general, polarization is produced even with an unpolarized beam.

- Spin-orbit interaction is mainly responsible for producing the polarization.

Exercise: Because of the *parity conservation*, only the p_y component takes a finite value. Why do the other p_x and p_z components become 0?

Parity inversion and conservation

Parity inversion (transformation) : P

In three dimensions, simultaneous flip in the sign of all three spatial coordinates:

$$P: \begin{pmatrix} x \\ y \\ x \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

P can be decomposed to the mirror reflection M and the π(180°) rotation R.

- For example, the reflection by the mirror on the x-y plane gives:

$$M_{xy}: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ -z \end{pmatrix}$$

- Then the rotation around the z-axis by θ=π(180°) gives: *(just changing your view point)*

$$R_z: \begin{pmatrix} x \\ y \\ -z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

Thus the parity inversion is physically same as the Mirror reflection.

Parity conservation of nuclear forces (strong interaction) means:

The probability of a process by nuclear forces
= The probability of the mirror-reflected process (=parity-inverted process)

Constraints on polarizations by parity conservation

The parity conservation gives some constraints on polarizations:

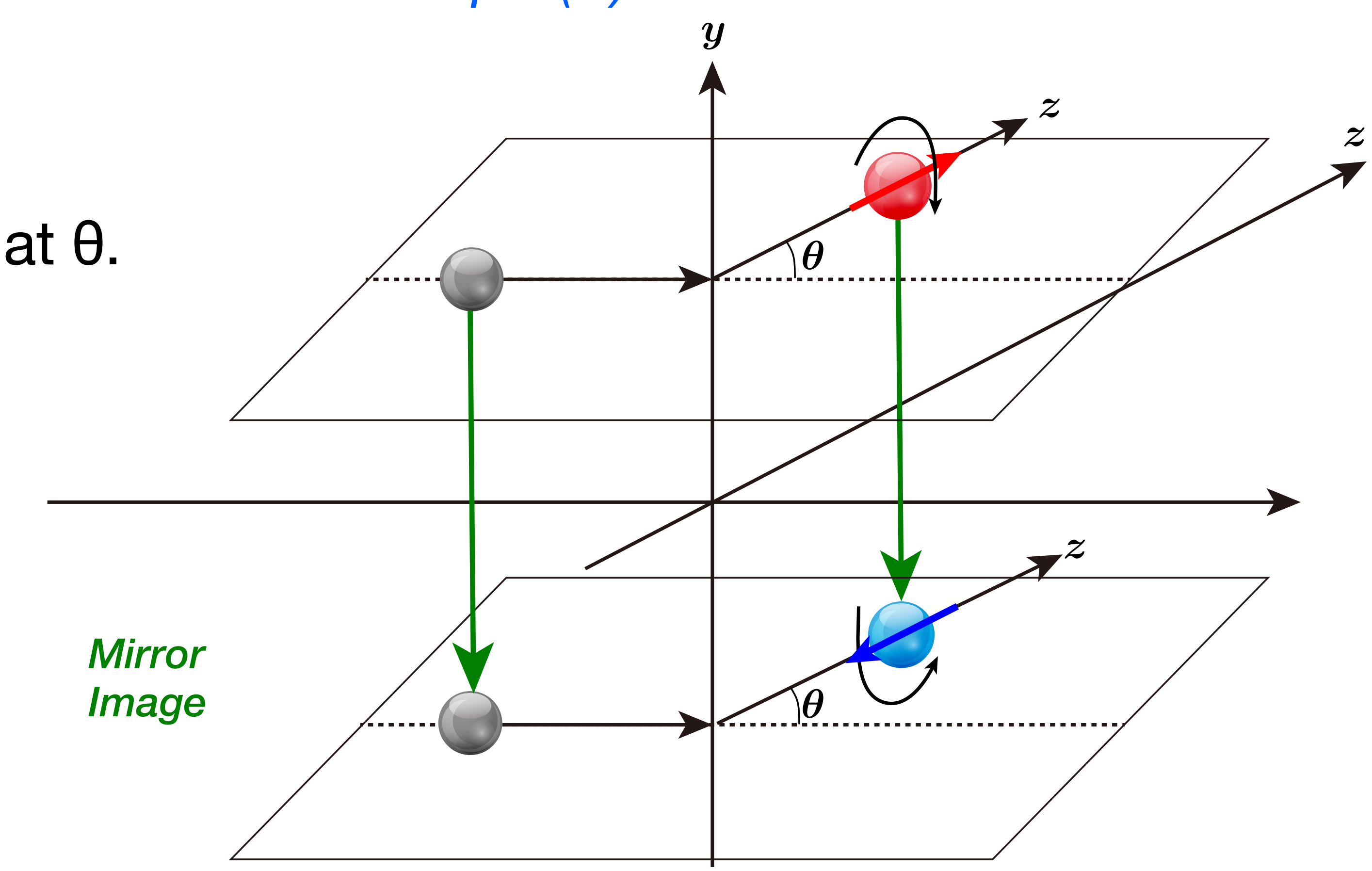
- For an illustrative purpose, it is convenient to describe the spin (polarization) as a spinning top (rotation).

up spin(↑)

down spin(↓)

P_z=0 can be shown as follows:

- Consider the following process:
  - An unpolarized nucleon is scattered at θ.
  - The scattered nucleon is polarized to the +z axis (helicity=+).
- Mirror reflection on the x-z plane (scattering plane).
- In mirror image:
  - The nucleon is also scattered at θ.
  - The scattered nucleon is polarized to the -z axis (helicity=-).



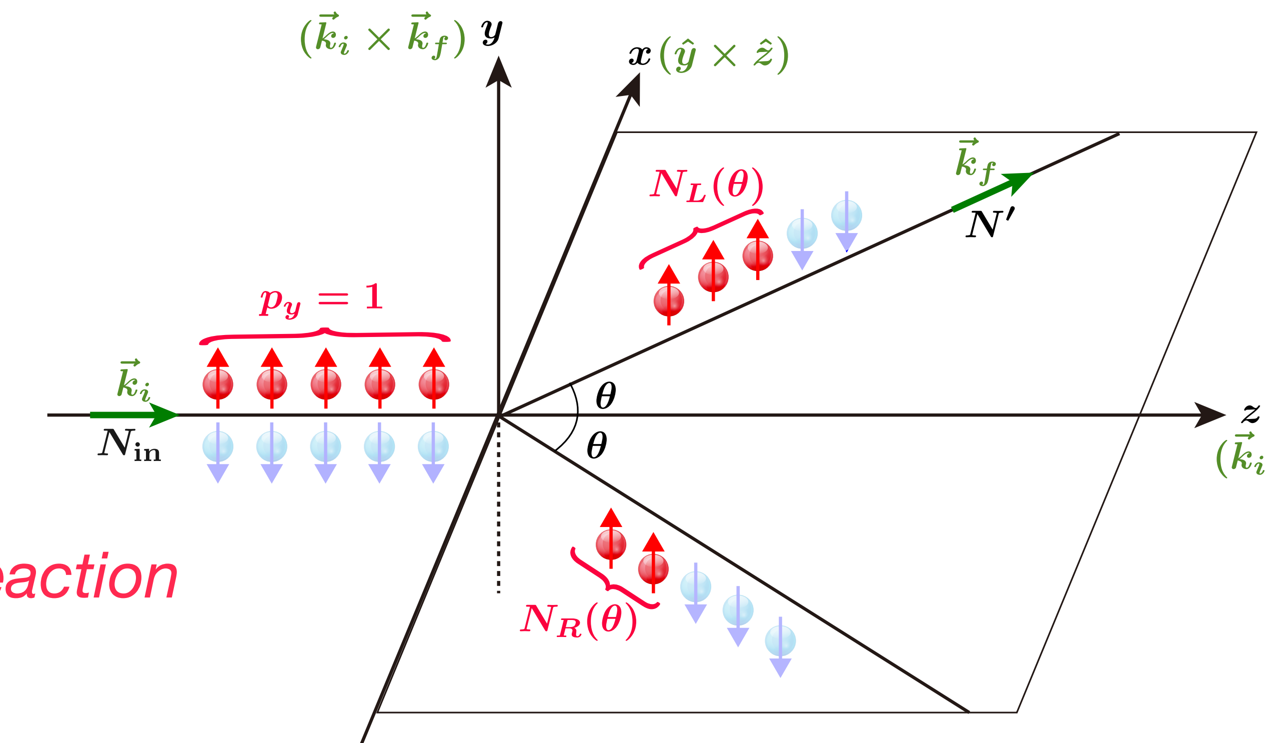
Both processes cause with same probability.
→ Nucleons are NOT polarized to z-axis (P_z=0)

Analyzing power A_y

When incoming beam N is polarized in A(N,N')B

- The numbers of scattered particles to left and right, N_L and N_R, are different in general
  - due to the spin-dependent interaction such as the spin-orbit interaction
- The left-right asymmetry A defined by

$$A(\theta) = \frac{N_L(\theta) - N_R(\theta)}{N_L(\theta) + N_R(\theta)}$$



  is proportional to both:
  - beam polarization : p_y
  - *analyzing power : A_y → specific to the reaction*
- Therefore, $A_y(\theta)$ is given by

$$p_y \cdot A_y(\theta) = \frac{N_L(\theta) - N_R(\theta)}{N_L(\theta) + N_R(\theta)}$$

Exercise: Find A_y in the case of N_L=1200 and N_R=800 for p_y = 0.6.

$$A_y = \frac{1}{p_y}\frac{N_L - N_R}{N_L + N_R} = \frac{1}{0.6}\frac{1200 - 800}{1200 + 800} = 0.33$$

Spin-dependence of yields

Numbers of scattered particles to left and right are expressed as

$$N_L(\theta) = \bar{N}_L(\theta)(1 + p_y A_y(\theta)) = I \cdot n \cdot \varepsilon_L \cdot \frac{d\sigma(\theta)}{d\Omega} \cdot \Delta\Omega_L (1 + p_y A_y(\theta))$$

$$N_R(\theta) = \bar{N}_R(\theta)(1 - p_y A_y(\theta)) = \underbrace{I \cdot n \cdot \varepsilon_R \cdot \frac{d\sigma(\theta)}{d\Omega} \cdot \Delta\Omega_R}_{\bar{N}_{L,R}} (1 - p_y A_y(\theta))$$

In general, $\bar{N}_L(\theta)$ and $\bar{N}_R(\theta)$ depend on

- numbers of incident and target particles: I and n
- cross section dσ/dΩ (for unpolarized beam)
- Solid angles and efficiencies of left and right detectors : $\Delta\Omega_{L/R}$ and $\varepsilon_{L/R}$

If $\bar{N}_L(\theta) = \bar{N}_R(\theta)$ for an ideal case, $A_y(\theta)$ can be easily deduced as

$$\bar{N}_L(\theta) = \frac{N_L}{(1 + p_y A_y)} = \bar{N}_R(\theta) = \frac{N_R}{(1 - p_y A_y)}$$

$$\longrightarrow \quad A_y = \frac{1}{p_y} \frac{N_L - N_R}{N_L + N_R}$$

Exercise : In practical, $\bar{N}_L(\theta) \neq \bar{N}_R(\theta)$.
In this case, how can we measure A_y precisely with small systematic uncertainty?

Absolute magnitude of polarization

Experimentally, an asymmetry $A(\theta) = p_y A_y(\theta)$ can be measured.

- If p_y is known, $A_y(\theta)$ can be obtained.
- If $A_y(\theta)$ is known, p_y can be deduced.

} How to obtain p_y or $A_y(\theta)$ firstly?

→ Double elastic-scattering method can be used.

Exercise 1 :

Explain how to obtain p_y or A_y firstly by the double scattering method referring Appendix B of this lecture.

Exercise 2 :

In the double scattering method, the P_y=A_y equality for elastic scattering of spin 1/2 particles from a spin-zero target is used. Proof this equality referring Appendix C of this lecture.

Spin transfer D_{ij}

Ref. Lecture by Ichimura-san

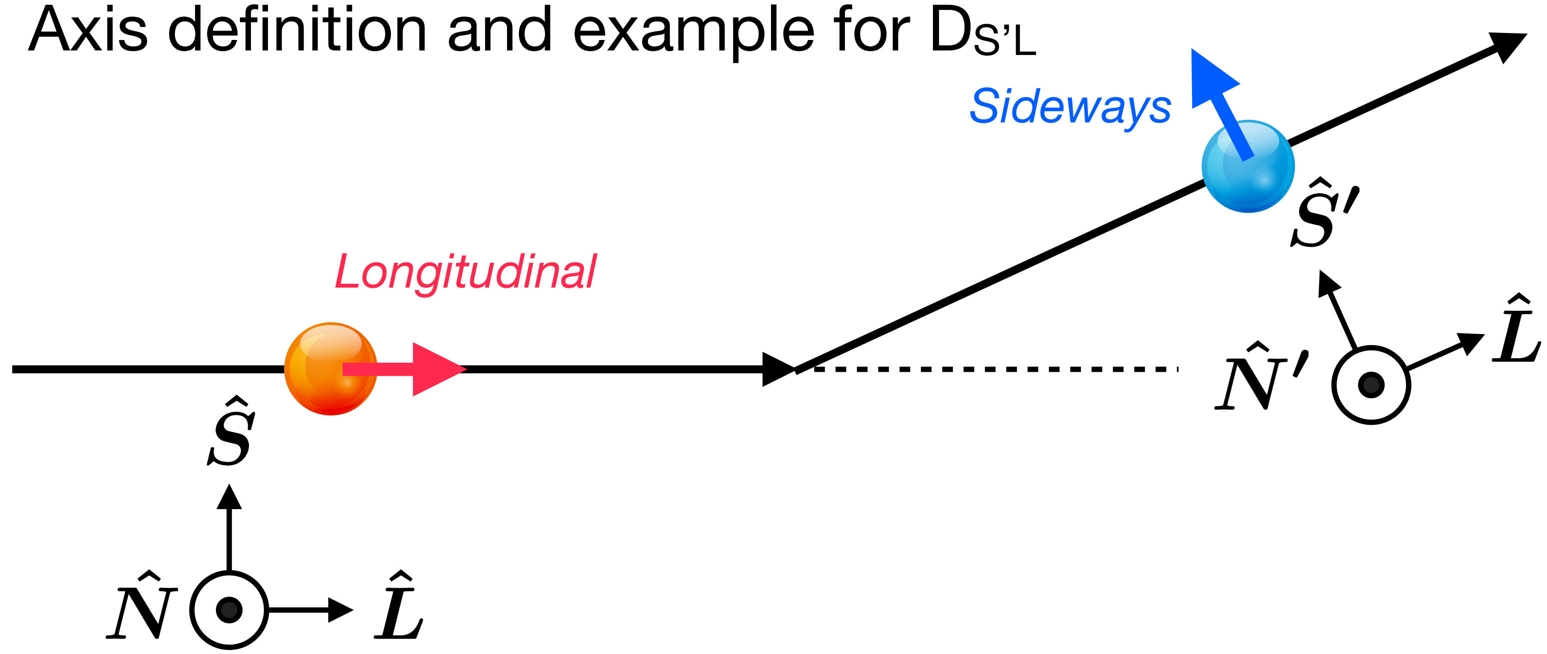
Polarization transfer D_{ij} in PWIA

M.Ichimura, H.Sakai, T.W., Prog. Part. Nucl. Phys. 56, 446 (2006).

Polarization transfer D_{ij} for $X(\vec{a},\vec{b})Y$ (also known as D_{ji} and K_{ij})

→ relate the i-axis component of polarization of outgoing nucleon
to j-axis component of incident nucleon

- Axis definition and example for $D_{S'L}$



Longitudinal $\hat{L} = \hat{k}$

Normal $\hat{N} = \dfrac{\hat{k}_i \times \hat{k}_f}{|\hat{k}_i \times \hat{k}_f|}$

Sideways $\hat{S} = \hat{N} \times \hat{L}$

In general, there are nine D_{ij}'s (3×3 matrix elements).

→The parity conservation allows finite values (≠0) only for five D_{ij}'s

$$\begin{pmatrix} D_{S'S} & D_{S'N} & D_{S'L} \\ D_{N'S} & D_{N'N} & D_{N'L} \\ D_{L'S} & D_{L'N} & D_{L'L} \end{pmatrix} \xrightarrow[\text{conservation}]{\text{parity}} \begin{pmatrix} D_{S'S} & 0 & D_{S'L} \\ 0 & D_{N'N} & 0 \\ D_{L'S} & 0 & D_{L'L} \end{pmatrix}$$

Exercise: Under the parity and rotation invariances, the polarization transfer $D_{S'N} = D_{N'S} = D_{N'L} = D_{L'N} = 0$. Proof this equality.

