Effective interactions of nuclei

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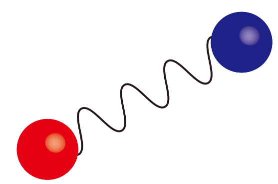
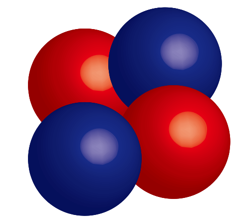
Construction of “Bridges”

Nuclear force **Light nuclei ~A≒10-20**

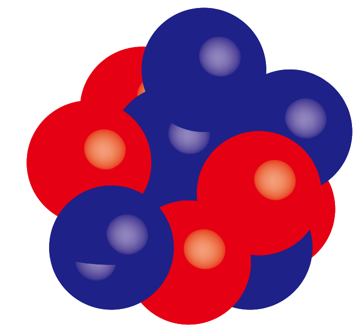
QCD





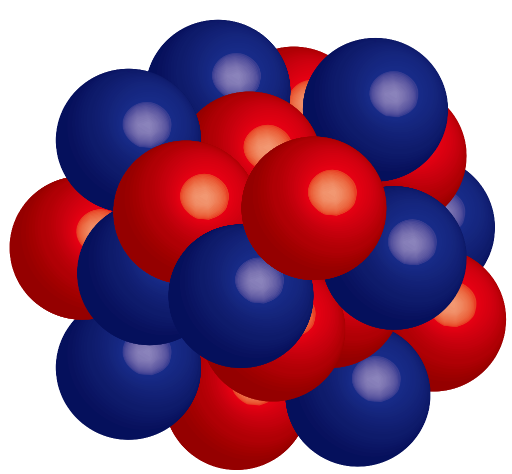
Lattice QCD
Effective Field Theory

Few body techniques
No core shell model
and many others…



This work

Medium mass nuclei~A≒20-100



shell model with core
via the **effective interaction**
derived from **nuclear force**

Purpose and scope of this lecture

- Understand effective interaction of nuclei
- Theory of “renormalization” of nuclear force
- Folded diagram method
- Current status of this line of research

Day1

- History of shell model calculation and effective interaction
    - Various effective interactions for shell model
- What is effective interaction?
    - Toy model
- Formal theories of effective interaction
    - KK method

Day2

- Review of time-dependent perturbation theory
  - Interaction picture
  - Dyson equation
- Folded diagram method
  - Factorization
- Implementation

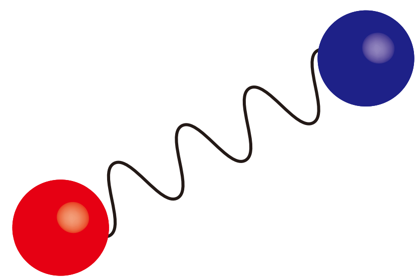
Day3

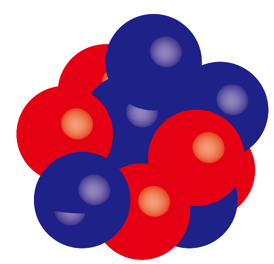
- Extended Kuo-Krenciglowa method
- Recent application of MBPT
  - Island of inversion
  - Comparison to latest experiments
  - Open problems

Nuclear force in medium

realistic
nucleon-nucleon
interaction

- two-body interaction in vacuum
- "bare" NN interaction
- determined by scattering experiment
- Non-central nature
- Not applicable to nuclear many-body problem directly





effective
nucleon-nucleon
interaction

- two-body interaction in nuclear medium
- "renormalized" NN interaction
- designed for chosen degrees of freedom
- determined by shell model fitting or microscopic theory
- effective Non-central nature

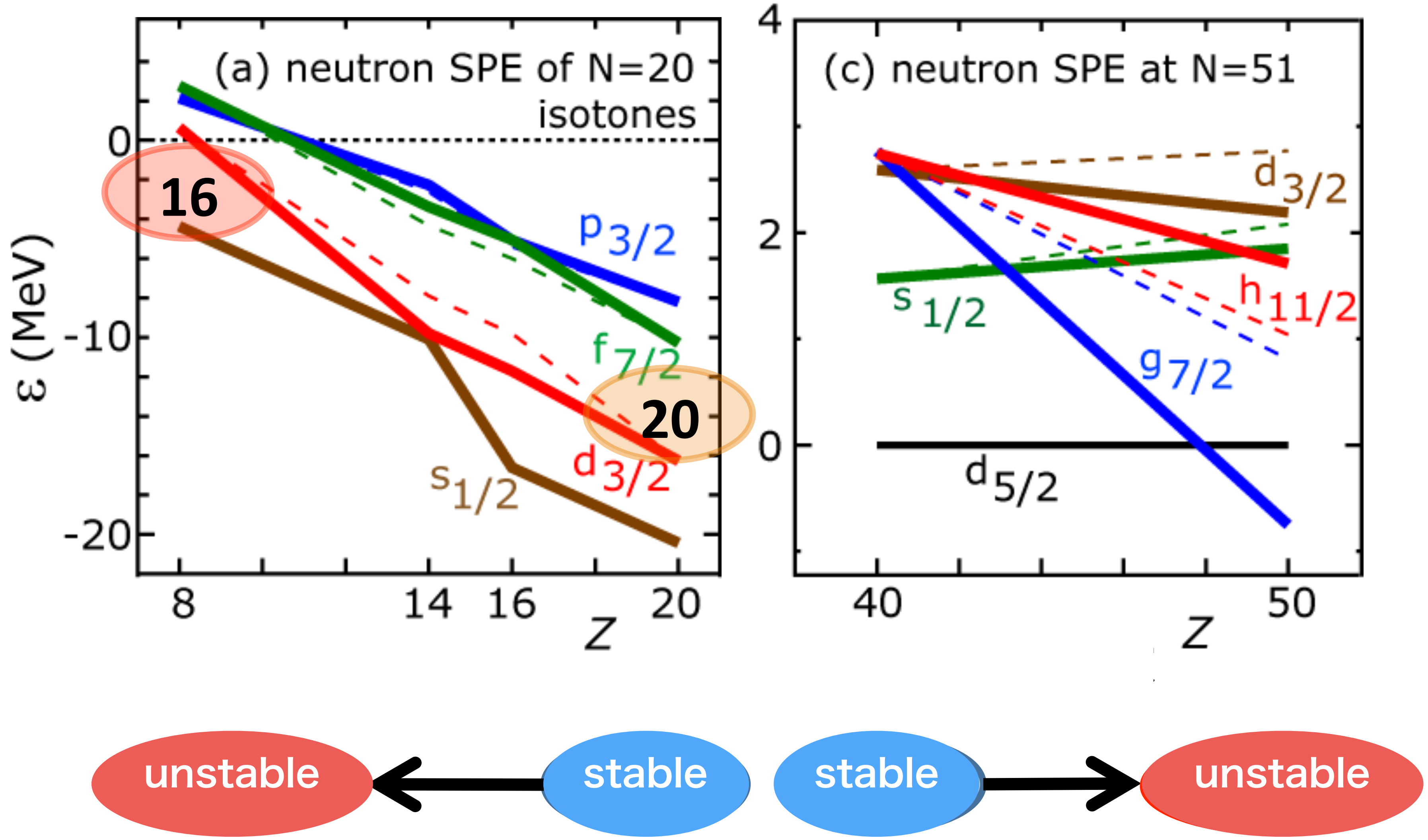
History of shell model and nuclear experiment

numerical capability	shell model		experiment
10^{11}~10^{23}		2010	neutron halo
			neutron/proton
large scale shell model calculation sdp3f7-shell	SDPF-M interaction	2000	drip-line
			nucleosynthesis
	GXPF1 interaction		
10^8 full pf-shell	KB3G interaction		RI beam facilites
full sd-shell	USD interaction		
	KK method	1970	
	Kuo-Brown		
single f7/2 shell			excitation states
	interacting shell model	1960	spin-parity assignment
10~100			......
	independent particle shell model		mass mesurement
	birth of shell model	1950	magnetic moment
			......
	Yukawa theory		
		1930	Invention of Cyclotron
		1910	Discovery of nucleus