T-matrix and NN amplitudes

D_{ij} is defined using T-matrix, T, and Pauli spin matrix, σ, as

$$D_{ij} = \frac{\mathrm{Tr}[T\sigma_j T^\dagger \sigma_i]}{\mathrm{Tr}[TT^\dagger]} \qquad \left(I \equiv \frac{d\sigma}{d\Omega} = \frac{1}{4}\mathrm{Tr}[TT^\dagger] \right)$$

T-matrix from the ground state $|0\rangle$ to the excited state $|m\rangle$ for N-A scattering in PWIA is given by

$$T(q) = \langle m|M(q)e^{-i\vec{q}\cdot\vec{r}}|0\rangle$$

where

- $\vec{q}\,(q)$: momentum transfer
- $M(q)$: nucleon-nucleon (NN) scattering amplitude

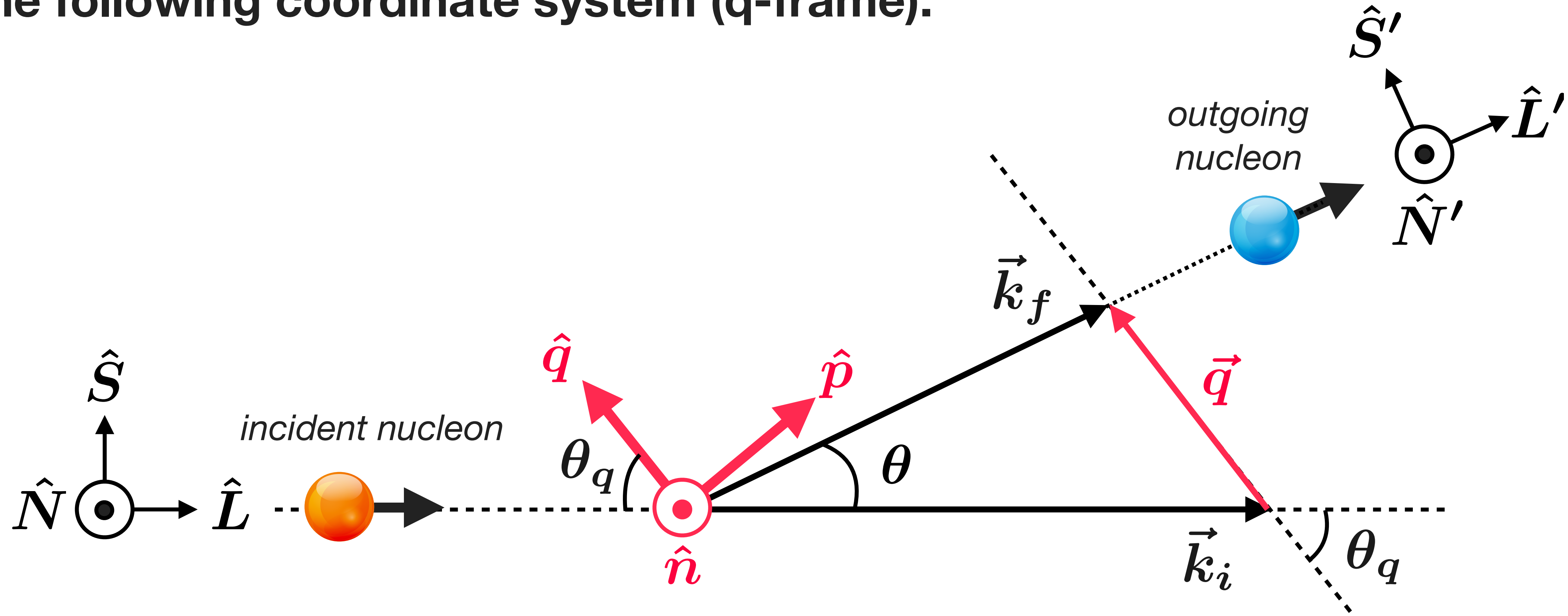
M.Ichimura, H.Sakai, T.W., Prog. Part. Nucl. Phys. 56, 446 (2006).

KMT notation and q-frame

In so-called KMT (Kerman-McManaus-Thaler) notation, M(q) is written as:

$$M(q) = A + B\sigma_{1\hat{n}}\sigma_{2\hat{n}} + C(\sigma_{1\hat{n}} + \sigma_{2\hat{n}}) + E\sigma_{1\hat{q}}\sigma_{2\hat{q}} + F\sigma_{1\hat{p}}\sigma_{2\hat{p}}$$

with the following coordinate system (q-frame).



$$\vec{q} = \vec{k}_f - \vec{k}_i$$

$$\vec{n} = \vec{k}_i \times \vec{k}_f$$

$$\hat{q} = \frac{\vec{q}}{|\vec{q}|}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$\hat{p} = \hat{q} \times \hat{n}$$

A.K.Kerman, H.McManus, and R.M.Thaler, Ann. Phys. 8, 551 (1959).

Spherical tensor expression of M(q)

A.K.Kerman, H.McManus, and R.M.Thaler, Ann. Phys. 8, 551 (1959).

It is convenient to take the q-direction as a quantization axis.

q is the direction of the "impact" to the target.

The NN amplitude is written with spherical tensor operators as:

$$M(q) = \underbrace{M_0}_{\text{spin-scalar}} + \underbrace{\sum_\mu (-1)^\mu \sigma_\mu^1 M_\mu^1}_{\text{spin-vector}}$$

- σ_μ^1 are tensor operators of rank-1 of the target nucleon defined by:

$$\sigma_0^1 = \sigma_{1\hat{q}} \qquad \sigma_1^1 = -\frac{1}{\sqrt{2}}(\sigma_{1\hat{n}} + i\sigma_{1\hat{p}}) \qquad \sigma_{-1}^1 = \frac{1}{\sqrt{2}}(\sigma_{1\hat{n}} - i\sigma_{1\hat{p}})$$

- M_0 and M_μ^1 are operators of the incident nucleon defined by:

$$M_0 = A + C\sigma_{2\hat{n}} \qquad M_1^1 = -\frac{1}{\sqrt{2}}(C + B\sigma_{2\hat{n}} - iF\sigma_{2\hat{p}})$$

$$M_0^1 = E\sigma_{2\hat{q}} \qquad M_{-1}^1 = \frac{1}{\sqrt{2}}(C + B\sigma_{2\hat{n}} + iF\sigma_{2\hat{p}})$$

Isospin

A nucleon (p or n) has the isospin (τ) degree of freedom.

→ Each amplitude in M(q) has isoscalar (IS) and isovector (IV) terms.

- For example, explicit form of A is

$$A = A_{\rm IS} + A_{\rm IV}\tau_1 \cdot \tau_2$$

→ *Here, we focus on the IV case ((p,n), etc), and thus express A_{IV} as A for simplicity.*

N-A T-matrix

N-A T-matrix for an isovector (IV) $0^+ \rightarrow J^\pi$ excitation is expressed as:

$$T(q) = \underbrace{\langle J | e^{-i\vec{q}\cdot\vec{r}} \tau^1_{\pm} | 0 \rangle \tau^1_{\mp} M_0}_{\text{spin-scalar}} + \underbrace{\sum_{\mu=-1}^{+1} \langle J | e^{-i\vec{q}\cdot\vec{r}} \sigma^1_\mu \tau^1_{\pm} | 0 \rangle \tau^1_{\mp} M^1_\mu}_{\text{spin-vector}}$$

spin-scalar $\Delta S = 0$

spin-vector $\Delta S = 1$

Target operators, $e^{i\vec{q}\cdot\vec{r}}$ **and** $e^{i\vec{q}\cdot\vec{r}}\sigma^1_\mu$ **, can be expressed in standard tensor forms:**

$$e^{-i\vec{q}\cdot\vec{r}} = \sum_\ell \rho_\ell Y^0_\ell$$ (plane-wave expansion/Rayleigh equation)

$$e^{-i\vec{q}\cdot\vec{r}}\sigma^1_\mu = \sum_{\ell J} \rho_\ell (1\ell\mu 0 | J\mu) T^\mu_J(\ell s)$$

$$\rho_\ell = \sqrt{2\ell+1}\sqrt{4\pi}(-i)^\ell j_\ell(qr)$$

$$T^\mu_J(\ell s) = \sum_{\mu'\mu''} (\ell s \mu' \mu'' | J\mu) Y^{\mu'}_\ell \sigma^1_{\mu''}$$

Tensor operator of rank J composed of operators $Y^{\mu'}_\ell$ *and* $\sigma^1_{\mu''}$

A.K.Kerman, H.McManus, and R.M.Thaler, Ann. Phys. 8, 551 (1959).

N-A T-matrix

A.K.Kerman, H.McManus, and R.M.Thaler, Ann. Phys. 8, 551 (1959).

Using the standard formulas for reduced matrix elements, for example we get the isovector spin-vector T-matrix as

$$T(q) = (-1)^{J-\mu} \frac{1}{\sqrt{2J+1}} (1\ell\mu 0|J\mu) Q_J^\ell M_\mu^1$$

with the reduced nuclear matrix element Q_J^l:

$$Q_J^\ell = \langle J || \rho_\ell T_J(\ell s) \tau^1 || 0 \rangle$$

Now we can calculate the specific observable for a $0^+ \rightarrow J^\pi$ transition with Q_J^l.

Example: ID_{nn} for $0^+ \rightarrow 2^-$ (J=2, L=1)

Note: $\sigma_i \sigma_j = i\varepsilon_{ijk}\sigma_k + \delta_{ij}$ $\{i,j,k\} = \{n,p,q\}$

$$ID_{nn} = \frac{1}{4}\mathrm{Tr}[T\sigma_n T^\dagger \sigma_n]$$

$$= (C^2 + B^2 - F^2)\frac{2\pi(J+1)}{2J+1}(Q_J^{\ell=J-1})^2 - E^2 \frac{2\pi \cdot 2J}{2J+1}(Q_J^{\ell=J-1})^2$$

$$= \left[\frac{3}{5}(C^2 + B^2 - F^2) - \frac{4}{5}E^2\right] 2\pi (Q_{J=2}^{\ell=1})^2$$

Polarization observables and transition densities

H.Sakai, "Lecture note at RIKEN Winter School" (1993).

Polarization observables in PWIA can be expressed with transition densities.

natural-parity, ΔS=0

$$X_0 = \sqrt{4\pi} Q_J$$

natural-parity, ΔS=1

$$X'_T = \sqrt{2\pi} Q_J^{\ell=J}$$

unnatural-parity, ΔS=1 (spin-longitudinal)

$$X_L = \sqrt{\frac{2\pi \cdot 2J}{2J+1}} Q_J^{\ell=J-1} - \sqrt{\frac{2\pi(2J+1)}{2J+1}} Q_J^{\ell=J+1}$$

unnatural-parity, ΔS=1 (spin-transverse)

$$X_T = \sqrt{\frac{2\pi(J+1)}{2J+1}} Q_J^{\ell=J-1} + \sqrt{\frac{2\pi \cdot J}{2J+1}} Q_J^{\ell=J+1}$$

observable	natural parity	unnatural parity
$I = \frac{d\sigma}{d\Omega}$	$(C^2+B^2+F^2)X_T'^2 + (A^2+C^2)X_0^2$	$(C^2+B^2+F^2)X_T^2 + E^2X_L^2$
ID_{qq}	$(C^2-B^2-F^2)X_T'^2 + (A^2-C^2)X_0^2$	$(C^2-B^2-F^2)X_T^2 + E^2X_L^2$
ID_{nn}	$(C^2+B^2-F^2)X_T'^2 + (A^2+C^2)X_0^2$	$(C^2+B^2-F^2)X_T^2 - E^2X_L^2$
ID_{pp}	$(C^2-B^2+F^2)X_T'^2 + (A^2-C^2)X_0^2$	$(C^2-B^2+F^2)X_T^2 - E^2X_L^2$
$ID_{qp} = -ID_{pq}$	$2\mathrm{Im}(BC^*)X_T'^2 - 2\mathrm{Im}(AC^*)X_0^2$	$2\mathrm{Im}(BC^*)X_T^2$
$ID_{n0} = ID_{0n}$ $(A_y = P)$	$2\mathrm{Re}(BC^*)X_T'^2 + 2\mathrm{Re}(AC^*)X_0^2$	$2\mathrm{Re}(BC^*)X_T^2$

Polarization transfer D_{ii} at 0 degrees

Note:

At 0°, the spin-longitudinal transition, X_L, is caused by the F-term in KMT.

From spatial symmetry,

- B = E (*two transverse directions are identical*) and C=0

Polarization transfers, D_{NN} and D_{LL}, in PWIA in laboratory frame.

Relations between polarization observables in q-frame and lab.-frame are given in Appendix D.

Polarization observables	natural parity (J=L)		unnatural parity (J=L±1)
	ΔS=0	ΔS=1	ΔS=1
D_{NN} (=D_{nn})	$+1$	0	$\frac{-F^2X_L^2}{2B^2X_T^2+F^2X_L^2}$
D_{LL} (=D_{qq})	$+1$	-1	$\frac{-2B^2X_T^2+F^2X_L^2}{2B^2X_T^2+F^2X_L^2}$
$2D_{NN}+D_{LL}$ (=$D_{qq}+D_{nn}+D_{pp}$)	$+3$	-1	-1

- In general, a natural parity transition is the mixed transitions of ΔS=0 and 1.
  - $D_{NN}(0°)$ = 0〜1 and $D_{LL}(0°)$ = -1〜1
- $\Delta J^{\pi}=0^{+}$(Fermi, IAS) is a special case with ΔS=0 → $D_{NN}(0°)=D_{LL}(0°)=+1$

H.Sakai, "Lecture note at RIKEN Winter School" (1993).

$D_{ii}(0°)$ in PWIA for several ΔJ^{π}

H.Sakai, "Lecture note at RIKEN Winter School" (1993).

$$\frac{X_T^2}{X_L^2} = \frac{J+1}{2J} \quad (J = J_> = L+1) \qquad \frac{X_T^2}{X_L^2} = \frac{J}{2(J+1)} \quad (J = J_< = L-1)$$

Transition	ΔJ^{π}	ΔS	X_T^2/X_L^2	$D_{NN}(0°)$	$D_{LL}(0°)$
Fermi	0^+	0	-	$+1$	$+1$
Gamow-Teller	1^+	1	1	$\frac{-F^2}{2B^2+F^2}$	$\frac{-2B^2+F^2}{2B^2+F^2}$
Dipole	1^-	0	-	$+1$	$+1$
Spin-Dipole	0^-	1	0	-1	$+1$
	1^-	1	-	0	-1
	2^-	1	3/4	$\frac{-2F^2}{3B^2+2F^2}$	$\frac{-3B^2+2F^2}{3B^2+2F^2}$

$D_{ii}(0°)$ in PWIA for several ΔJ^{π}

H.Sakai, "Lecture note at RIKEN Winter School" (1993).

If the central NN interactions are dominant and the tensor interactions are negligible.

- B=F (central only)
- which is appropriate at $T_p < 200$ MeV (at $T_p > 200$ MeV, tensor int. are significant)

Transition	ΔJ^{π}	ΔS	X_T^2/X_L^2	$D_{NN}(0°)$	$D_{LL}(0°)$
Fermi	0^+	0	-	$+1$	$+1$
Gamow-Teller	1^+	1	1	$-\frac{1}{3}$	$-\frac{1}{3}$
GD	1^-	0	-	$+1$	$+1$
Spin-Dipole	0^-	1	0	-1	$+1$
	1^-	1	-	0	-1
	2^-	1	3/4	$-\frac{2}{5}$	$-\frac{1}{5}$

Spin measurements

Polarimeter (FOM)

Polarization (*p*) analysis in a polarimeter

- Measure the left-right (up-down) asymmetry: A
- $A = p \cdot A_{y;eff}$ $A_{y;eff}$ = effective analyzing power of a polarimeter (A_y of polarimetry)

In general, polarization is measured as follows:

- A polarized beam (particles) bombards on an analyzer target.
- Incident particles are scattered to left or right, and detected with efficiency ε
  - Here ε is defined as

$$\varepsilon = \frac{\textit{Number of detected particles}}{\textit{Number of incident particles}}$$

- The analyzing reaction produces the left-right asymmetry due to its effective analyzing power $A_{y;eff}$

Dependence of the polarimeter performance (ability for determining *p*) on ε and $A_{y;eff}$?

Polarimeter performance

For $2n_0$ incident particles, detected numbers of left and right detectors of a polarimeter are given by

$$N_L = \varepsilon n_0 (1 + p_y A_{y;\text{eff}})$$
$$N_R = \varepsilon n_0 (1 - p_y A_{y;\text{eff}})$$
$$\Bigg\} \quad A = p_y A_{y;\text{eff}} = \frac{N_L - N_R}{N_L + N_R}$$

Statistical uncertainty of A is given by

$$(\Delta A)^2 = \left(\frac{\partial A}{\partial N_L}\right)^2 (\Delta N_L)^2 + \left(\frac{\partial A}{\partial N_R}\right)^2 (\Delta N_R)^2$$
$$= \frac{4 N_L N_R}{(N_L + N_R)^3}$$

Statistical uncertainty of p is given by

$$\Delta p_y = \frac{\Delta A}{A_{y;\text{eff}}} \simeq \frac{1}{A_{y;\text{eff}}} \frac{1}{\sqrt{\varepsilon}} \cdot \frac{1}{\sqrt{2n_0}}$$

Statistical term (incident particle number)

intrinsic to polarimeter → *Determine the performance of a polarimeter*

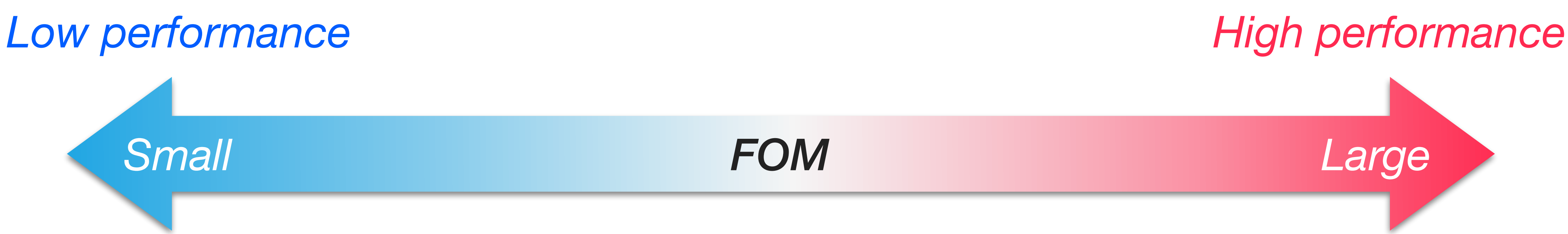
Figure of merit of a polarimeter

Since Δp_y is given by

$$\Delta p_y = \frac{1}{\sqrt{2n_0}} \cdot \frac{1}{\sqrt{\varepsilon} \cdot A_{y;\mathrm{eff}}}$$

the “Figure Of Merit” (FOM) of a polarimeter can be defined as

$$\mathrm{FOM} \equiv \varepsilon \cdot A_{y;\mathrm{eff}}^2 \quad \leftrightarrow \quad \Delta p = \frac{1}{\sqrt{2n_0}} \frac{1}{\sqrt{\mathrm{FOM}}}$$



Typical performances of polarimeters

Typical/designed values:

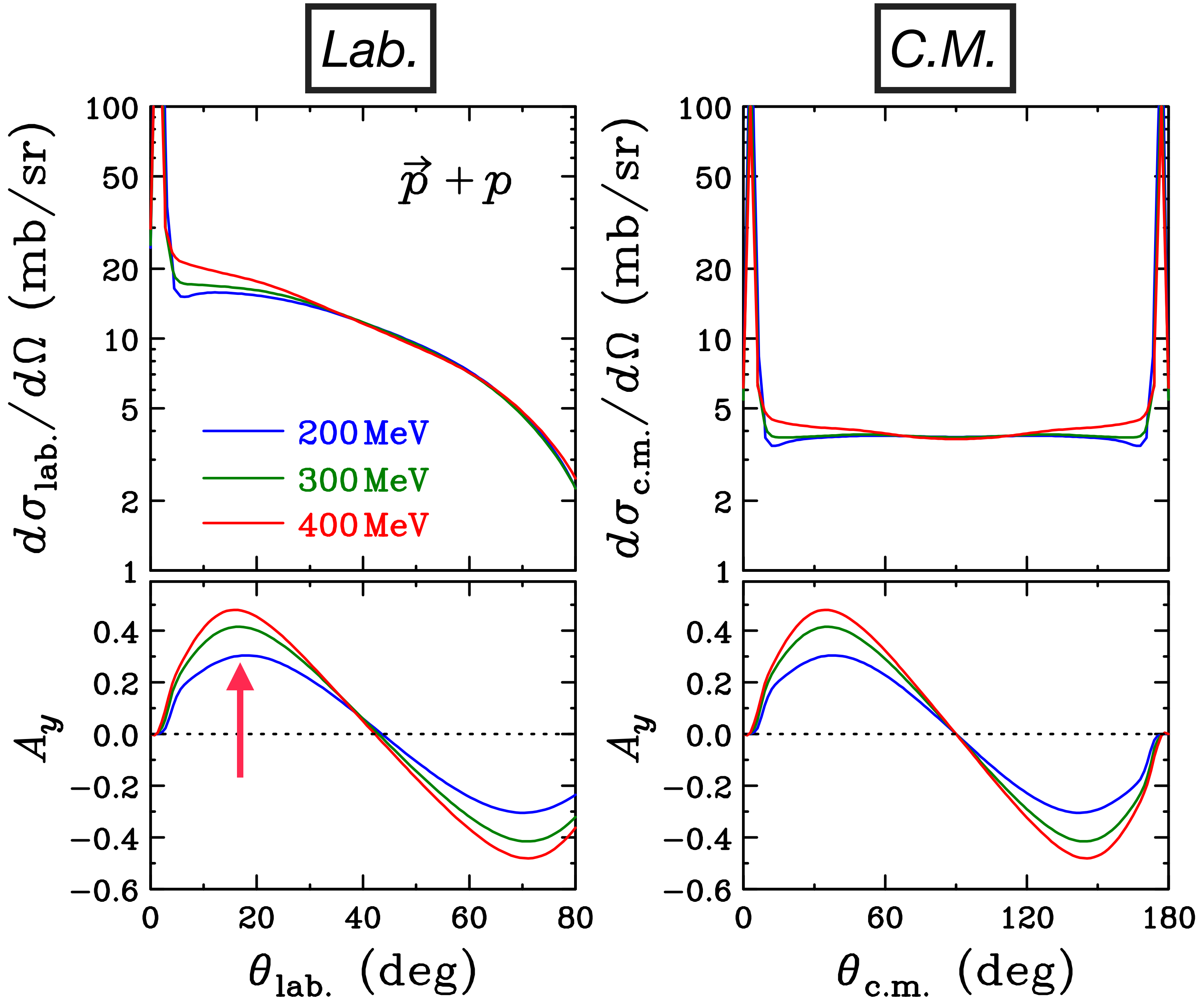
- $\varepsilon = 10^{-1}$ (for protons) ∼ 10^{-4} (for neutrons)
- $A_{y;\mathrm{eff}} = 0.1$ (intermediate energy for neutrons) ∼ 0.9 (low energy)

Proton “beam” polarimeter

p+p scattering is generally used because

- moderate A_y(∼0.4) and dσ/dΩ
- easy to measure

dσ/dΩ and A_y for p+p at T_p=200-400 MeV



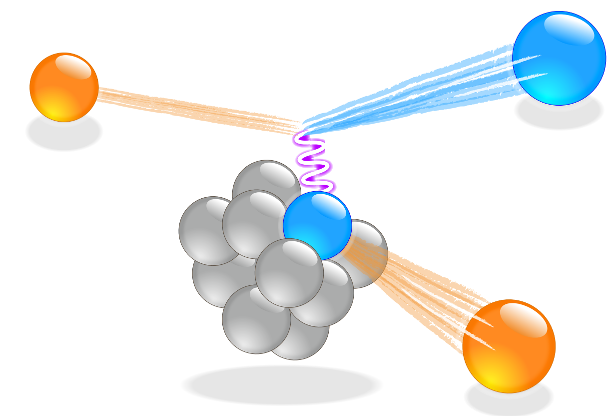
A_y takes a maximum at θ_{lab}=17° for 200-400 MeV

http://gwdac.phys.gwu.edu

B.G. in the polarization analysis

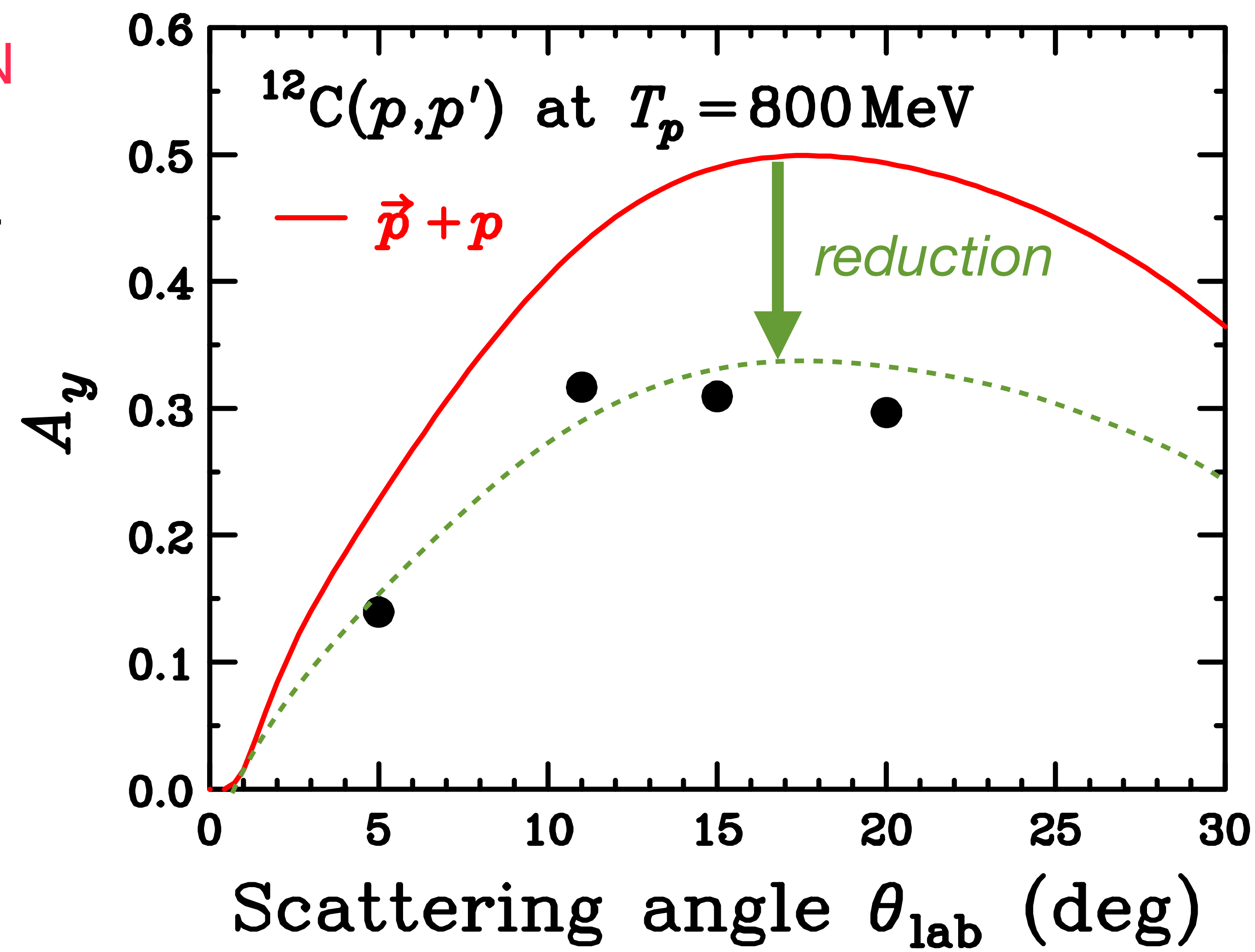
In general, a polyethylene sheet (CH_2) is used as a hydrogen target

- B.G. from the C-target
- At θ=17°, quasi-elastic scattering (QES) is dominant (p+p in C)



A_y for (p,p')-QES on ^{12}C at LAMPF/TRIUMF/RCNP

- Systematically smaller than A_y for p+N
- QES on ^{12}C should be suppressed
  to maximize the FOM of a polarimeter



C.J.Horowitz, M.J.Iqbal, Phys. Rev. C 33 (1986) 2059.

J.A.McGill et.al., Phys. Lett. B 134 (1983) 157.

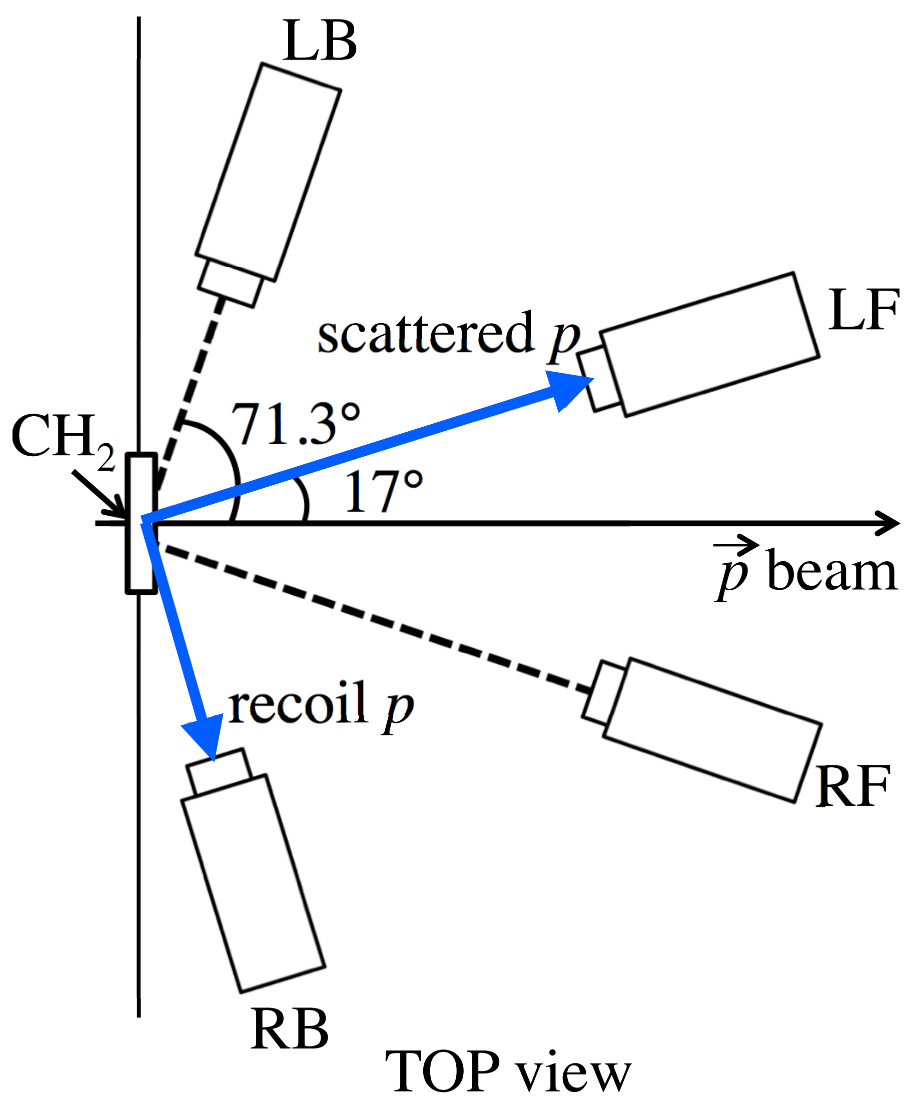
Kinematical coincidence

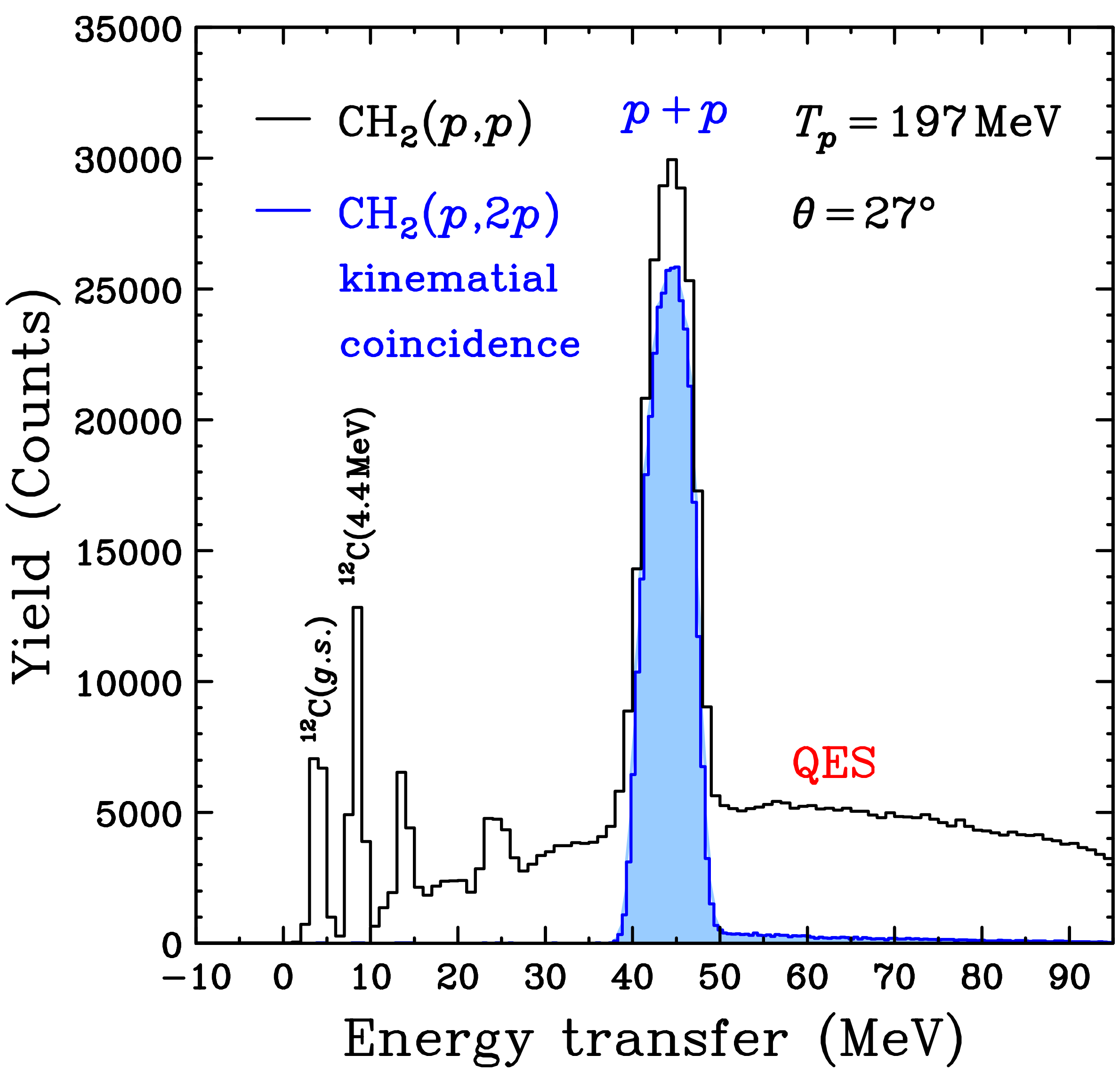
Kinematical coincidence is useful to suppress the QES background

- p+p scattering : 2-body scattering → Recoil angle θ_R : fixed
- $^{12}C(p,pp)^{11}B$: 3-body scattering → θ_R : varied (due to Fermi motion of target-N)

Measure scattered(θ) and recoiled(θ_R) protons "in coincidence"

- QES events can be significantly suppressed.