Shell evolution

Single particle energies of neutron

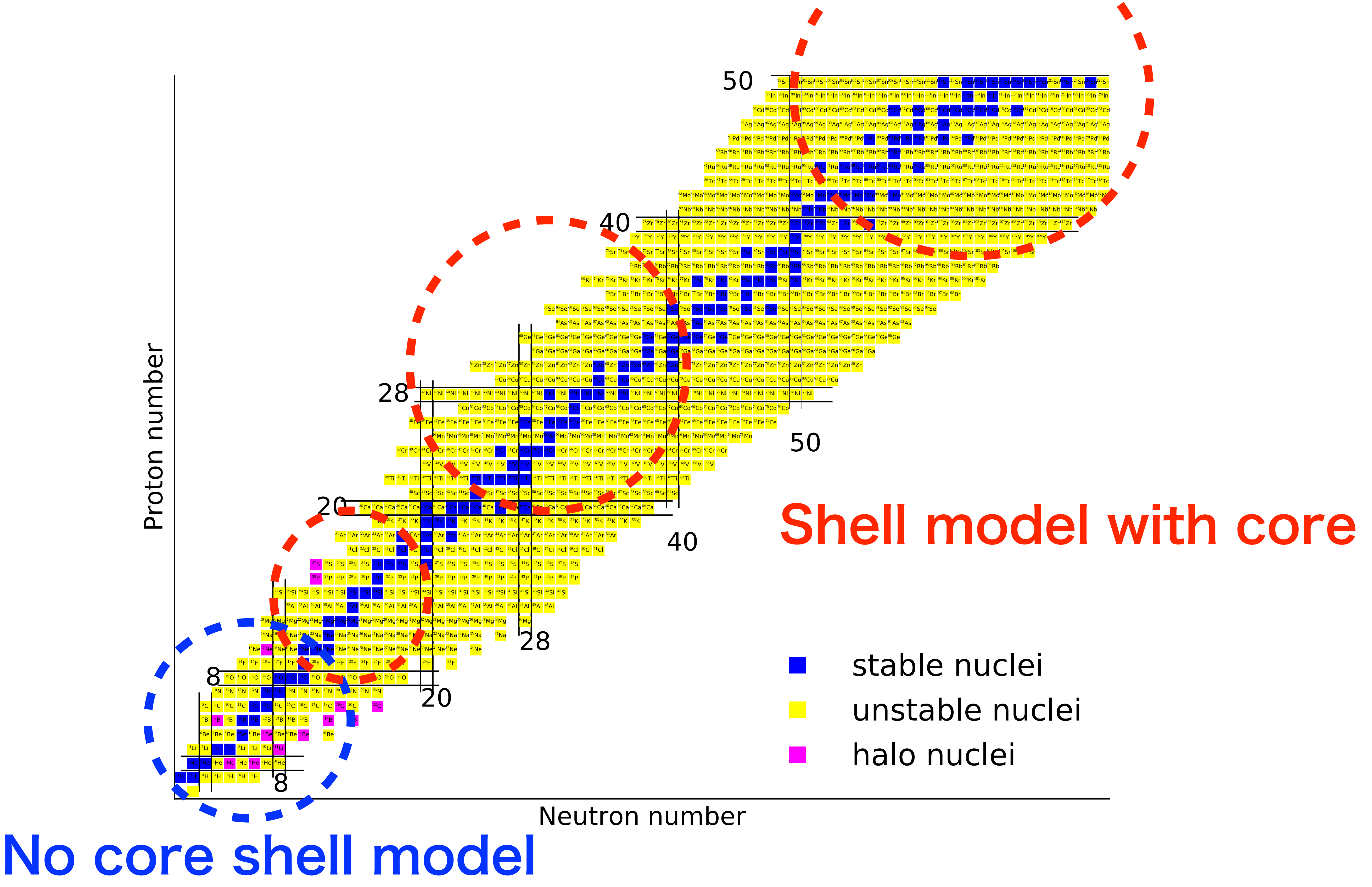
Figure 3 from PRL **104,** 012501 (2010)



Shell structure is changed drastically with the proton number

→ key ingredient is effective nucleon-nucleon interaction

Nuclear chart



Early stage of effective interaction to nuclei

[1] T. T. S. Kuo and G. E. Brown, Nucl. Phys. A 114, 241 (1968).

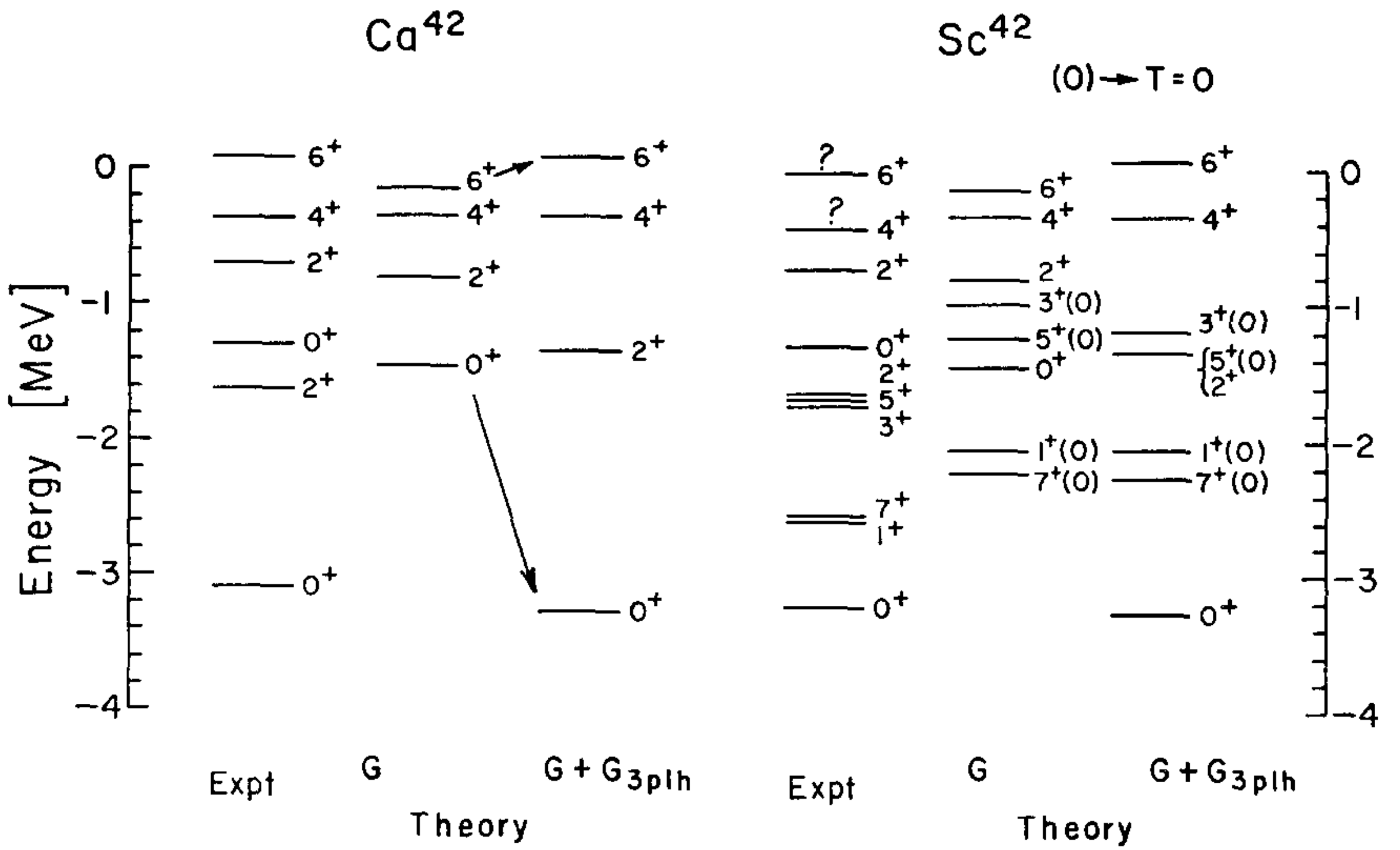


Fig. 3. Energy levels of ^{42}Ca and ^{42}Sc calculated with G and G_{3p1h}. The experimental level schemes ai from refs. [13,14]).

- G-matrix from Hamada-Jonston int.
- 2hw excitation
- 3p1h diagram included

Early stage of effective interaction to nuclei

[1] H. M. Sommermann, et al., Physical Review C 23, 1765 (1981).