Neutron polarimeters

In general, a neutron polarimeter consists of analyzer and catcher planes:

Both planes are made of scintillator (H+C)

- can measure arrival time and 2D position

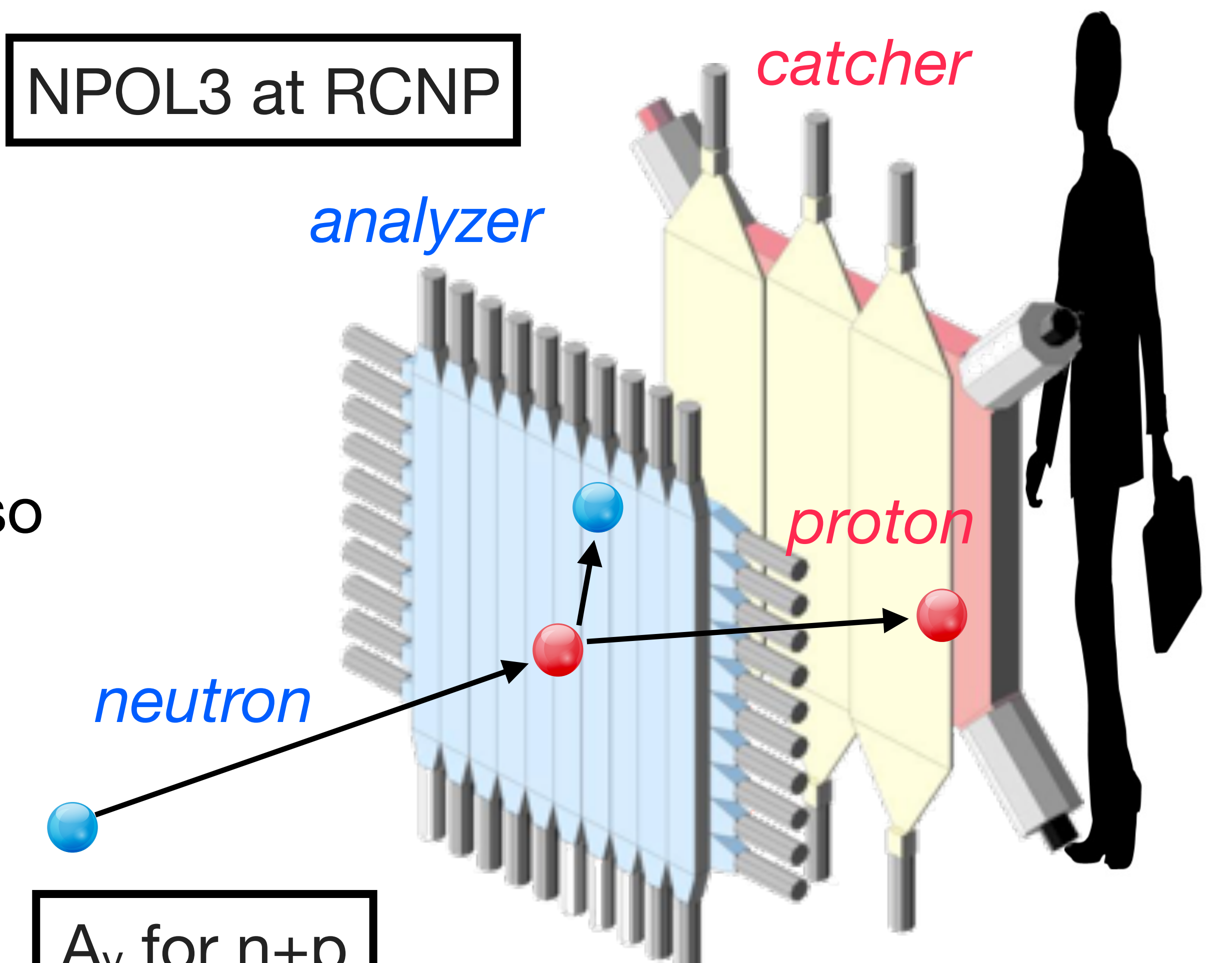
In the analyzer, n+p scattering will occur

- arrival time → *neutron Time-Of-Flight (TOF)*

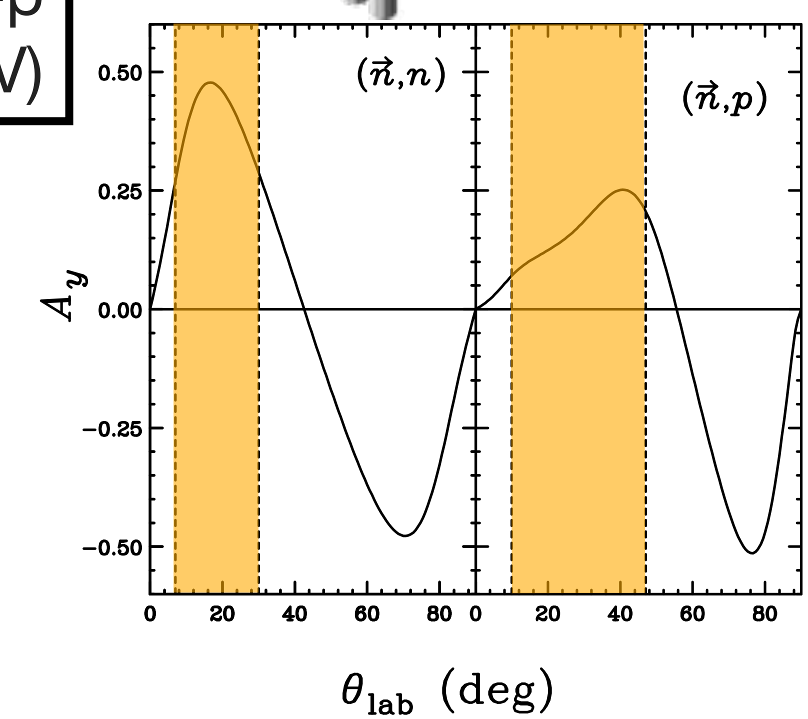
Doubly scattered neutron or recoil proton is also measured in the following catcher

- arrival time difference
  → *TOF of double scattering particle.*
- 2D positions in analyzer and catcher
  → *double scattering angles (θ,ϕ)*

NPOL3 at RCNP



A_y for n+p (346 MeV)



Left/Right/Up/Down scatterings can be defined by (θ,ϕ)
→ *Left-right asymmetry* → *$p_{n,N'}$ (normal)*
→ *Up-Down asymmetry* → *$p_{n,S'}$ (sideways)*

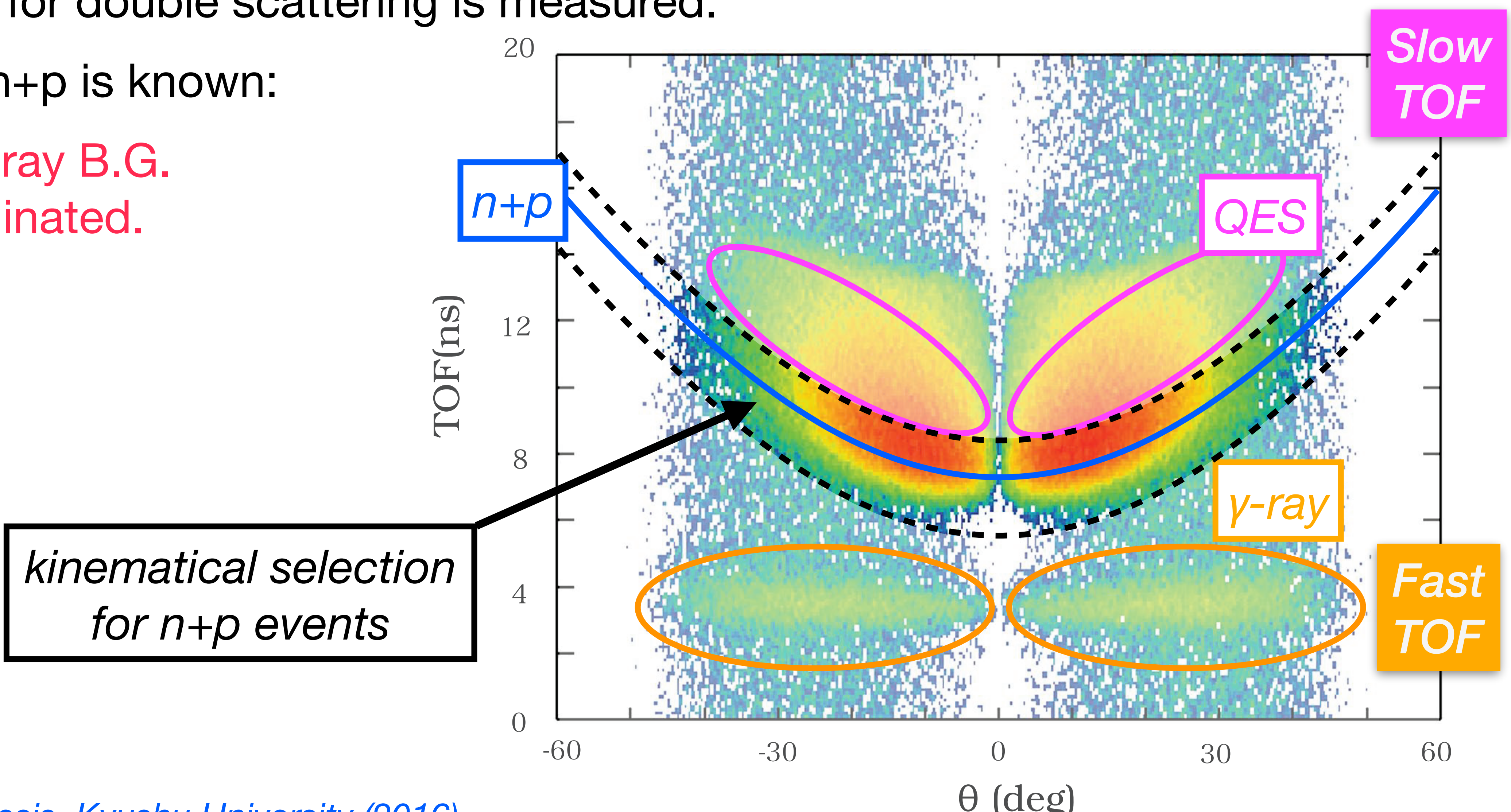
Kinematical selection

The analyzer is made of scintillator including H and C.

- The n+C events including QES become B.G..
- The FOM should be maximized
  by eliminating these events.

Kinematical selection for n+p events

- TOF and (θ,ϕ) for double scattering is measured.
- TOF vs. θ for n+p is known:
  - QES and γ-ray B.G.
    can be eliminated.



R.Nakajima, Graduation thesis, Kyushu University (2016).

FOM of neutron polarimeters

FOM of modern neutron polarimeters

FOM = 2〜5×10^{-4}

Note:
Calibrations methods of a neutron polarimeter are described in Appendix E of this lecture.

One in a few thousand neutrons entering a polarimeter is effective for polarization analysis.

Facility	Tn range (MeV)	TOF path length (m)	FOM×10^4 (Tn)	Ref.
RCNP	150-400	100	**4.94 (291 MeV)**	[1,2,3]
IUCF	80-200	120	1.73 (194 MeV)	[4,5]
LAMPF	300-800	600	2.00 (318 MeV)	[6,7,8]

[1] H.Sakai et al., Nucl. Instrum. Methods Phys. Res. A 320, 479 (1992).
[2] H.Sakai et al., Nucl. Instrum. Methods Phys. Res. A 369, 120 (1996).
[3] T. Wakasa et al., Nucl. Instrum. Methods Phys. Res. A 404, 355 (1998).
[4] C.D.Goodman et al., IEEE Trans. Nucl. Sci. 25, 2248 (1979).
[5] M.Palarczyk et al., Nucl. Instrum. Methods Phys. Res. A 457, 309 (2001).
[6] J.B.McClelland et al., Nucl. Instrum. Methods Phys. Res. A 276, 35 (1989).
[7] D.J.Mercer, Ph.D. Thesis, University of Colorado, 1993.

Experimental investigation for $\Delta S=0$ and $\Delta S=1$ strengths using D_{ij}

Power of spin transfers

Polarization transfer observable D_{ij}:

- Direct measure of the spin transfer

PWIA predictions at T_p < 200 MeV *(Central components of the NN interaction are dominant):*

Transition	ΔJ^π	ΔS	X_T^2/X_L^2	$D_{NN}(0°)$	$D_{LL}(0°)$
Fermi	0^+	0	-	$+1$	$+1$
GT	1^+	1	1	$-\frac{1}{3}$	$-\frac{1}{3}$

Examples at 0° and T_p=120-200 MeV for $^{14}C(p,n)^{14}N$:

Well-known Fermi and GT transitions:

- Fermi (ΔS=0) at 2.3 MeV → $D_{NN} = D_{LL} = 1$ > 0
- GT (ΔS=1) at 3.9 MeV → $D_{NN} = D_{LL} = -1/3$ < 0

Are polarization transfers D_{ij} really useful for distinguishing Fermi and GT states?
(consistent with PWIA predictions?)

Demonstration : $^{14}C(p,n)^{14}N$

Fermi IAS (0^+) and Gamow-Teller (1^+) peaks are observed.

Fermi IAS (0^+) :

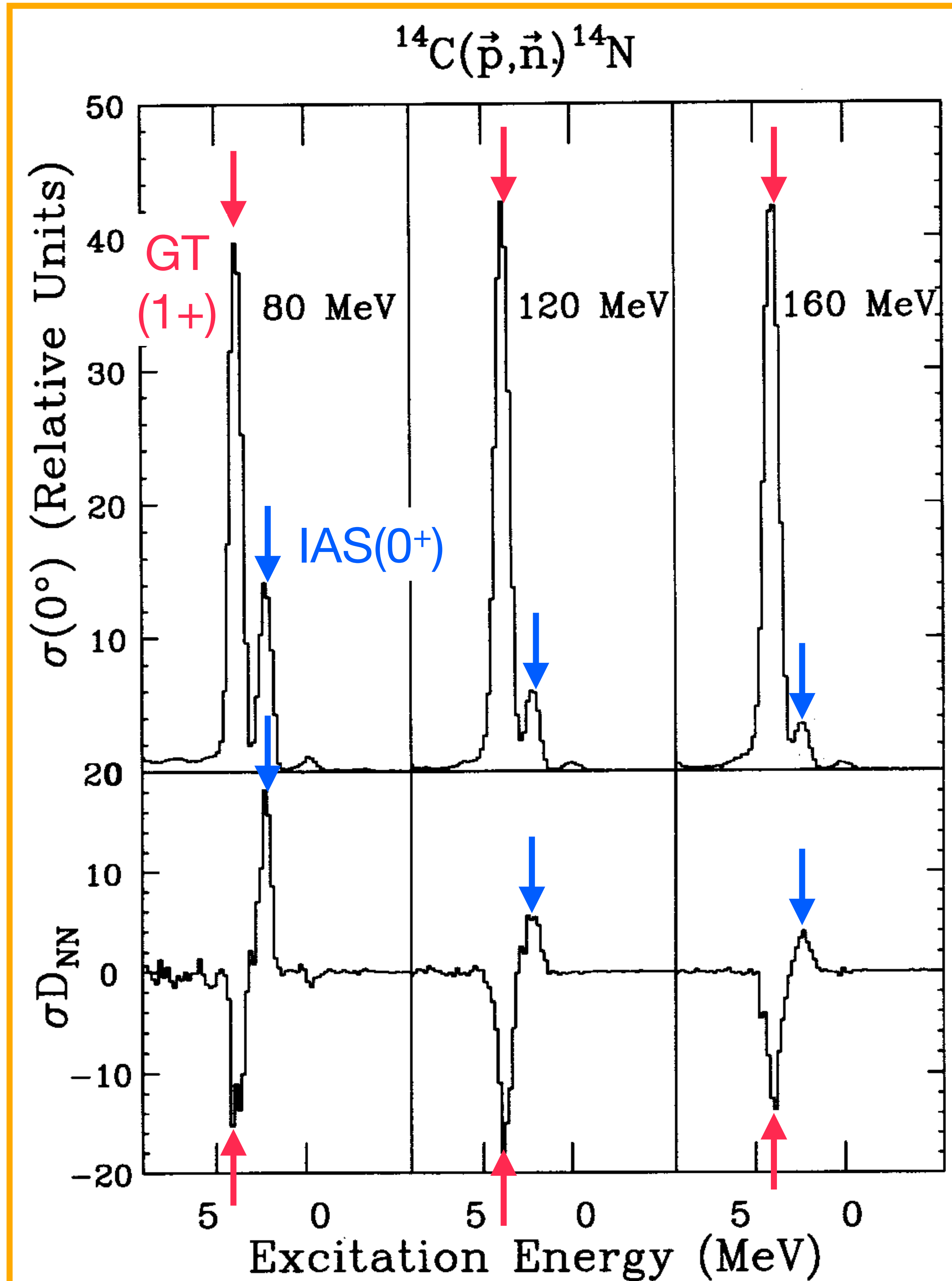
- $\sigma D_{NN} > 0$ ($\because D_{NN} = +1$)

Gamow-Teller (1^+) :

- $\sigma D_{NN} < 0$ ($\because D_{NN} = -1/3$)

We can identify F and GT transitions with D_{ij}.

How about $^{90}Zr(p,n)$ in which GTR was observed?



J.Rapaport and E.Sugarbaker, Ann. Rev. Nucl. Part. Sci. 44, 109 (1994).

Spin-vector dominance for ^{90}Zr(p,n)

In PWIA/DWIA

- GT ($\Delta J^{\pi}=1^{+}$)

$$D_{NN} \simeq -0.3$$

- IAS ($\Delta J^{\pi}=0^{+}$)

$$D_{NN} = +1.0$$

At 300 MeV

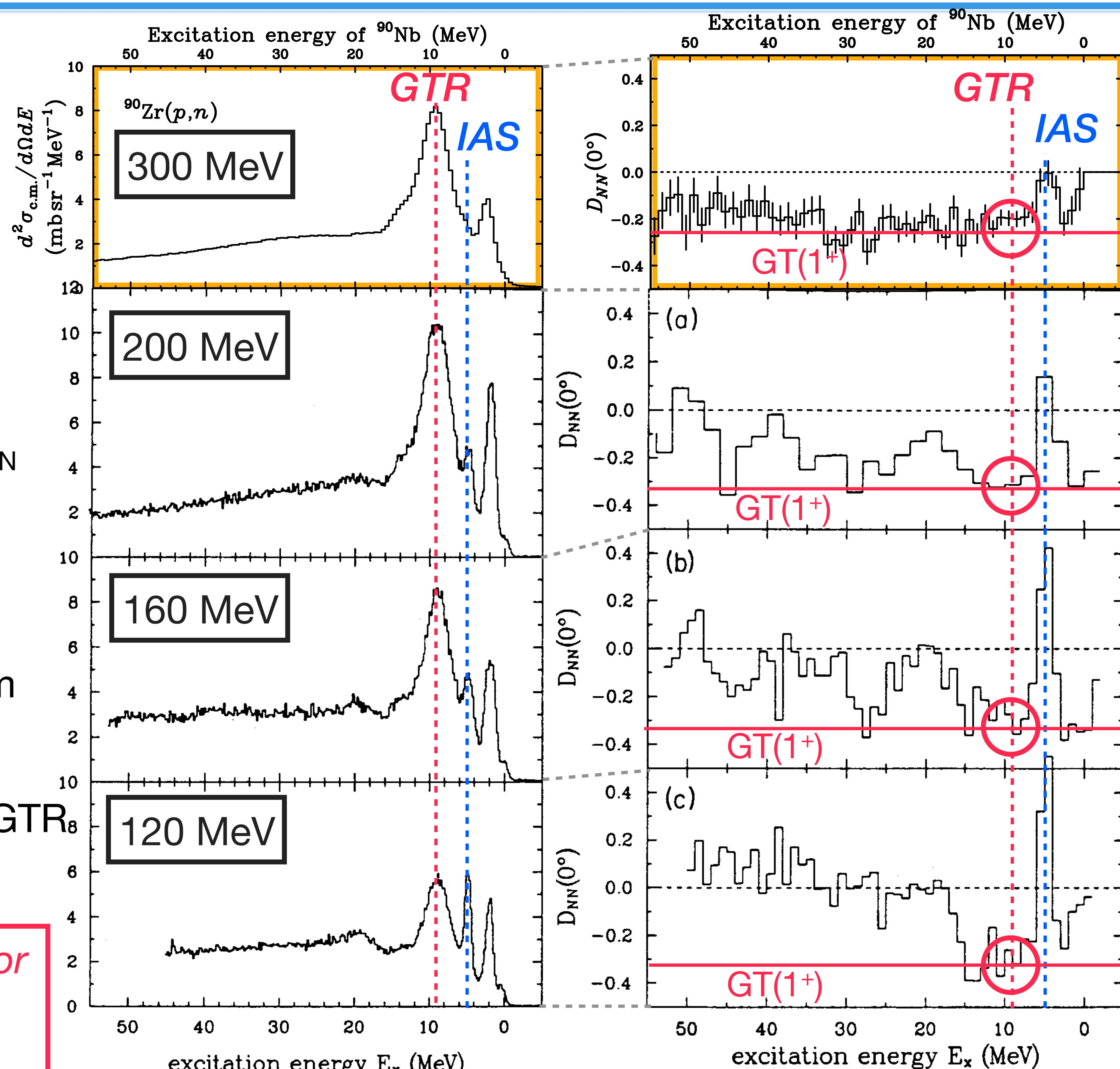
IAS can be identified in D_{NN} whereas it is not seen in σ

- D_{NN} is powerful to identify ΔS=0 and ΔS=1
- IAS is relatively minimum

Continuum beyond GTR

- D_{NN} is similar to that of GTR
- ΔS=1 dominance

The 300 MeV data is ideal for searching the GT strength in the continuum



T.N.Taddeucci, Can. J. Phys. 65, 557 (1987), T.W. et al., J. Phys. Soc. Jpn. 73, 1611 (2004).

Homework #2

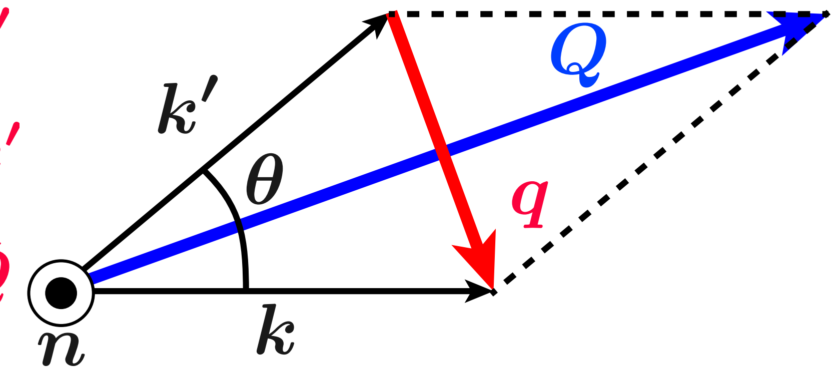
Homework #2

1. Show that, in a proton-nucleus scattering with unpolarized protons, the scattered protons would be polarized due to the spin-orbit interaction.
2. Under the parity invariance and rotation invariances, the polarization transfer $D_{L'N} = 0$. Proof this equality.
3. In an analyzing power measurement, $\bar{N}_L(\theta) \neq \bar{N}_R(\theta)$ in general since it is very difficult to set $\Delta\Omega_L = \Delta\Omega_R$.
   How can we measure A_y precisely with small systematic uncertainty? (*Hint: see Appendix A of this lecture*).
4. Explain how to obtain p_y or A_y firstly by the double scattering method referring Appendix B of this lecture.
5. In the double scattering method, the P_y=A_y equality for elastic scattering of spin 1/2 particles from a spin-zero target is used. Proof this equality referring Appendix C of this lecture.
6. Under the parity invariance and rotation invariances, the polarization transfer $D_{L'N} = 0$. Proof this equality.

Homework #2 (cont'd)

7. There are several conventions for the NN scattering amplitude. In the following conventions, express the α–ε terms by using the A–F terms in the KMT convention.

$$\begin{cases} \boldsymbol{q} \equiv \boldsymbol{k} - \boldsymbol{k}' \\ \boldsymbol{Q} = \boldsymbol{k} + \boldsymbol{k}' \\ \hat{\boldsymbol{n}} \equiv \hat{\boldsymbol{q}} \times \hat{\boldsymbol{Q}} \end{cases}$$



(1)Love and Franey

central: spin-singlet, spin-triplet; spin-orbit

$$M(E_{\rm CM}, \theta) = \alpha \frac{1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{4} + \beta \frac{3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{4} + \gamma(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{n}}$$

$$+\delta S_{12}(\hat{\boldsymbol{q}}) + \epsilon S_{12}(\hat{\boldsymbol{Q}})$$

tensor: direct, exchange

tensor operator

$$S_{12}(\hat{\boldsymbol{u}}) \equiv 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{u}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{u}}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

(2)Love

non-spin, spin-orbit, spin-longitudinal

$$M(E_{\rm CM}, \theta) = \alpha + \beta(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{n}} + \gamma(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{q}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{q}})$$

$$+\delta(\boldsymbol{\sigma}_1 \times \hat{\boldsymbol{q}}) \cdot (\boldsymbol{\sigma}_2 \times \hat{\boldsymbol{q}})$$

$$+\epsilon \left[(\boldsymbol{\sigma}_1 \times \hat{\boldsymbol{n}}) \cdot (\boldsymbol{\sigma}_2 \times \hat{\boldsymbol{n}}) - (\boldsymbol{\sigma}_1 \times \hat{\boldsymbol{Q}}) \cdot (\boldsymbol{\sigma}_2 \times \hat{\boldsymbol{Q}}) \right]$$

spin-transverse (δ and ε terms)

Homework #2 (cont'd)

8. In general, the quantization axis of the polarized proton beam is the normal direction (normal to the bending plane of the beam line). In order to measure a complete set of polarization transfer D_{ij}, we also need the proton beams polarized to longitudinal and sideways directions. At RCNP (Osaka, Japan), these polarized beams can be made by using one 45° dipole and two solenoid magnets as shown in Fig.1. Please explain how can we obtain the longitudinally and sideways polarized proton beams by using these magnets referring to the Appendix F of this lecture. Assume that the proton beam energy is 60 MeV.

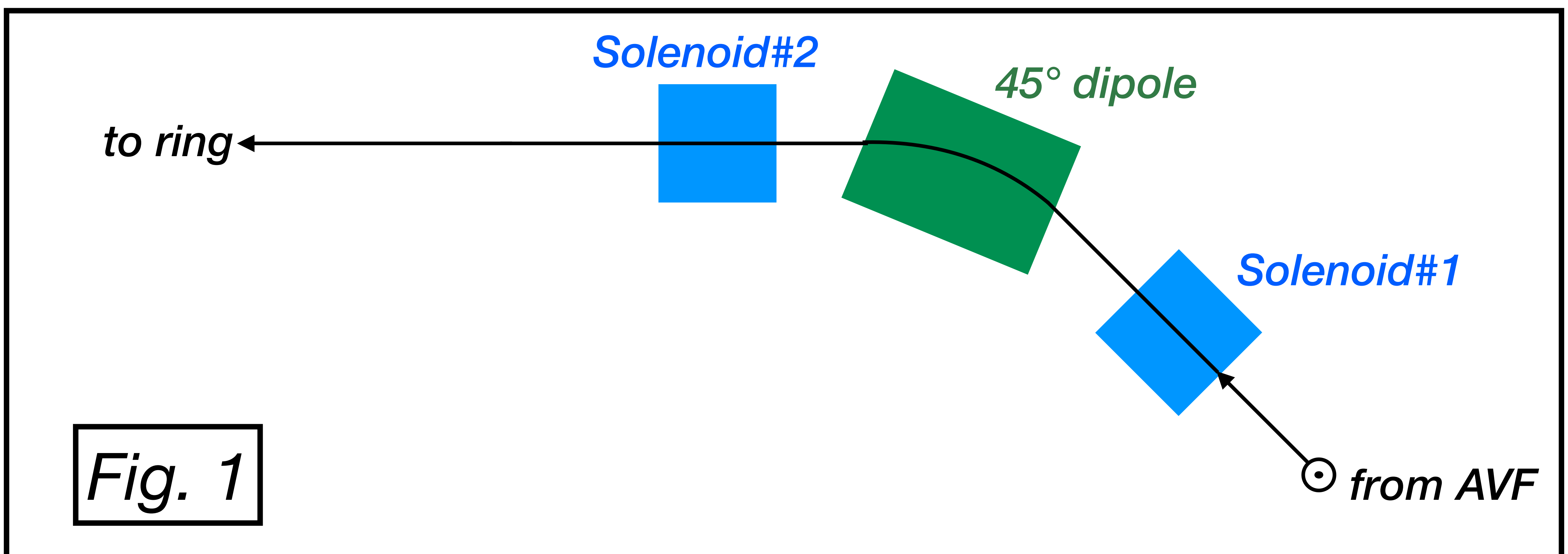


Fig. 1

Appendix A

Practical measurement of A_y

Practical measurement of A_y

In practical, $\bar{N}_L(\theta) \neq \bar{N}_R(\theta)$ **since it is very difficult to set $\Delta\Omega_L = \Delta\Omega_R$**

Thus, we need the data as follows for two different polarizations: p_y^1 and p_y^2

$$\left.\begin{aligned} N_L^1 &= \bar{N}_L(\theta)(1 + p_y^1 A_y) \\ N_R^1 &= \bar{N}_R(\theta)(1 - p_y^1 A_y) \end{aligned}\right\} \text{for } p_y^1 \qquad \left.\begin{aligned} N_L^2 &= \bar{N}_L(\theta)(1 + p_y^2 A_y) \\ N_R^2 &= \bar{N}_R(\theta)(1 - p_y^2 A_y) \end{aligned}\right\} \text{for } p_y^2$$

If we set $p_y^1 = -p_y^2 = p_y$ by tuning a PIS, the double ratio Y becomes

$$Y \equiv \frac{N_L^1/N_L^2}{N_R^1/N_R^2} = \left(\frac{1 + p_y A_y}{1 - p_y A_y}\right)^2$$

which is *"independent"* of $\bar{N}_L(\theta)$ and $\bar{N}_R(\theta)$.

Then we can get A_y as

$$\longrightarrow \quad A_y = \frac{1}{p_y}\frac{\sqrt{Y} - 1}{\sqrt{Y} + 1}$$

- This method has an exp. advantage *since it does not need I, n, ε, ΔΩ.*
  - Systematic uncertainty in A_y can be largely reduced.

Appendix B

Absolute magnitude of polarization

Absolute magnitude of polarization

Experimentally, an asymmetry $A(\theta) = p_y A_y(\theta)$ can be measured.

- If p_y is known, $A_y(\theta)$ can be obtained.
- If $A_y(\theta)$ is known, p_y can be deduced.

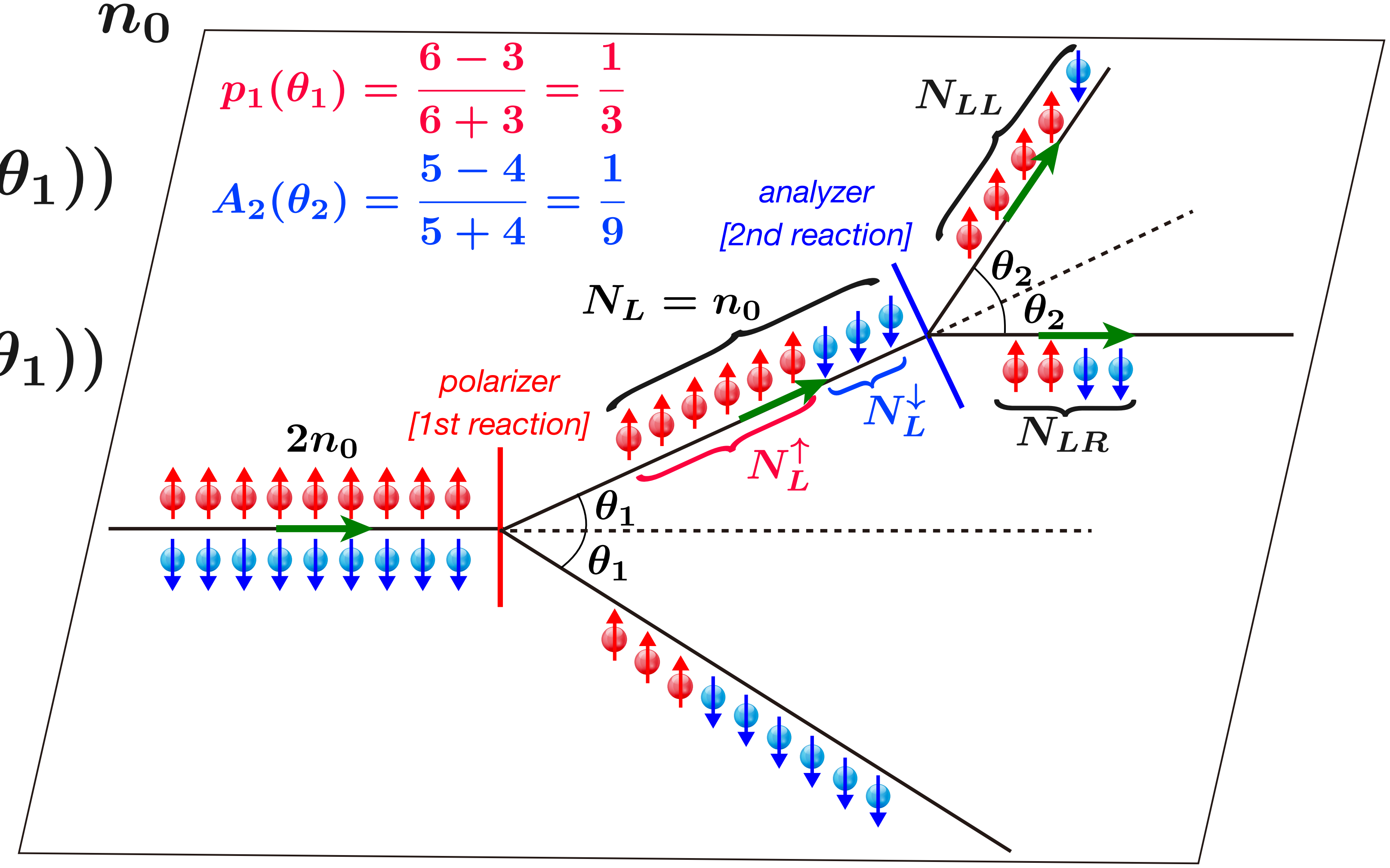
} How to obtain p_y or $A_y(\theta)$ firstly?

→ Double elastic-scattering method can be used.

Firstly, produce the polarized beam, $p_1(\theta_1)$, in 1st reaction with $2n_0$ "unpolarized" beam.

$$p_1(\theta_1) = \frac{N_L^{\uparrow} - N_L^{\downarrow}}{N_L^{\uparrow} + N_L^{\downarrow}} = \frac{N_L^{\uparrow} - N_L^{\downarrow}}{n_0}$$

$$\rightarrow \begin{cases} N_L^{\uparrow} = \dfrac{n_0}{2}(1 + p_1(\theta_1)) \\ N_L^{\downarrow} = \dfrac{n_0}{2}(1 - p_1(\theta_1)) \end{cases}$$



Absolute magnitude of polarization

Secondly, the p_y pol. beam is scattered in 2nd reaction and measure asymmetry A_2.