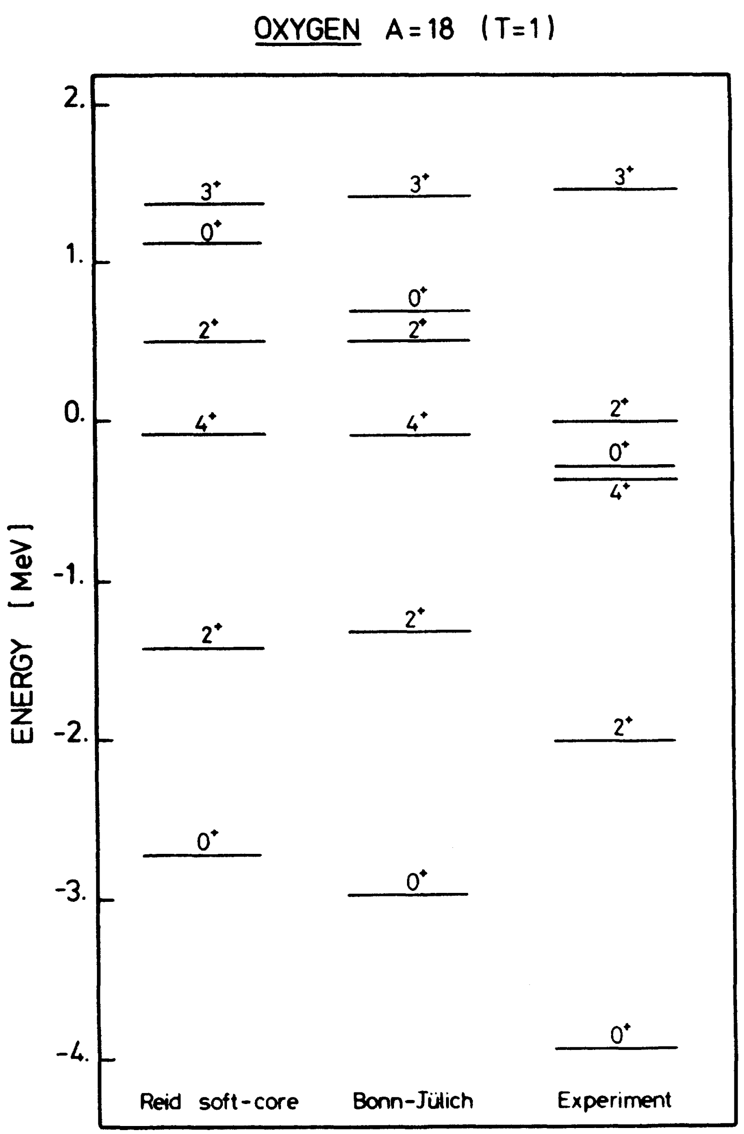


FIG. 6. Spectrum of ^{18}O ($T=1$). This spectrum was calculated with the inclusion of the folded diagrams as well as the contribution from G^{T}_{3p1h} with high momentum intermediate particle states.

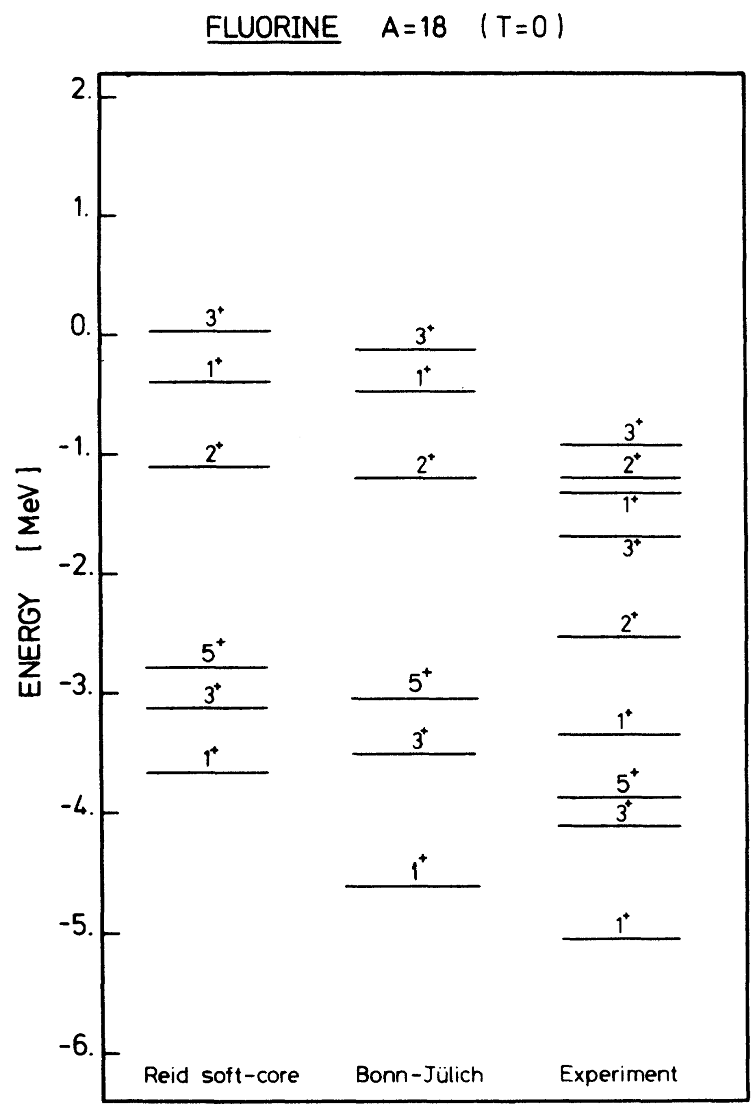
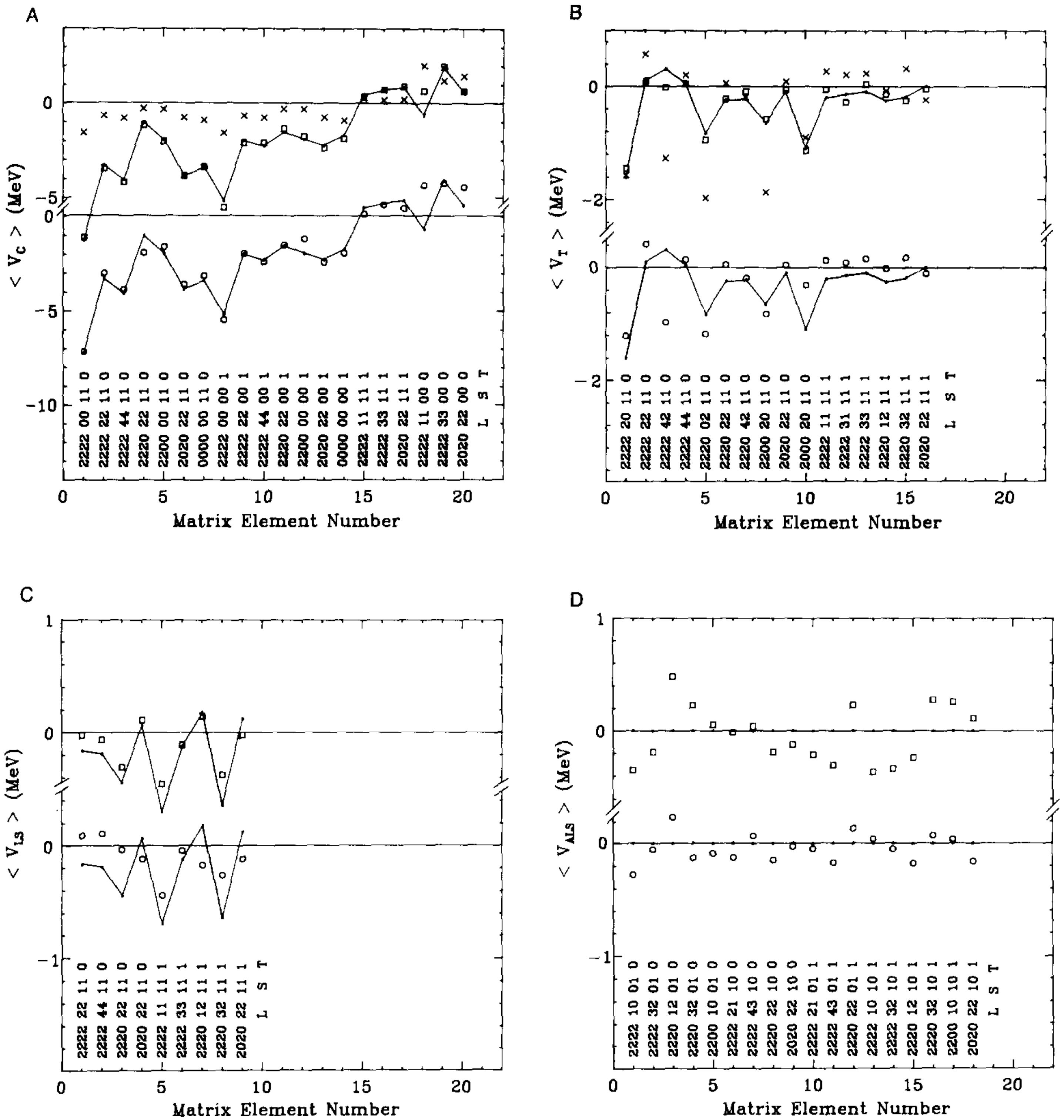


FIG. 7. Spectrum of ^{18}F ($T=0$). See explanation of Fig. 6.