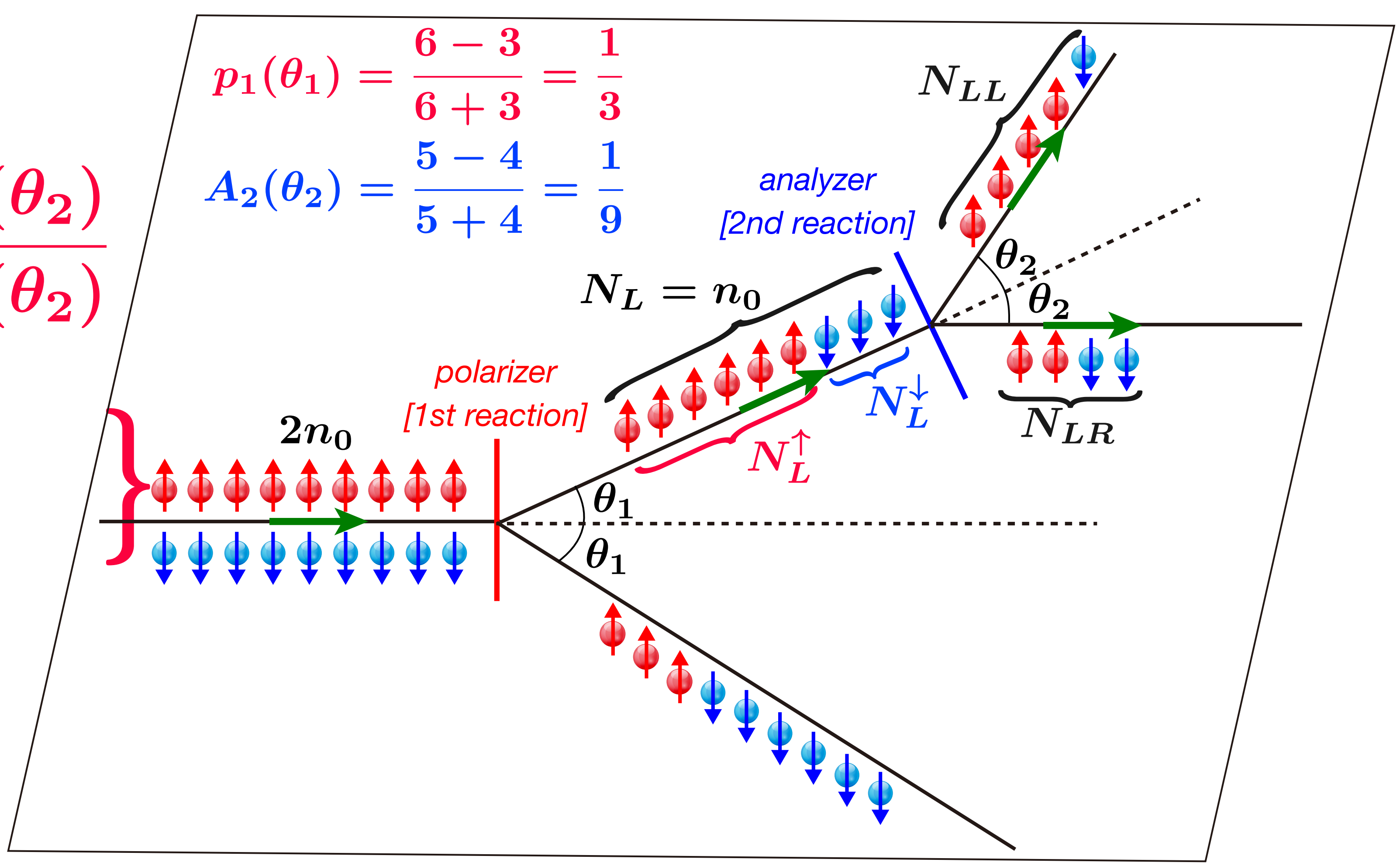
$$
\begin{aligned}
N_{LL}(\theta_2) &= N_L^{\uparrow}\ (1 + A_y(\theta_2)) + N_L^{\downarrow}(1 - A_y(\theta_2)) \\
&= \frac{n_0}{2}(1 + p_y(\theta_1))(1 + A_y(\theta_2)) + \frac{n_0}{2}(1 - p_y(\theta_1))(1 - A_y(\theta_2)) \\
&= n_0(1 + p_y(\theta_1)A_y(\theta_2))
\end{aligned}
$$

$$
N_{LR}(\theta_2) = n_0(1 - p_y(\theta_1)A_y(\theta_2))
$$

Left-right asymmetry in 2nd reaction

$$
A_2(\theta_2) \equiv \frac{N_{LL}(\theta_2) - N_{LR}(\theta_2)}{N_{LL}(\theta_2) + N_{LR}(\theta_2)}
$$

$$
= p_y(\theta_1)A_y(\theta_2)
$$



Absolute magnitude of polarization

$$A_2(\theta_2) \equiv \frac{N_{LL}(\theta_2) - N_{LR}(\theta_2)}{N_{LL}(\theta_2) + N_{LR}(\theta_2)} = p_y(\theta_1)A_y(\theta_2)$$

If we arrange *for the 1st and 2nd elastic scatterings:*

- Same target nuclei ($p_y = A_y$)
- Same scattering angles ($\theta_1 = \theta_2 = \theta$)

The measured asymmetry in 2nd scattering can be expressed as:

$$A_2(\theta) = p_y(\theta)A_y(\theta) = [p_y(\theta)]^2 = [A_y(\theta)]^2$$

$$\longrightarrow |p_y(\theta)| = |A_y(\theta)| = \sqrt{A_2(\theta)}$$

- *Absolute values of pol. and A_y can be obtained by just measuring the asymmetry.*
  - In order to determine the sign, an interference effect between Coulomb and nuclear interactions is used.

Exercise: Proof the $P_y = A_y$ equality for elastic scattering of spin 1/2 particles from a spin-zero target.

Appendix C

Polarization-Asymmetry equality

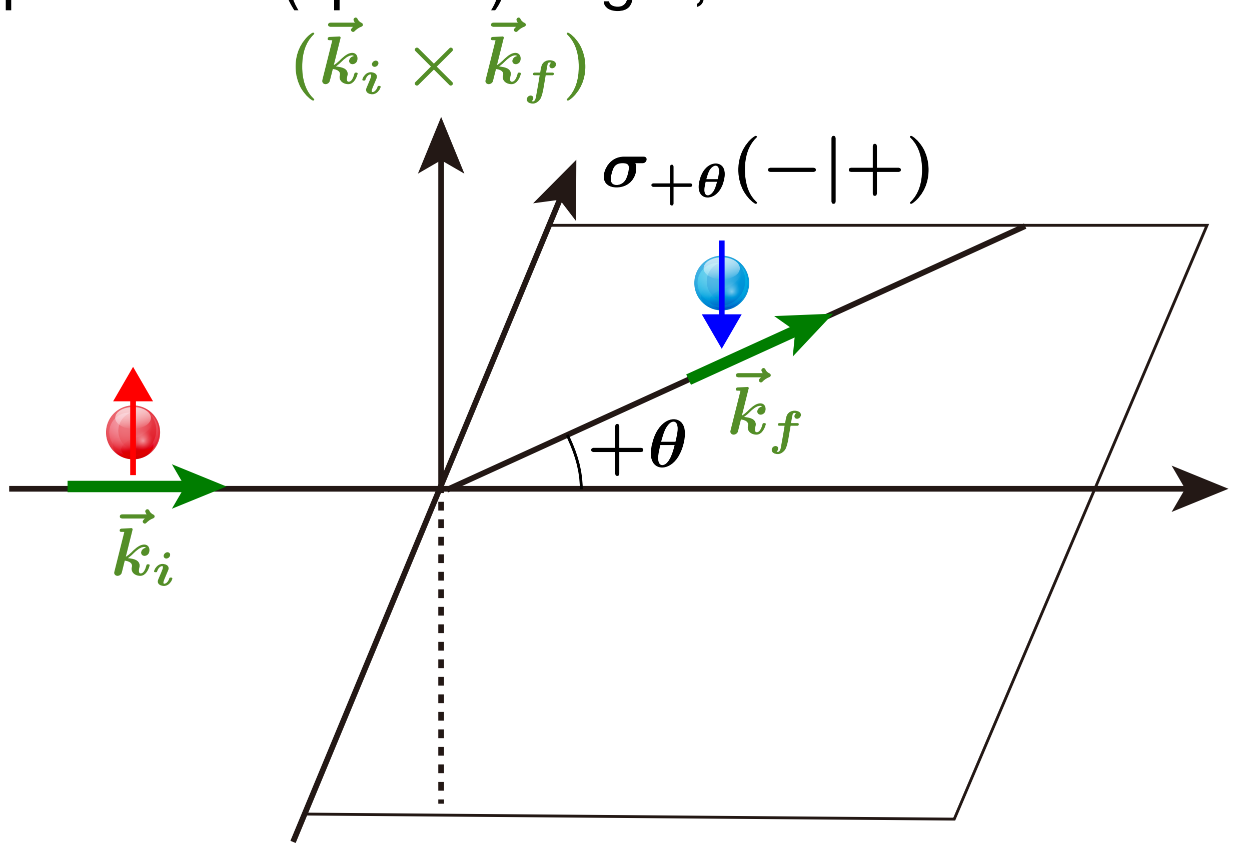
Polarization-Asymmetry equality

J.S.Bell and F.Mandl, Proc. Phys. Soc. 71, 272 (1958).

Exercise: Under the time-reversal and rotation invariances, vector polarization *P* and analyzing power A_y are identically equal for elastic scattering of spin 1/2 particles from a spin-zero target. Proof this equality.

Consider an incident unpolarized beam of spin 1/2 nucleons with momentum $\vec{k}_i$

- scattered to an angle +θ (left side) from an unpolarized (spin=0) target,
- the final momentum is $\vec{k}_f$
- the quantization axis
  = normal to the reaction plane, $(\vec{k}_i \times \vec{k}_f)$
- the cross section at θ
  from the initial spin state $m = \pm 1/2 \equiv \pm$
  to the final spin state $m' = \pm 1/2 \equiv \pm$
  is described as:
  $\sigma_\theta(m'|m)$



The polarization of scattered beam is given by

final spin state m'=+ | final spin state m'=-

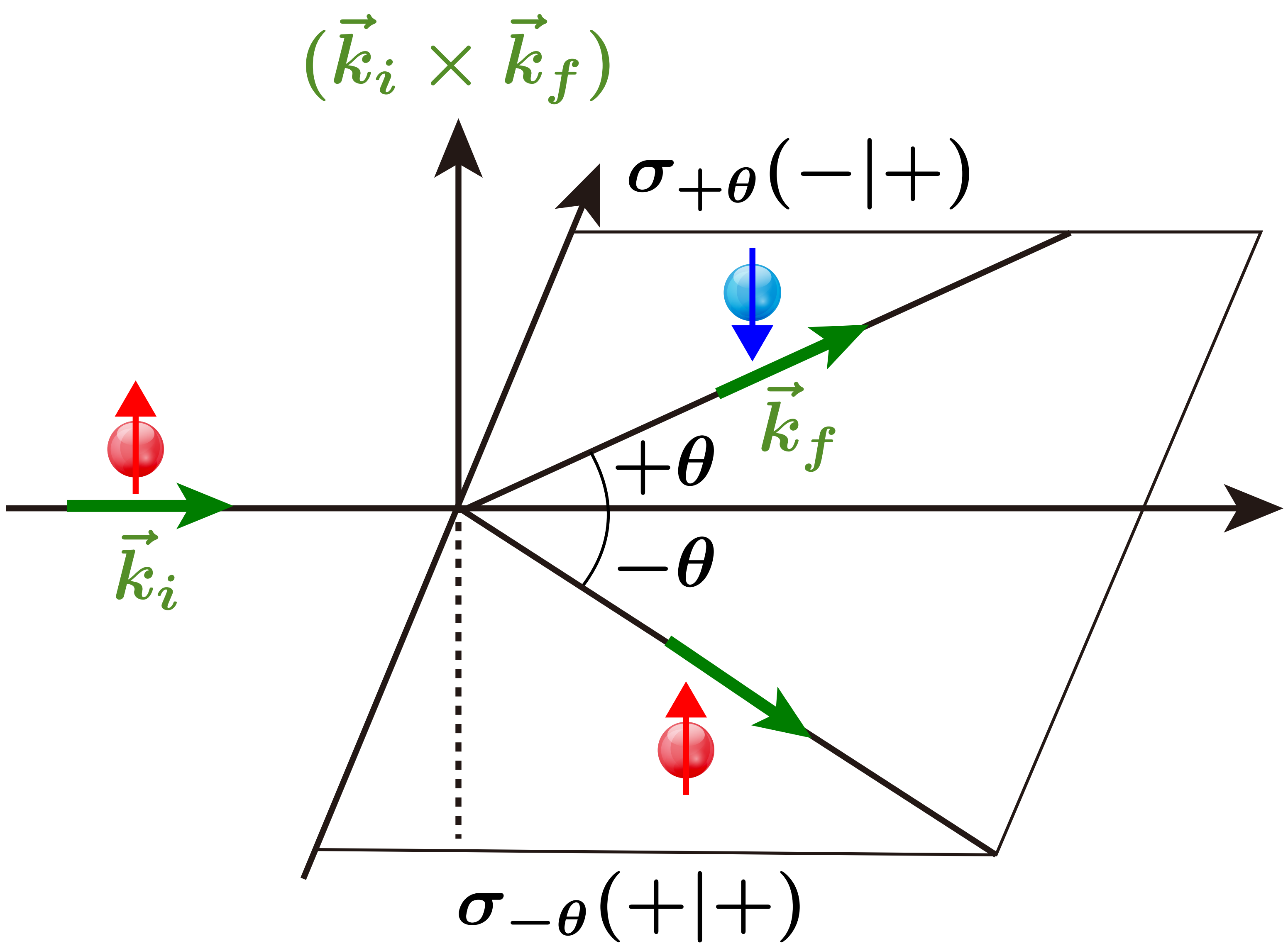
$$P(\theta) = \frac{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(+|-)] - [\sigma_{+\theta}(-|+) + \sigma_{+\theta}(-|-)]}{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(+|-)] + [\sigma_{+\theta}(-|+) + \sigma_{+\theta}(-|-)]}$$

Polarization-Asymmetry equality

Correspondingly, the asymmetry due to scattering a fully polarized beam (m=+) is:

at +θ (left side) *at -θ (right side)*

$$A_y(\theta) = \frac{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(-|+)] - [\sigma_{-\theta}(+|+) + \sigma_{-\theta}(-|+)]}{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(-|+)] + [\sigma_{-\theta}(+|+) + \sigma_{-\theta}(-|+)]}$$



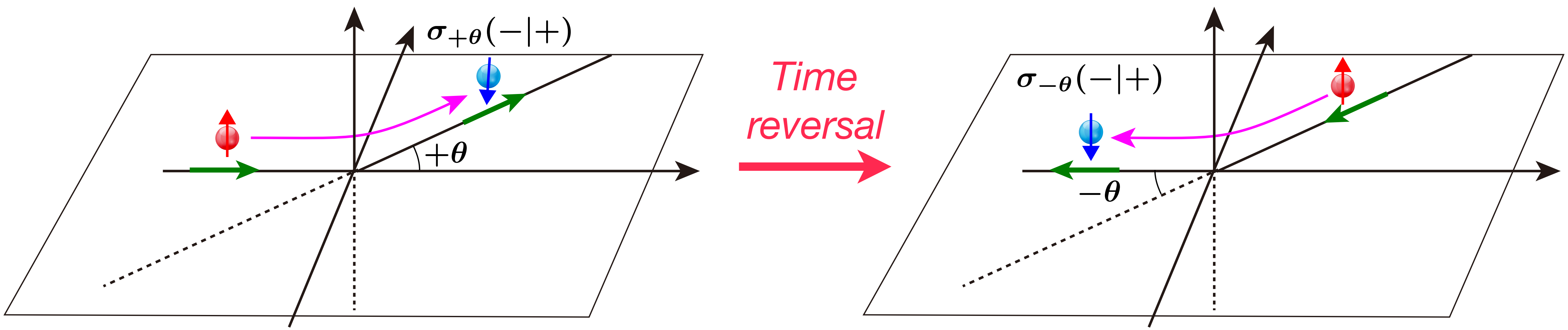
Polarization-Asymmetry equality

Time reversal means:

- interchanging initial and final states,
- changing the signs of all spins and momenta.