- G-matrix from Hamada-Jonston int.
- 22 hw excitation
- 3p1h diagram included
- with folded diagrams

[1] B. A. Brown et al., Annals of Physics 182, 191 (1988).



- Famous USD interaction
- Kuo-Brown interaction is modified (called renormalized G-matrix)

MBPT for single major shell

[1] M. Hjorth-Jensen et al. , Phys. Rep. 261, 125 (1995).

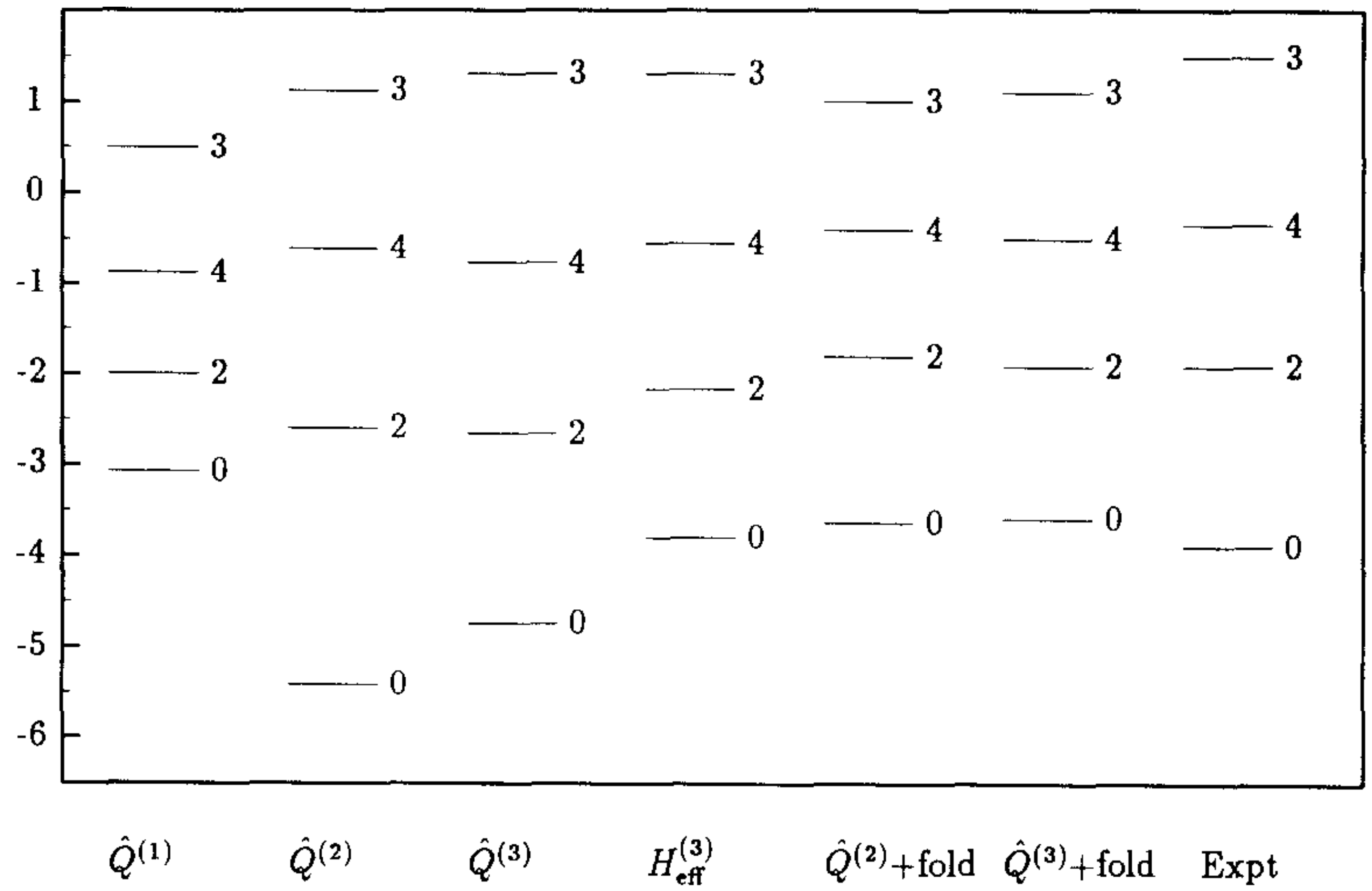


Fig. 42. The low-lying spectra for ^{18}O with the Bonn C potential.

- G-matrix from Bonn A, B, C pot.
- many hw excitation
- 2nd, 3rd order and folded diagrams

Multi-shell effective interactions

[1] Y. Utsuno et al., Phys. Rev.C C60, 054315 (1999).

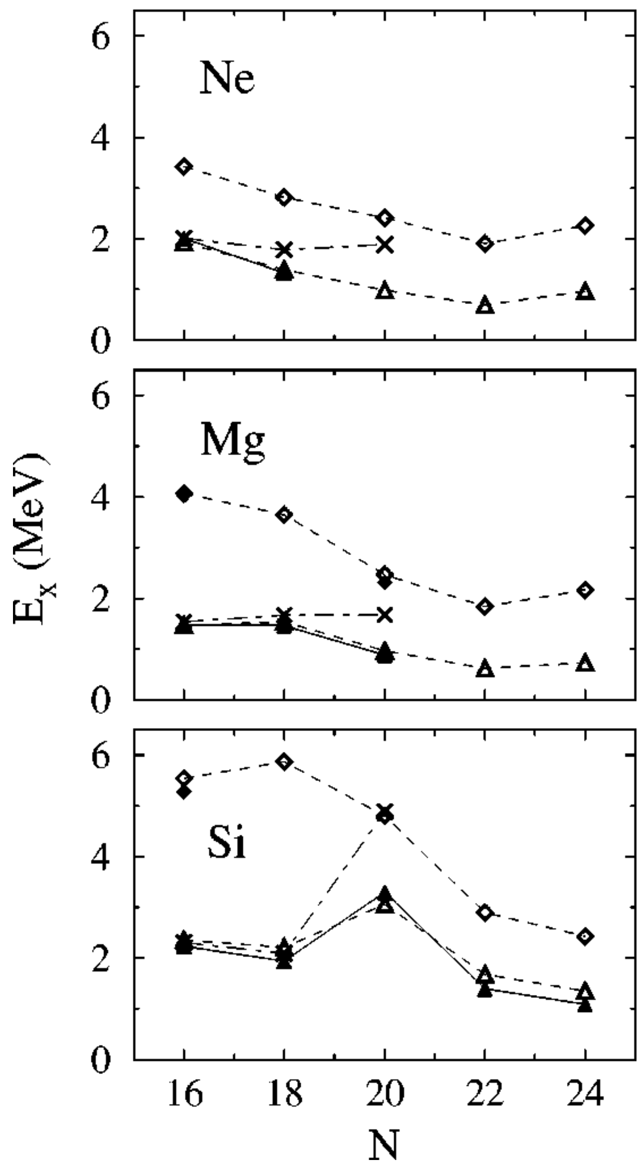


FIG. 6. Yrast levels of Ne (top), Mg (center), and Si (bottom) isotopes. The filled (open) triangles, diamonds are the experimental (calculated) 2_1^+ and 4_1^+ levels, respectively. The crosses mean $E_x(2_1^+)$ calculated by the sd-shell model.

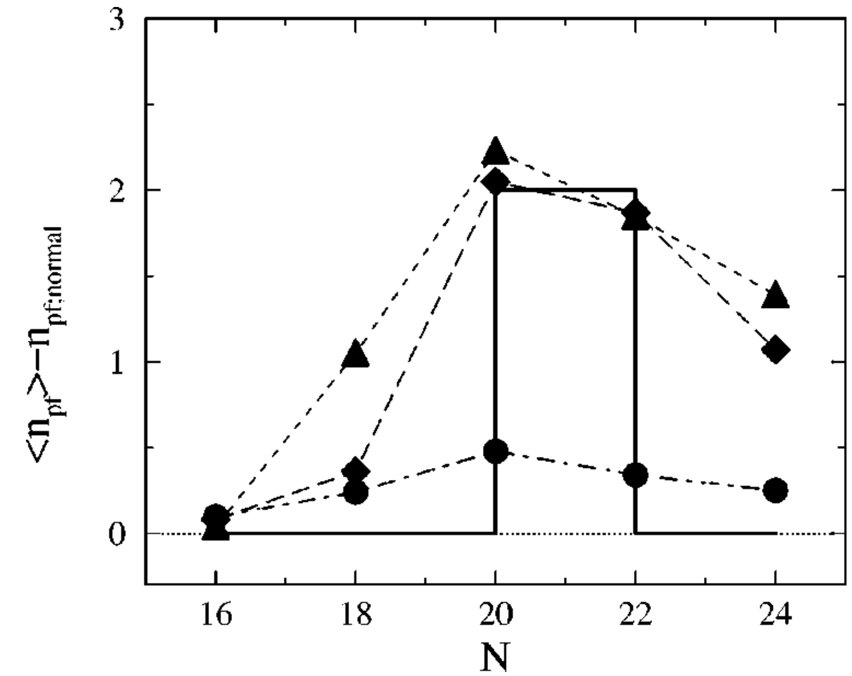


FIG. 8. Average number of neutrons in the pf shell subtracted by the corresponding number in the normal (i.e., filling) configuration. The triangles, diamonds, and circles stand for the values of Ne, Mg, and Si isotopes, respectively. The solid line denotes the corresponding value of Ne and Mg isotopes as predicted by the ''island of inversion'' of [11].

- sdpf-m int.
- island of inversion
- N=20 gap
- sd+pf shell

Multi-shell effective interactions

[1] E. Caurier, et al., Phys. Rev. C 90, 014302 (2014).

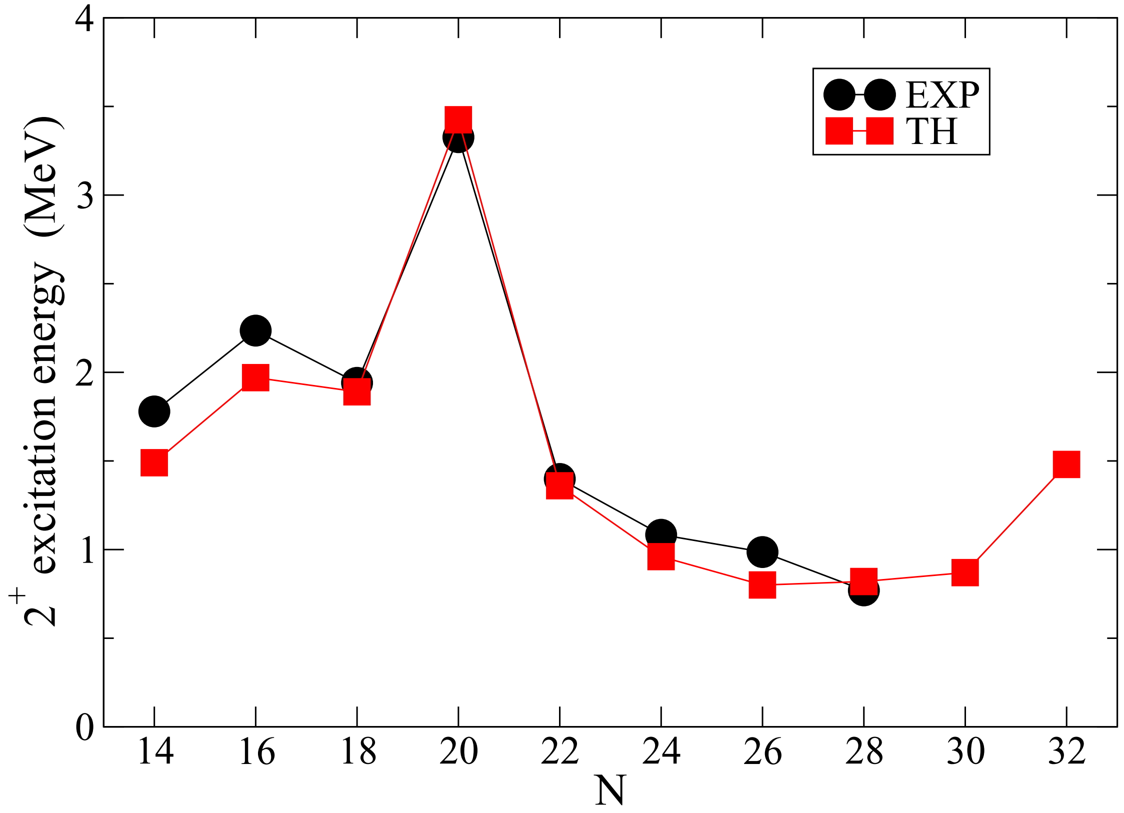


FIG. 11. (Color online) Excitation energies of the first 2^+ states in the silicon isotopes (see caption of Fig. 6).

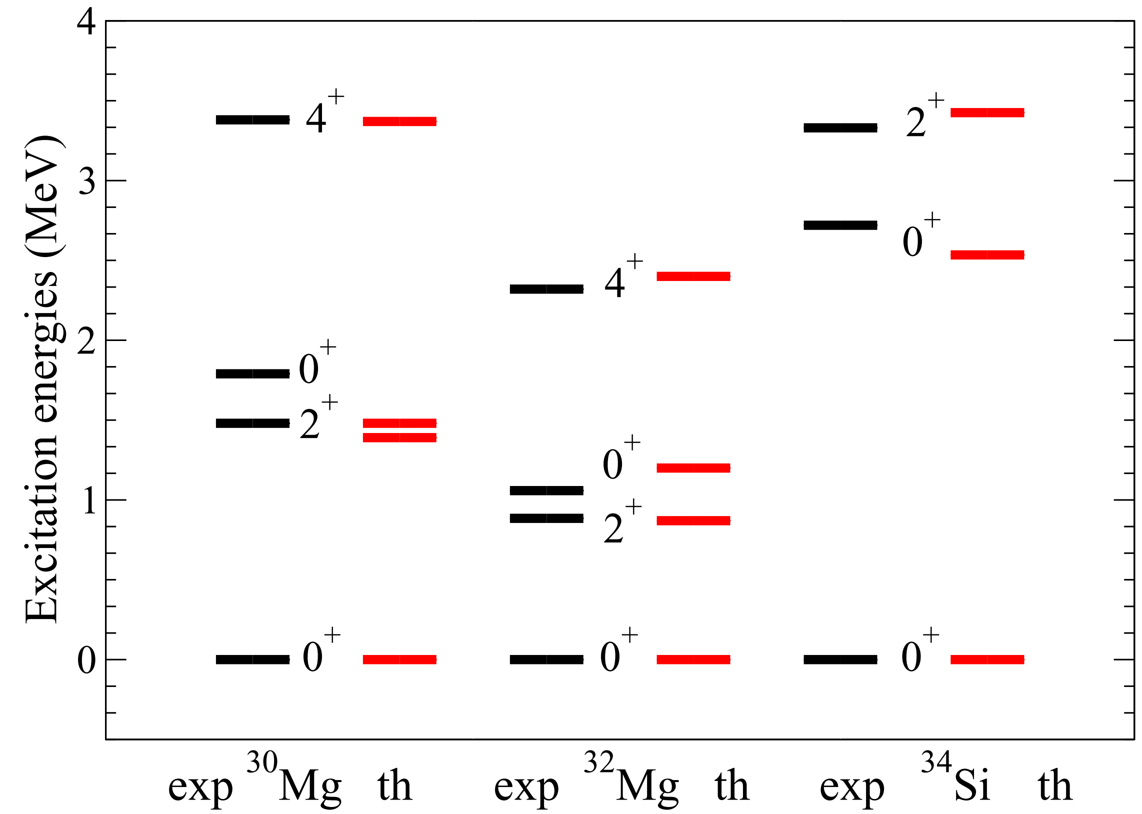
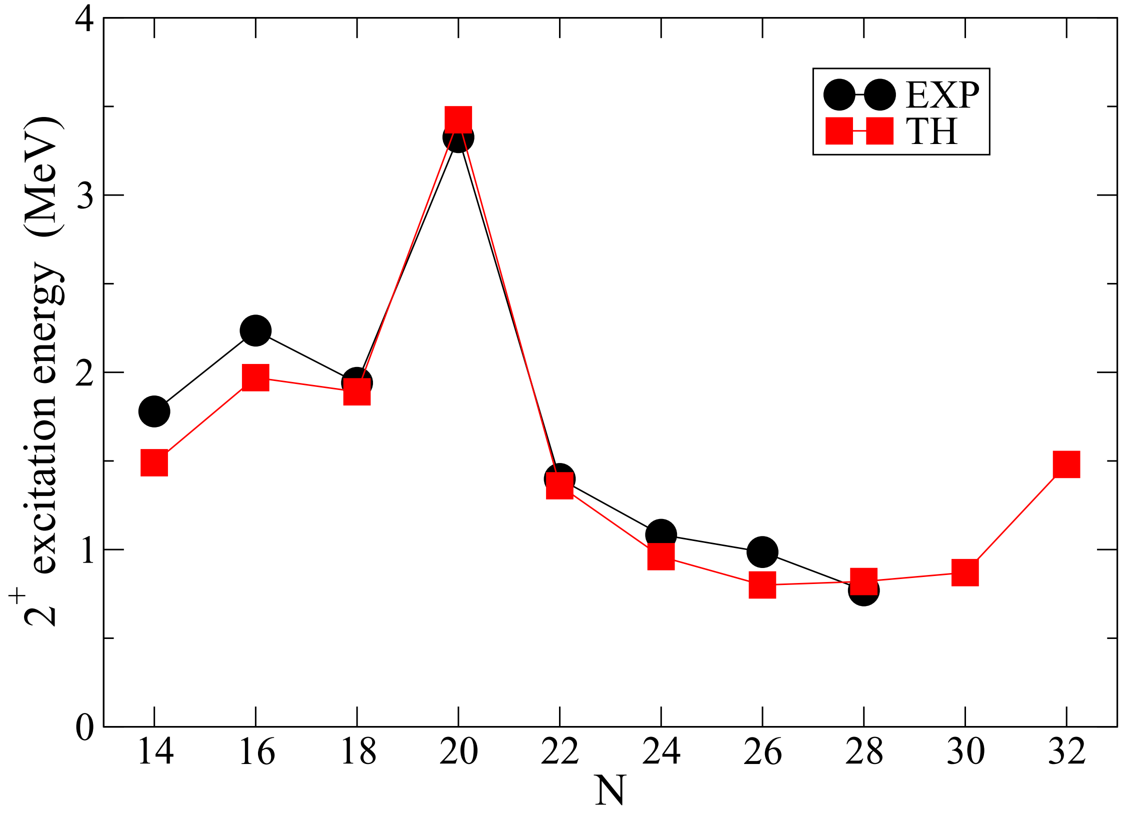