Under time reversal:

$$\sigma_{+\theta}(m'|m) \mapsto \sigma_{-\theta}(-m|m')$$



Assuming invariance of time reversal, the polarization P becomes:

$$P(\theta) = \frac{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(+|-)] - [\sigma_{+\theta}(-|+) + \sigma_{+\theta}(-|-)]}{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(+|-)] + [\sigma_{+\theta}(-|+) + \sigma_{+\theta}(-|-)]}$$

Time reversal →

$$P(\theta) = \frac{[\sigma_{-\theta}(-|-) + \sigma_{-\theta}(+|-)] - [\sigma_{-\theta}(-|+) + \sigma_{-\theta}(+|+)]}{[\sigma_{-\theta}(-|-) + \sigma_{-\theta}(+|-)] + [\sigma_{-\theta}(-|+) + \sigma_{-\theta}(+|+)]}$$

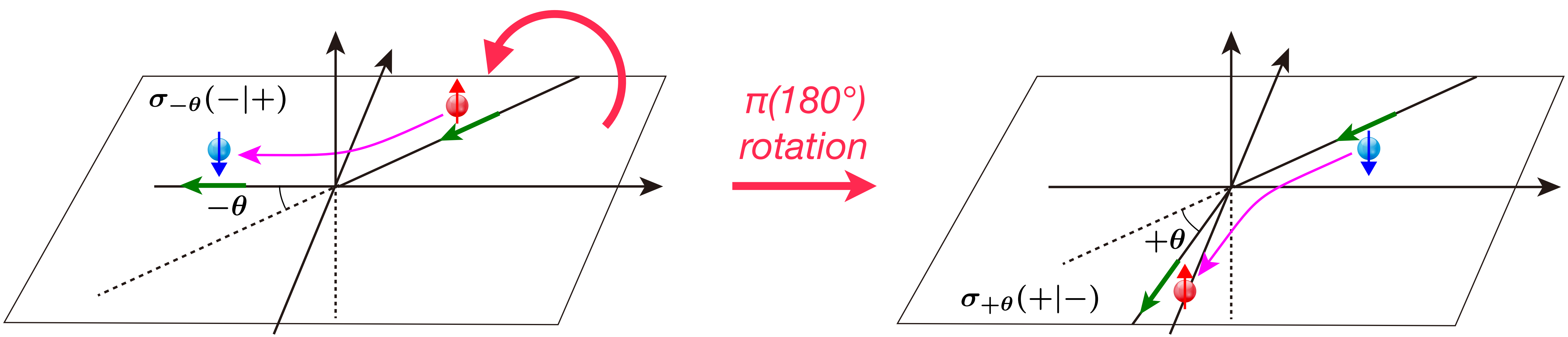
Polarization-Asymmetry equality

Then carry out a rotation through π (180°) about

- changes $\pm\theta$ to $\mp\theta$
- changes the spin states as $\pm m \to \mp m$ and $\pm m' \to \mp m'$

Under the rotation around k:

$$\sigma_{+\theta}(m'|m) \mapsto \sigma_{-\theta}(-m'|-m)$$



Assuming rotation invariance, the polarization P becomes:

$$P(\theta) = \frac{[\sigma_{-\theta}(-|-) + \sigma_{-\theta}(+|-)] - [\sigma_{-\theta}(-|+) + \sigma_{-\theta}(+|+)]}{[\sigma_{-\theta}(-|-) + \sigma_{-\theta}(+|-)] + [\sigma_{-\theta}(-|+) + \sigma_{-\theta}(+|+)]}$$

rotation

$$\longrightarrow P(\theta) = \frac{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(-|+)] - [\sigma_{-\theta}(-|+) + \sigma_{-\theta}(+|+)]}{[\sigma_{+\theta}(+|+) + \sigma_{+\theta}(-|+)] + [\sigma_{-\theta}(-|+) + \sigma_{-\theta}(+|+)]} = A_y(\theta)$$

Appendix D

Relation between polarization observables
in different frames

Polarized cross sections

It is useful to use the polarized cross sections, ID_i, introduced by Bleszynski et al.

- spin-scalar : $ID_0 = \frac{I}{4}[1 + D_{nn} + D_{qq} + D_{pp}]$
- spin-longitudinal (q-direction) : $ID_q = \frac{I}{4}[1 - D_{nn} + D_{qq} - D_{pp}]$
- spin-transverse (p-direction) : $ID_p = \frac{I}{4}[1 - D_{nn} - D_{qq} + D_{pp}]$
- spin-transverse (n-direction) : $ID_n = \frac{I}{4}[1 + D_{nn} - D_{qq} - D_{pp}]$

E.Bleszynski, M.Bleszynski, and J.C.A.Whitten, Phys .Rev. C 26, 2063 (1982).

C.M. and Lab. frames

Polarization observabels, D_i and D_{ij}, are defined in C.M. frame [p,n,q].

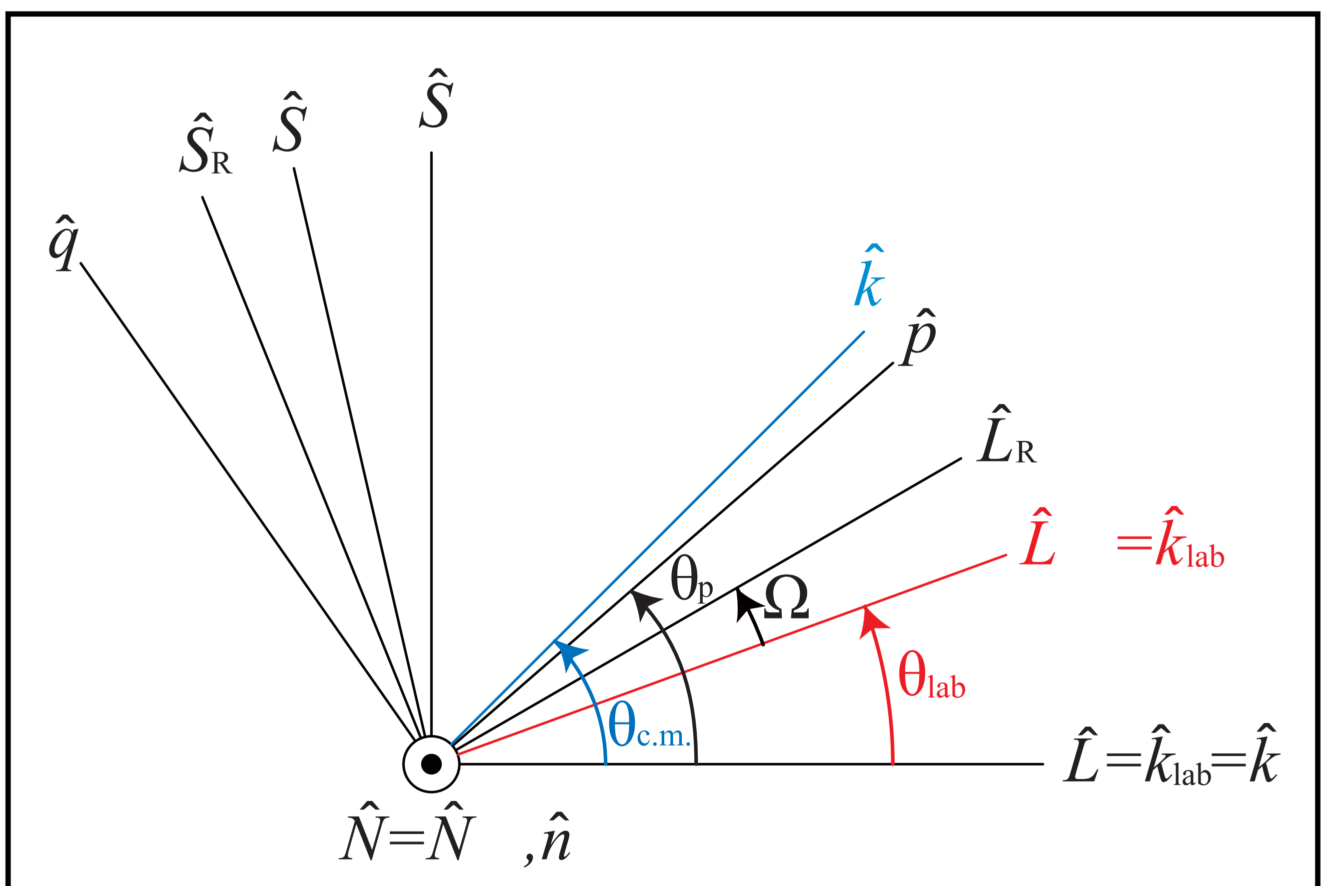
- Experimentally, polarization observables D_{ij} are measured in lab. frame.

$$(i,j) \in \begin{cases} S : Sideways \\ N : Normal \\ L : Longitudinal \end{cases}$$

[S,N,L] : incident nucleon

[S',N',L'] : outgoing nucleon

Relation between C.M. [p,n,q] and lab. [S,N,L] frames is as follows:



- Θ_{lab} : *lab. scattering angle for X(a,b)Y*
- $\Theta_{c.m.}$: *c.m. scattering angle*
- Ω : *relativistic spin-rotation angle*
- Θ_p : *angle between ki and p-direction*

C.M. and Lab. frames

Relation between polarization observables in C.M. and lab. frames becomes:

$$\begin{cases} D_0 = \frac{1}{4}[1 + D_{NN} + (D_{S'S} + D_{L'L})\cos\alpha_1 + (D_{L'S} - D_{S'L})\sin\alpha_1], \\ D_n = \frac{1}{4}[1 + D_{NN} - (D_{S'S} + D_{L'L})\cos\alpha_1 - (D_{L'S} - D_{S'L})\sin\alpha_1], \\ D_q = \frac{1}{4}[1 - D_{NN} + (D_{S'S} - D_{L'L})\cos\alpha_2 - (D_{L'S} + D_{S'L})\sin\alpha_2], \\ D_p = \frac{1}{4}[1 - D_{NN} - (D_{S'S} - D_{L'L})\cos\alpha_2 + (D_{L'S} + D_{S'L})\sin\alpha_2], \end{cases}$$

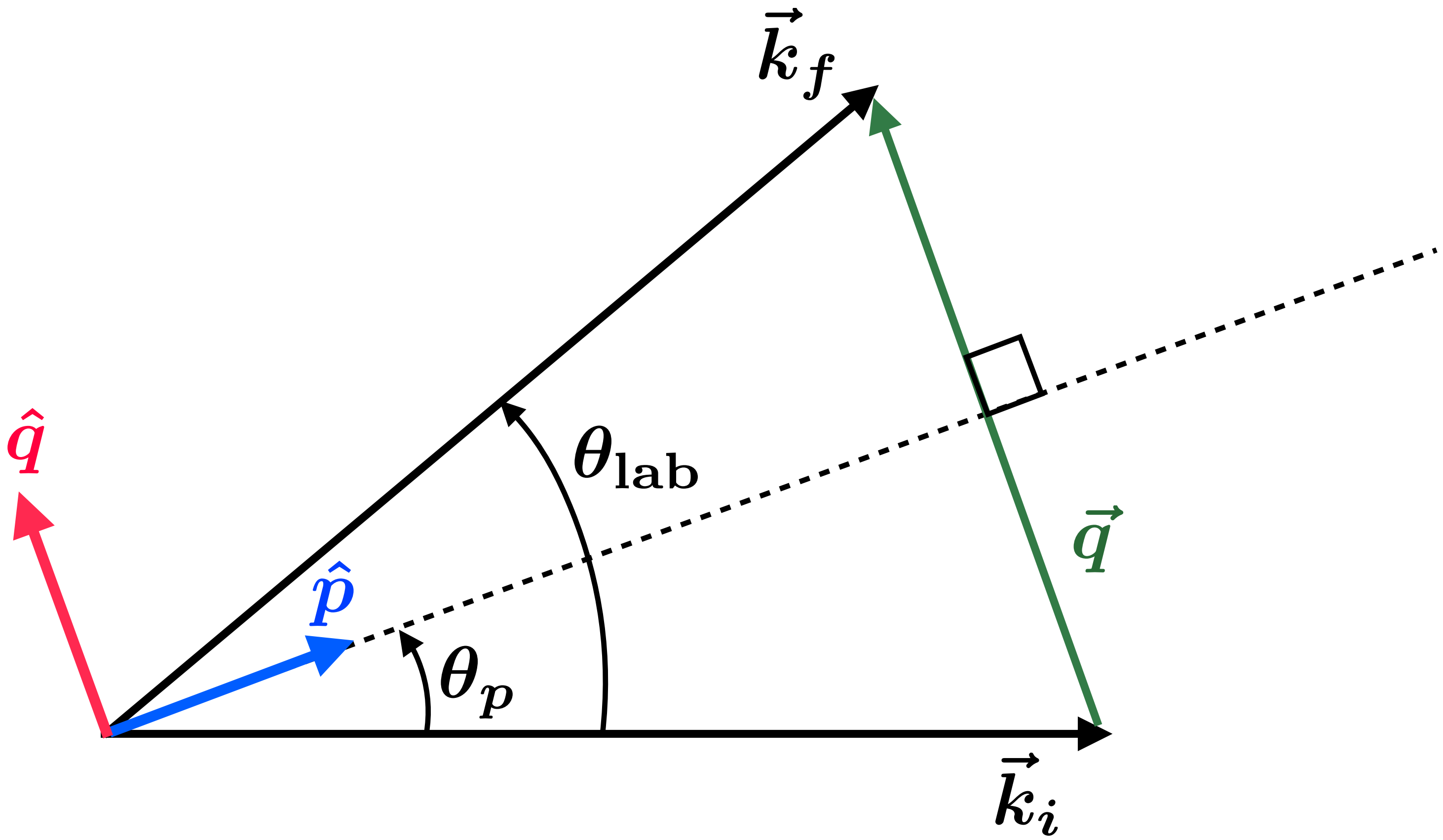
where $\alpha_1 = \theta_{lab}+\Omega$ and $\alpha_2 = 2\theta_p-\theta_{lab}-\Omega$.

In the elastic scattering ($D_{pq}=-D_{qp}$)and non-relativistic limit ($\Omega=0$), relation becomes:

$$\begin{cases} D_0 = \frac{1}{4}[1 + D_{NN} + (D_{S'S} + D_{L'L})\cos\theta_{\rm lab} + (D_{L'S} - D_{S'L})\sin\theta_{\rm lab}] \\ D_n = \frac{1}{4}[1 + D_{NN} - (D_{S'S} + D_{L'L})\cos\theta_{\rm lab} - (D_{L'S} - D_{S'L})\sin\theta_{\rm lab}] \\ D_q = \frac{1}{4}[1 - D_{NN} + (D_{S'S} - D_{L'L})\cos(2\theta_p - \theta_{\rm lab}) - (D_{L'S} + D_{S'L})\sin(2\theta_p - \theta_{\rm lab})] \\ D_p = \frac{1}{4}[1 - D_{NN} - (D_{S'S} - D_{L'L})\cos(2\theta_p - \theta_{\rm lab}) + (D_{L'S} + D_{S'L})\sin(2\theta_p - \theta_{\rm lab})] \end{cases}$$

Special case#1: Infinitely heavy target and Q=0

Relation in C.M. and lab. frames becomes:



$$\begin{cases} k_i = k_f \\ \theta_{\text{lab}} = \theta_{\text{c.m.}} \\ \theta_p = \dfrac{\theta_{\text{lab}}}{2} \\ \alpha_2 = 2\theta_p - \theta_{\text{lab}} - \Omega = 0 \end{cases}$$

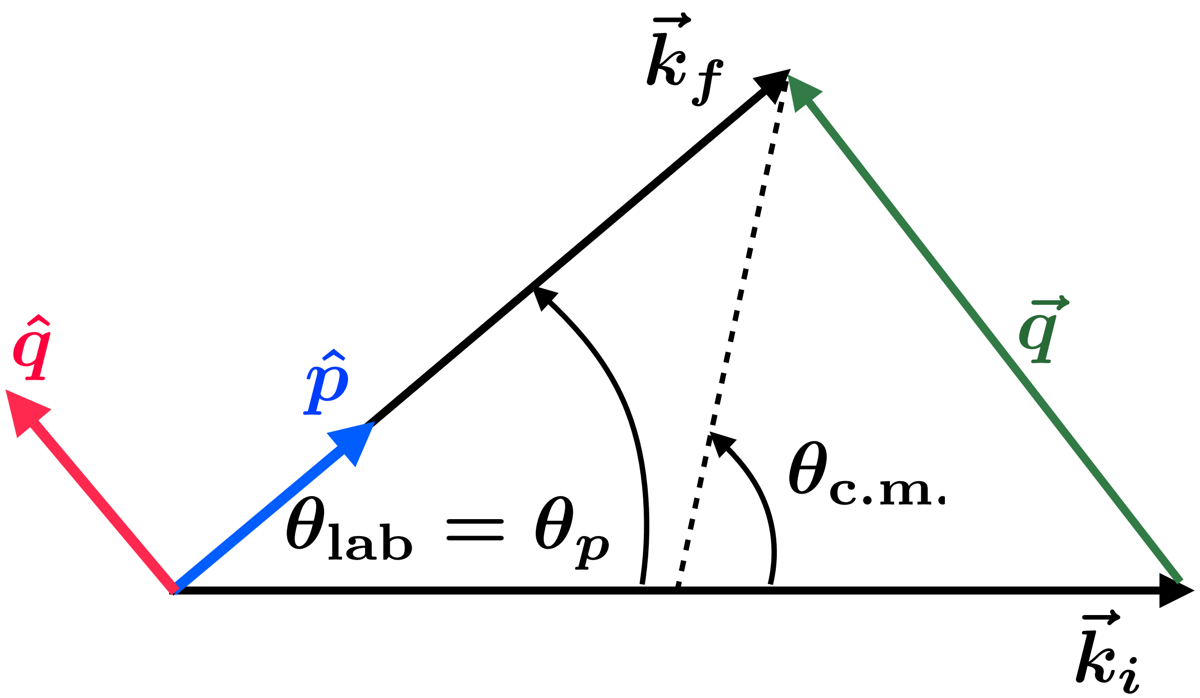
Relation between polarization observables becomes:

$$ID_q = \frac{I}{4}[1 - D_{NN} + D_{S'S} - D_{L'L}] = E^2 X_L^2$$

$$ID_p = \frac{I}{4}[1 - D_{NN} - D_{S'S} + D_{L'L}] = F^2 X_T^2$$

Special case#2: nucleon-nucleon (NN) scattering

Relation in C.M. and lab. frames becomes:



$$\begin{cases} \theta_{\rm lab} = \theta_p \\ \theta_{\rm c.m.} = 2\theta_{\rm lab} \\ \alpha_2 = 2\theta_p - \theta_{\rm lab} - \Omega = \theta_{\rm lab} \end{cases}$$

Relation between polarization observables becomes:

$$ID_q = \frac{I}{4}[1 - D_{NN} + (D_{S'S} - D_{L'L})\sec\theta_{\rm lab}]$$

$$ID_p = \frac{I}{4}[1 - D_{NN} - (D_{S'S} - D_{L'L})\sec\theta_{\rm lab}]$$

$$(\because \quad (D_{S'S} - D_{L'L})\cos\theta_{\rm lab} - (D_{L'S} + D_{S'L})\sin\theta_{\rm lab} = (D_{S'S} - D_{L'L})\sec\theta_{\rm lab})$$

Appendix E

Calibration of neutron polarimeters

Calibration of neutron polarimeter

In order to calibrate $A_{y;\mathrm{eff}}$ of a neutron polarimeter NPOL:

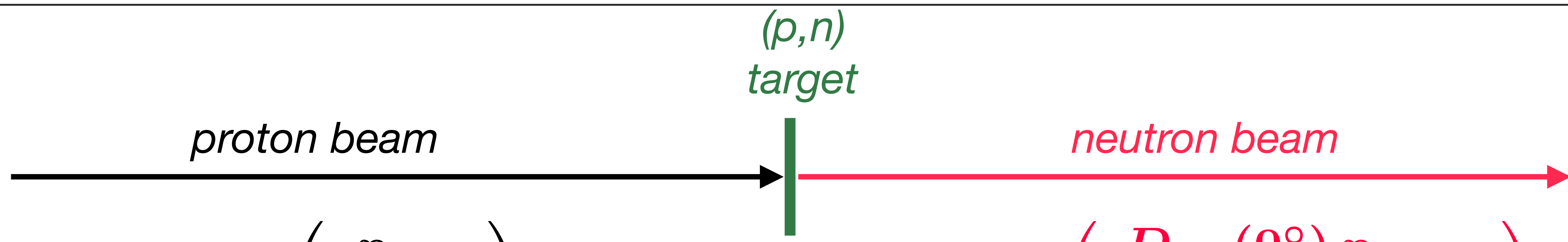
- We need a neutron beam whose polarization is known.
- In general, the neutron beam is produced by a charge exchange (p,n) reaction at 0°

The neutron beam with a known known polarization can be produced as follows:

- proton polarization $\vec{p}_p$ is known (i=S, N, L).
- $D_{ii}(0°)$ of the (p,n) reaction is known.