FIG. 12. (Color online) Comparison between experiment and theory for the most important low lying states in ^{30}Mg, ^{32}Mg, and ^{34}Si.

- sdpf-U-mix int.
- merging island of inversion and N=28 gap
- sd+pf shell

Multi-shell effective interactions

[1] E. Caurier, et al., Phys. Rev. C 90, 014302 (2014).



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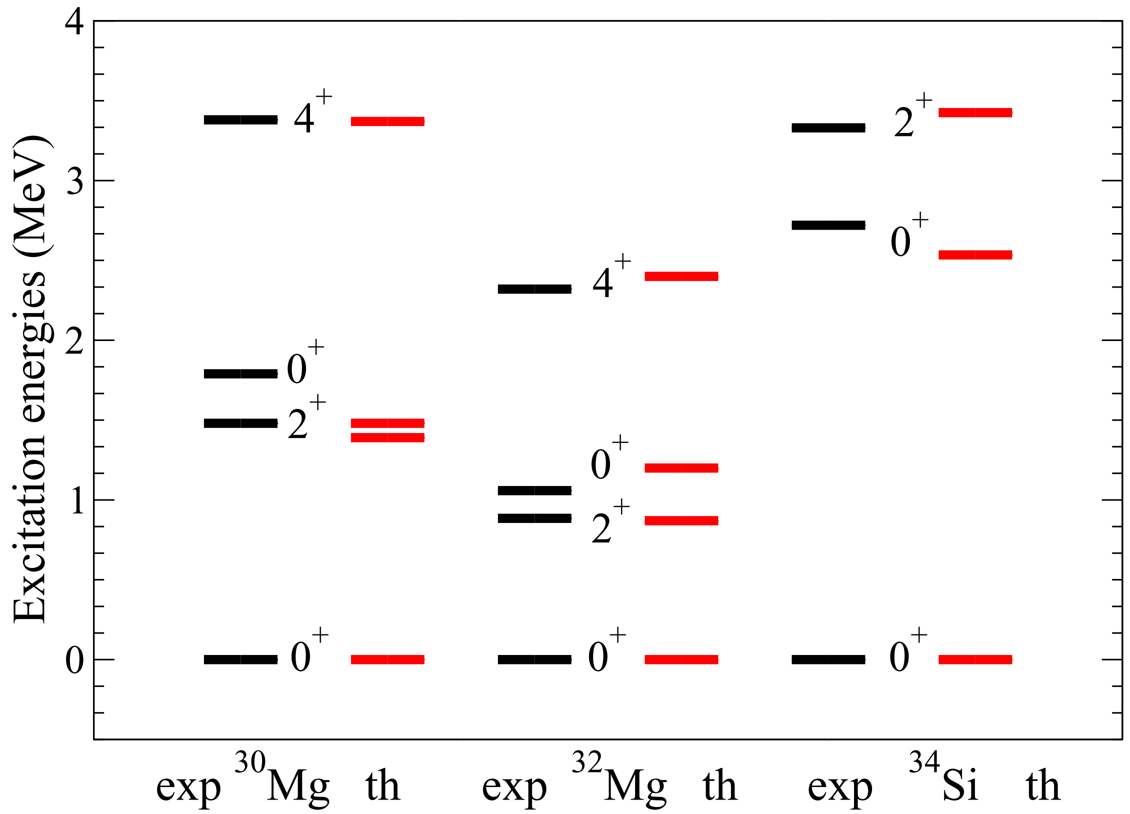


FIG. 12. (Color online) Comparison between experiment and theory for the most important low lying states in ^{30}Mg, ^{32}Mg, and ^{34}Si.

- sdpf-U-mix int.
- merging island of inversion and N=28 gap
- sd+pf shell

Multi-shell effective interactions

[1] Y. Tsunoda et al., Phys. Rev. C 89, 031301 (2014).

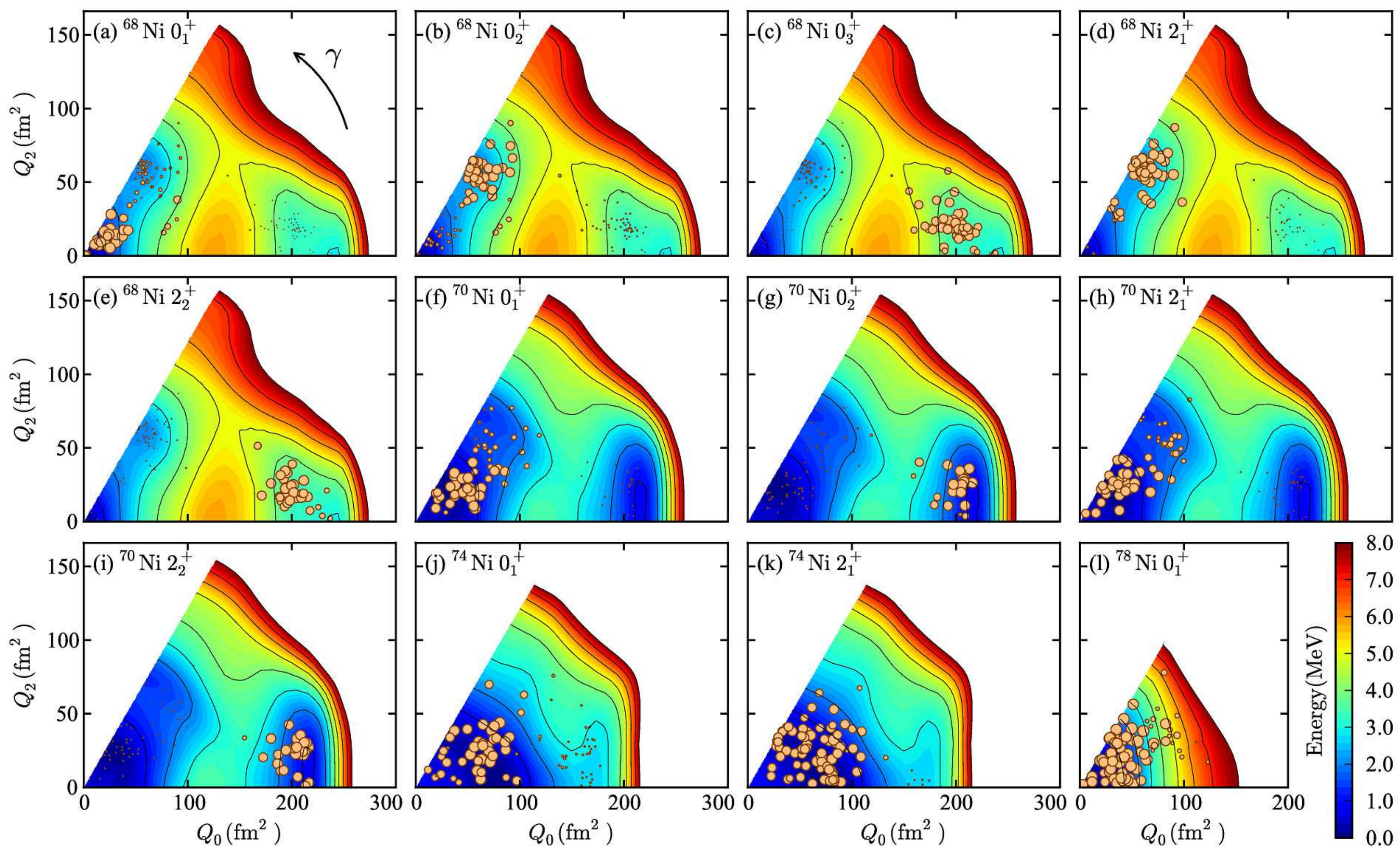


FIG. 3. (Color online) Potential energy surfaces (PES) of Ni isotopes, coordinated by usual Q_0 and Q_2 (or γ). The energy relative to the minimum is shown by contour plots. Circles on the PES represent shapes of MCSM basis vectors (see the text).

- pf-g9/2-d5/2-shell
- Ni isotopes and shape transition
- Invention of T-plot

Multi-shell effective interactions

[1] T. Togashi et al., Phys. Rev. Lett. 117, 172502 (2016).

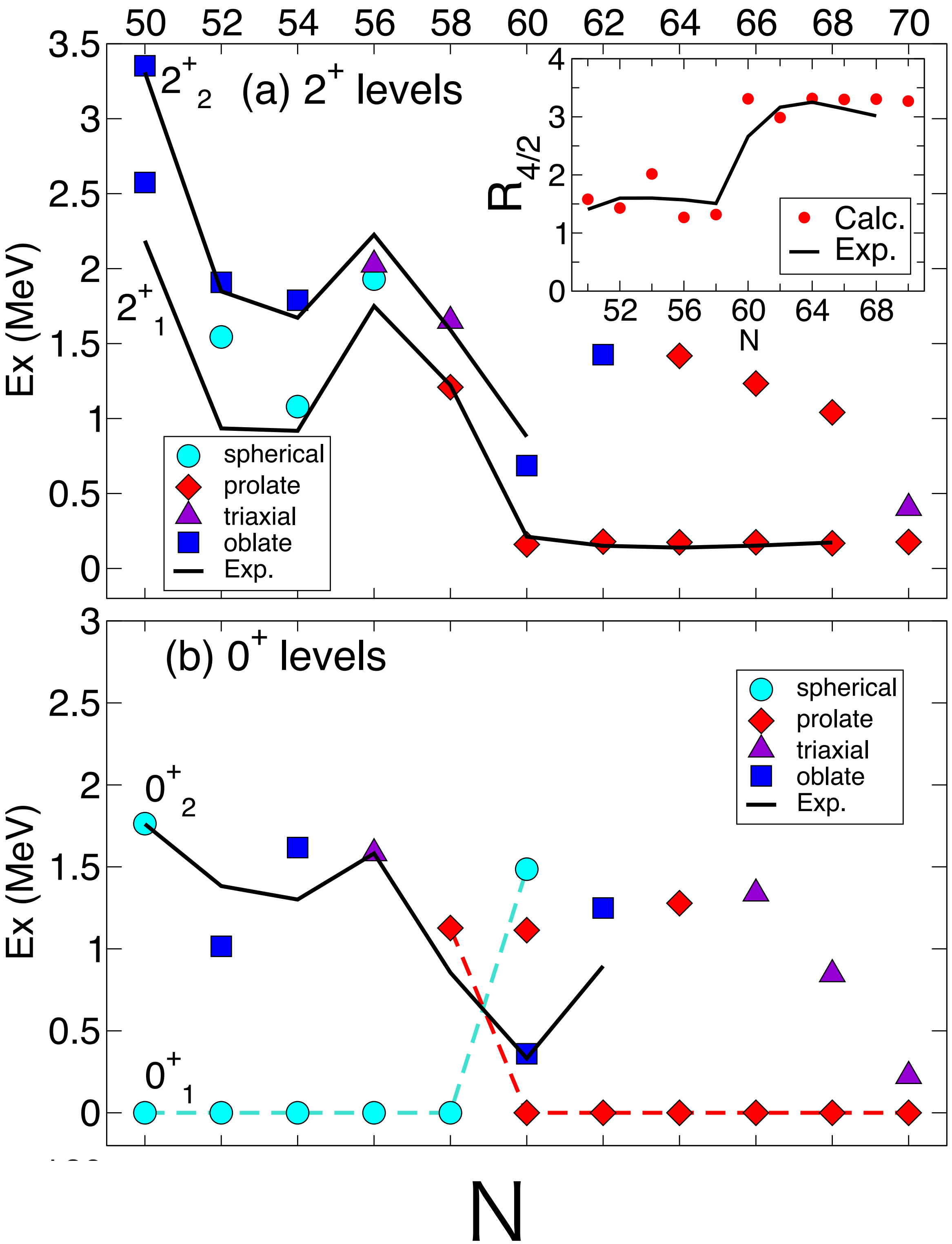


TABLE I. Model space for the shell model calculation.

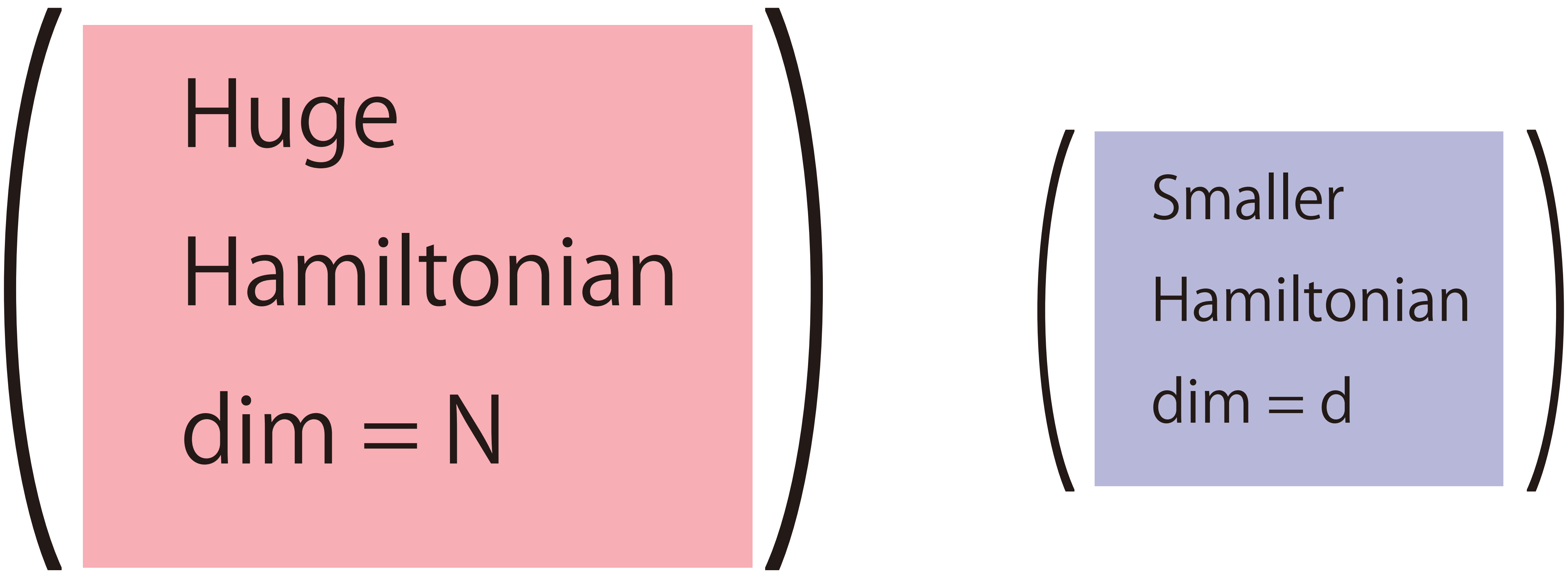
proton orbit	magic number	neutron orbit
-		$1f_{7/2}, 2p_{3/2}$
	82	
- $0g_{7/2}, 1d_{5/2,3/2}, 2s_{1/2}$		$0h_{11/2}$ $0g_{7/2}, 1d_{5/2,3/2}, 2s_{1/2}$
	50	
$0g_{9/2}$ $0f_{5/2}, 1p_{3/2,1/2}$		$0g_{9/2}$ -

- Large model space
- sudden change of 2+ and 0+ level w.r.t. N
- Quantum phase transition

Shell model effective interactions

- Attempt to obtain effective interactions for the shell model calculation first done by Kuo and Brown in 1968.
- There exists many effective interactions for shell model calculation
- Recent calculations mainly based on the effective interaction obtained by fit
- Fitting usually done on top of so-called G-matrix or MBPT.