The neutron polarization $\vec{p}_n$ can be deduced as

$$p_{n,i} = D_{ii}(0^\circ)\, p_{p,i} \quad (i = S, N, L)$$



$$\vec{p}_{p,i} = \begin{pmatrix} p_{p,S} \\ p_{p,N} \\ p_{p,L} \end{pmatrix} \qquad \vec{p}_{n,i} = \begin{pmatrix} D_{SS}(0^\circ)\, p_{p,S} \\ D_{NN}(0^\circ)\, p_{p,N} \\ D_{LL}(0^\circ)\, p_{p,L} \end{pmatrix}$$

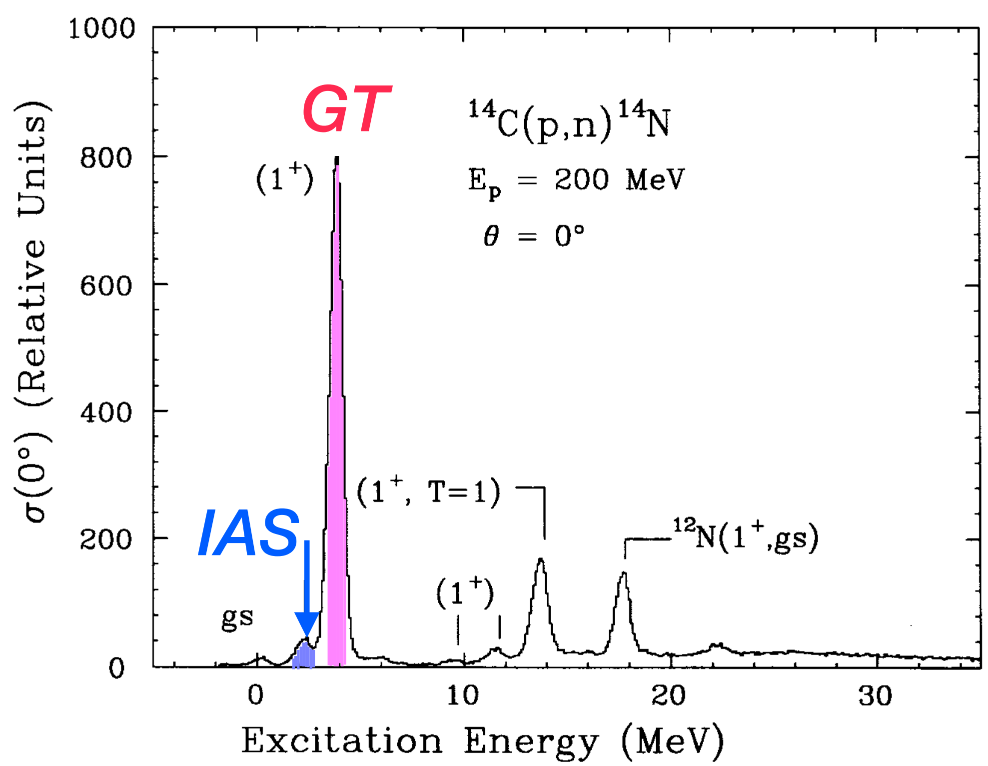
Calibration method #1

$^{14}C(p,n)^{14}N(0^+$; 2.31 MeV)

- $D_{ii}(0°)$ = 1 for the $0^+ \rightarrow 0^+$ IAS transition
- $p_{n,i} = D_{ii}(0°)\, p_{p,i} = p_{p,i} \quad (i = S, N, L)$
  - Neutron beam polarization = Proton beam polarization

→ An ideal reaction to produce a polarized neutron beam

- Some disadvantages:
  - ^{14}C is a radioisotope (difficult to use as a target).
  - IAS at E_x=2.31 MeV is weakly excited whereas GT 1^+ at 3.95 MeV is strongly excited.
  - A good energy resolution of ΔE≦500 keV is required.



NPOL at IUCF was calibrated by this method.

J.Rapaport and E.Sugarbaker, Ann. Rev. Nucl. Part. Sci. 44, 109 (1994).

Calibration method #2

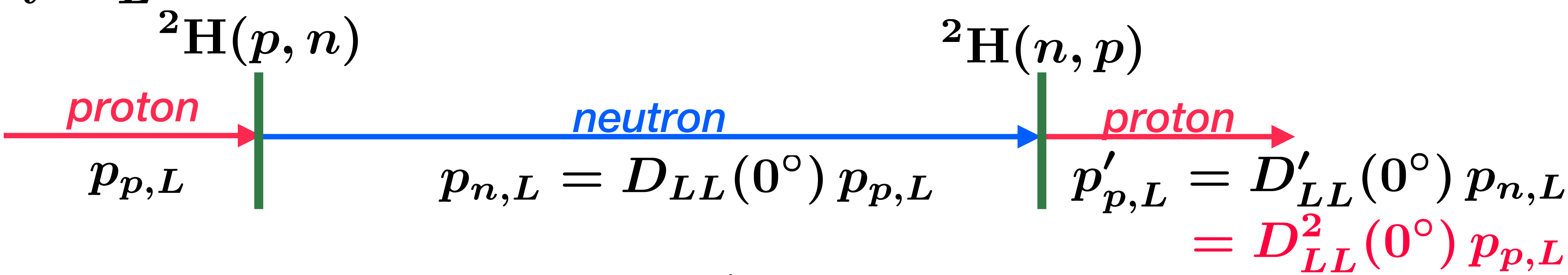
^{2}H(p,n)pp at 0° (GT $1^{+} \rightarrow 0^{+}$)

Under the charge symmetry:

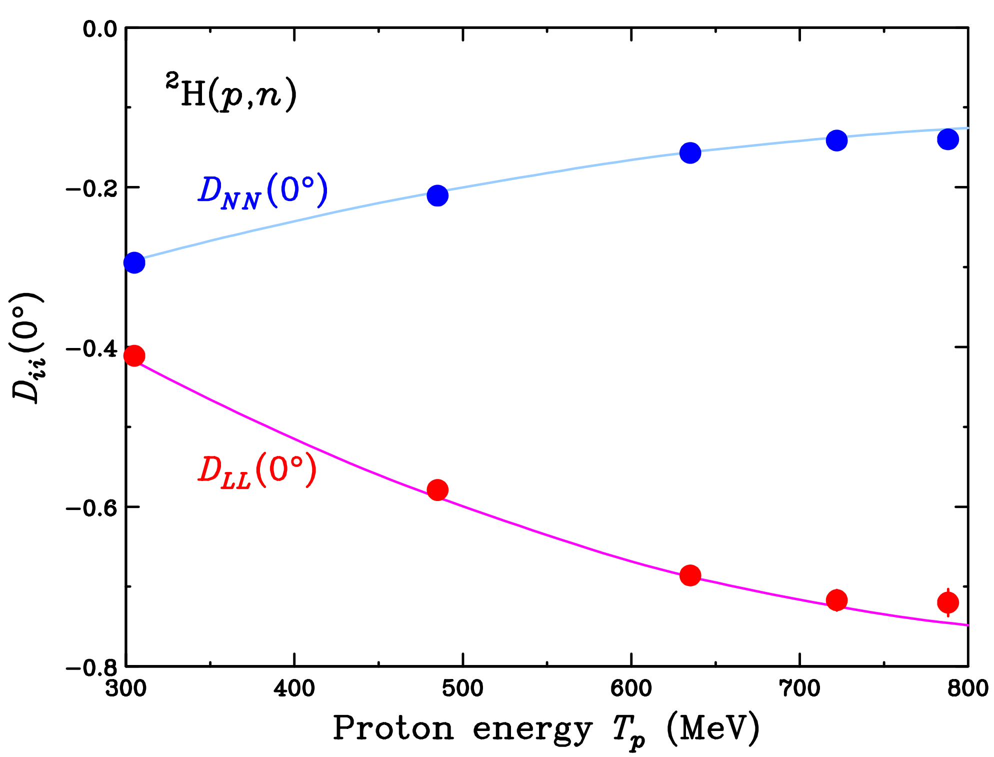
$$D_{ii}(0^\circ)\ \text{for}\ ^{2}\mathrm{H}(p,n) \quad = \quad D'_{ii}(0^\circ)\ \text{for}\ ^{2}\mathrm{H}(n,p)$$

- Double scattering measurement → $D_{ii}(0^\circ)$ can be obtained

Example: $i = \hat{L}$



- By measuring proton polarizations, $p_{p,L}$ and $p'_{p,L}$, $D_{LL}(0^\circ)$ can be deduced.



$$D_{NN}(0^\circ) = \frac{-1 - D_{LL}(0^\circ)}{2}$$

$$p_{n,i} = D_{ii}(0^\circ)\, p_{p,i} \quad (i = S, N, L)$$

→ NPOL's at LAMPF and RCNP were calibrated by this method.

GT $^{12}C(p,n)^{12}N(1^+)$ transition w/o knowing D_{ii}

For a spin-flip ΔS=1 GT transition, $D_{ii}(0°)$'s satisry

$$2D_{SS}(0^\circ) + D_{LL}(0^\circ) = -1\,, \qquad (D_{NN}(0^\circ) = D_{SS}(0^\circ))$$

Prepare $\vec{p}_p$ beam with $\hat{S}$ and $\hat{L}$ components.

$$\vec{p}_p = (p_{p,S}, 0, p_{p,L})$$

- The L-component is measured as the S-component at BLP.

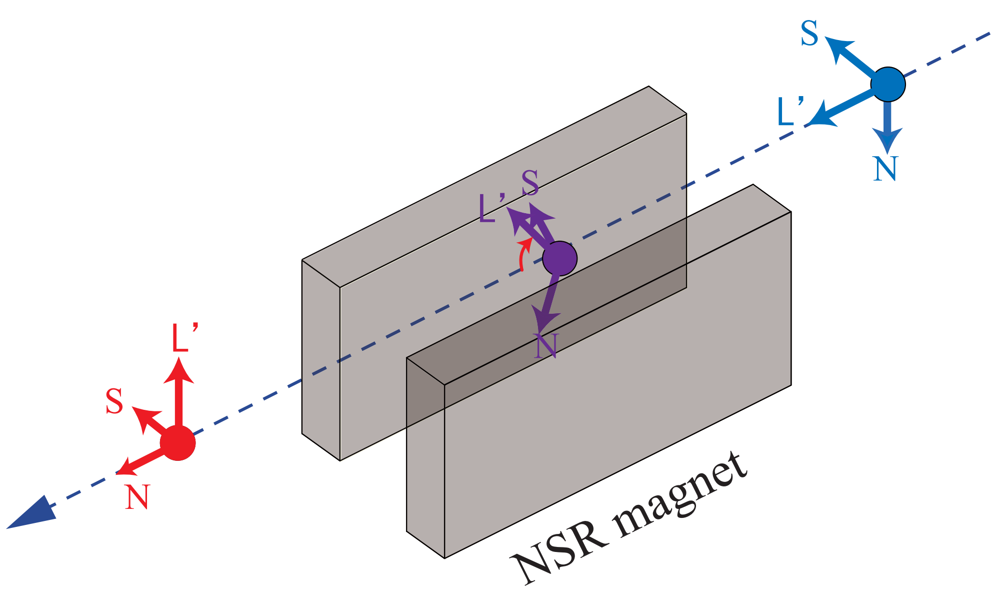
After (p,n) at 0°, neutron polarization $\vec{p}_n$ is:

$$\vec{p}_n = (p_{n,S}, p_{n,N}, p_{n,L}) = (D_{SS}(0^\circ)\, p_{p,L}, 0, D_{LL}(0^\circ)\, p_{p,L})$$

With a dipole field, $p_{n,L}$ is rotated into $p_{n,N}$

$$\vec{p}'_n = (p'_{n,S}, p'_{n,N}, p'_{n,L})$$
$$= (D_{SS}(0^\circ)\, p_{p,S}, D_{LL}(0^\circ)\, p_{p,L}, 0)$$

- rotation in the N-L plane.



GT $^{12}C(p,n)^{12}N(1^+)$ transition w/o knowing D_{ii}

$$\vec{p}\,'_n = (D_{SS}(0^\circ)\, p_{p,S},\, D_{LL}(0^\circ)\, p_{p,L},\, 0)$$

Left-Right and Up-Down asymmetries, A_{LR} & A_{UD}, by $p'_{n,N}$ & $p'_{n,S}$ are measured:

- $A_{LR} = p'_{n,N}\, A_{y;\mathrm{eff}} = D_{LL}(0^\circ)\, p_{p,L}\, A_{y;\mathrm{eff}}$
- $A_{UD} = p'_{n,S}\, A_{y;\mathrm{eff}} = D_{SS}(0^\circ)\, p_{p,S}\, A_{y;\mathrm{eff}}$

Because $2D_{SS}(0^\circ)+D_{LL}(0^\circ)=-1$, $A_{y;\mathrm{eff}}$ can be deduced as

$$2\frac{A_{UD}}{p_{p,S}\, A_{y;\mathrm{eff}}} + \frac{A_{LR}}{p_{p,L}\, A_{y;\mathrm{eff}}} = -1$$

$$\longrightarrow A_{y;\mathrm{eff}} = -\left[2\frac{A_{UD}}{p_{p,S}} + \frac{A_{LR}}{p_{p,L}}\right]$$

- $A_{y;\mathrm{eff}}$ can be calibrated w/o knowing $D_{ii}(0^\circ)$ beforehand.

$D_{ii}(0^\circ)$ can be deduced as:

$$D_{LL}(0^\circ) = \frac{A_{LR}}{p_{p,L}\, A_{y;\mathrm{eff}}} \quad D_{SS}(0^\circ) = \frac{A_{UD}}{p_{p,S}\, A_{y;\mathrm{eff}}}$$

NPOL3 at RCNP was calibrated by this method.

Appendix F

Proton spin precession in magnetic fields

Proton spin precession in a magnetic field #1

In dipole, the relation between the proton momentum k and the magnetic field B is:

$$\vec{k} \perp \vec{B} \quad (B_\perp = \vec{k} \times \vec{B})$$

- Spin is precessed in the medium plane (in the bending plane).
- A relative spin precession angle θ is given by

$$\theta = \gamma \left(\frac{g_p}{2} - 1 \right) \Theta$$

  - g_p = 5.586 : proton g-factor
  - Θ : bending angle
  - γ : Lorentz factor

Example (injection line from AVF to ring @ RCNP)

- θ≒90° for Tp≒60 MeV and Θ=45°
- By bending Θ=45°, the polarization vector is precessed by 90°. Therefore

$$\hat{L} \rightarrow \hat{S} \qquad \hat{S} \rightarrow \hat{L}$$

Proton spin precession in a magnetic field #2

In solenoid, the relation between the proton momentum k and the magnetic field B is:

$$\hat{k} \parallel \vec{B} \quad (B_{\parallel} = \hat{k} \cdot \vec{B})$$

- Spin is precessed around k (perpendicular to the beam direction).
- Spin precession angle ϕ is given by

$$\phi = g_p \frac{\mu_N \cdot B_{\parallel} L}{\beta\gamma \cdot \hbar c}$$

  - $g_p = 5.586$: proton g-factor
  - $\mu_N = 3.15 \times 10^{-14}\,\mathrm{MeV \cdot T^{-1}}$: nucleon magneton
  - $B_{\parallel} L$ in unit of Tm : magnetic field × length
  - $\beta\gamma$: Lorentz factors

Example (injection line from AVF to ring @ RCNP)

- ϕ=90° for Tp=53 MeV and BL=0.600 Tm
- By passing in BL=0.6 Tm, the polarization vector is precessed by 90°. Therefore

$$\hat{L} \rightarrow \hat{S} \text{ or } \hat{N}$$