What is “effective” interaction



- Solve same physics for chosen degrees of freedom
- Same d eigen values and eigen vectors
- Typically low energy physics

What is “effective” interaction?

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 5 \end{pmatrix}$$ 3x3 matrix

Eigenvalues and eigenvectors

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} -1.08 \\ 0.88 \\ 5.19 \end{pmatrix} \quad \begin{pmatrix} 0.92 \\ -1.00 \\ 0.038 \end{pmatrix} \begin{pmatrix} -1.00 \\ -0.88 \\ 0.20 \end{pmatrix} \begin{pmatrix} 0.16 \\ 0.22 \\ 1.00 \end{pmatrix}$$

Suppose if we are interested in only lowest two states.

Projected matrix

P-space $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

$$PVP = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} -1.0 \\ 1.0 \end{pmatrix} \quad \begin{pmatrix} 1.0 \\ -1.0 \end{pmatrix} \begin{pmatrix} -1.0 \\ -1.0 \end{pmatrix}$$

Different eigenvalues and eigenenergies

What is "effective" interaction?

Then, what is the interaction which yields the same eigenvalues and eigenvectors?

$$\begin{pmatrix} 1.2 * 10^{-5} & 1 - 1.5 * 10^{-5} \\ 1 - 0.48 * 10^{-1} & -0.19 \end{pmatrix}$$ 2x2 matrix

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} -1.08 \\ 0.88 \end{pmatrix} \quad \begin{pmatrix} 0.92 \\ -1.00 \end{pmatrix} \begin{pmatrix} -1.00 \\ -0.88 \end{pmatrix}$$

<u>Same eigenvalues and eigenvectors for lowest 2 states.</u>

—> different from PVP, but quite similar.

What was the point?

Separation of the scale

Orignial matrix

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 5 \end{pmatrix}$$

Renormalized matrix

$$\begin{pmatrix} 1.2 * 10^{-5} & 1 - 1.5 * 10^{-5} \\ 1 - 0.48 * 10^{-1} & -0.19 \end{pmatrix}$$

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} -1.08 \\ 0.88 \\ 5.19 \end{pmatrix}$$

P-space

Q-space

Separation of the scale

- If the P-space are well separated from Q-space, renormalized matrix is not far from PVP
- Non hermiticity is not large

Model space, P-space

Hamiltonian

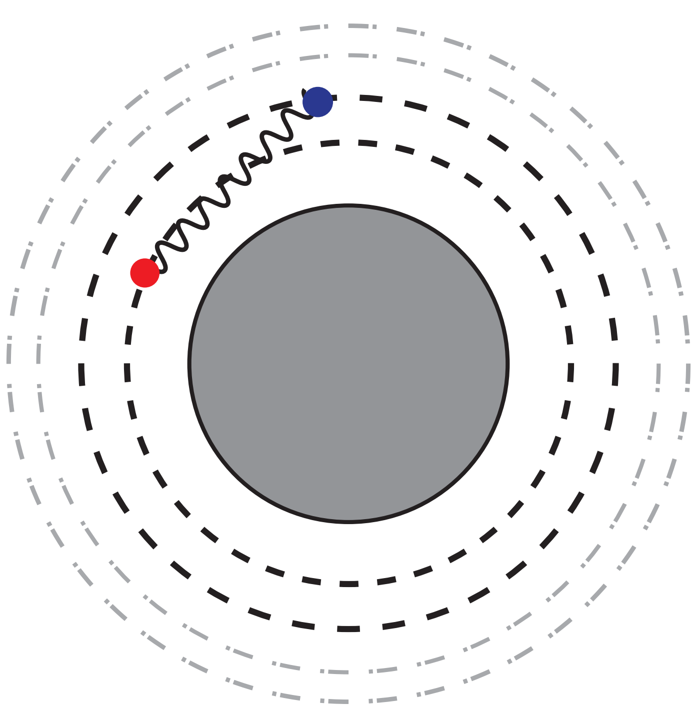
$$H = H_0 + V, \qquad H_0|\phi_i\rangle = E_i|\phi_i\rangle$$

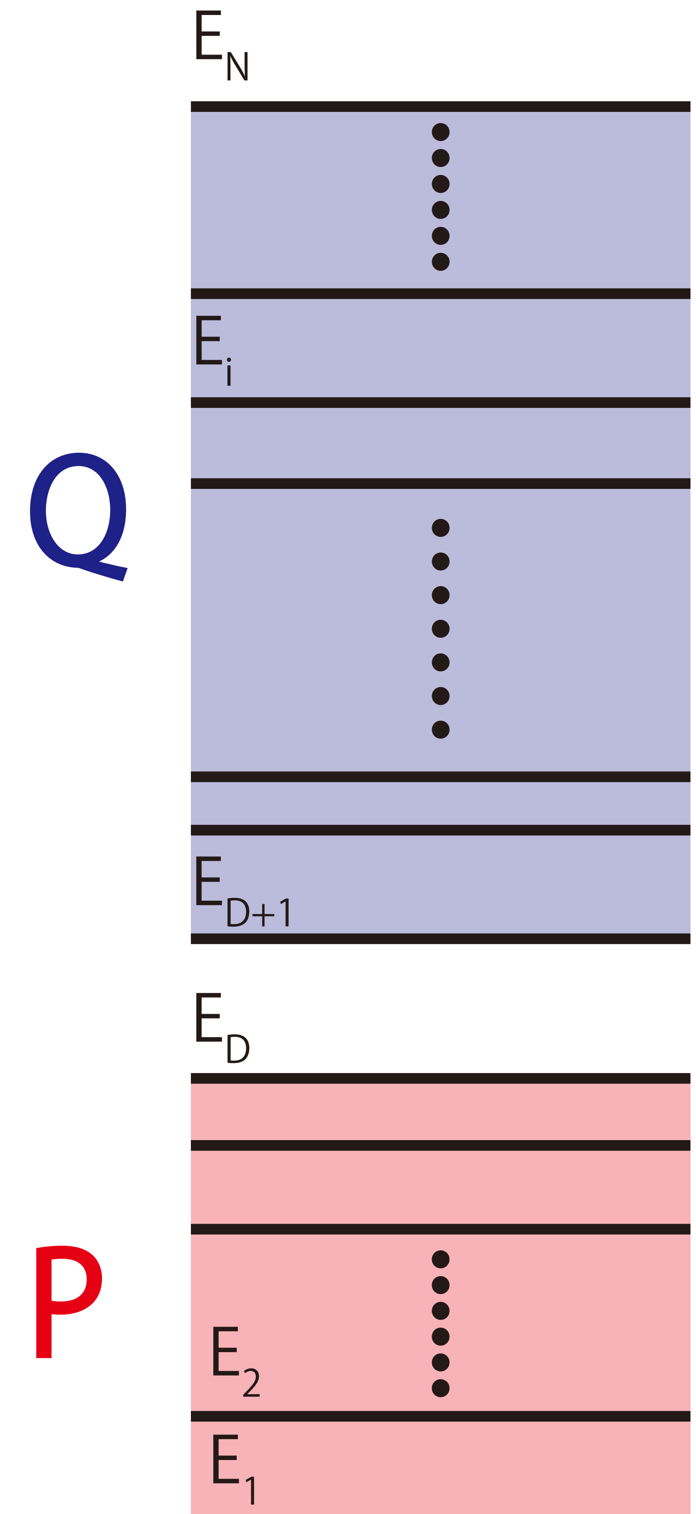
Projection operator

$$[P, H_0] = [Q, H_0] = 0. \qquad P^2 = P, \quad Q^2 = Q$$

$$PQ = QP = 0,$$

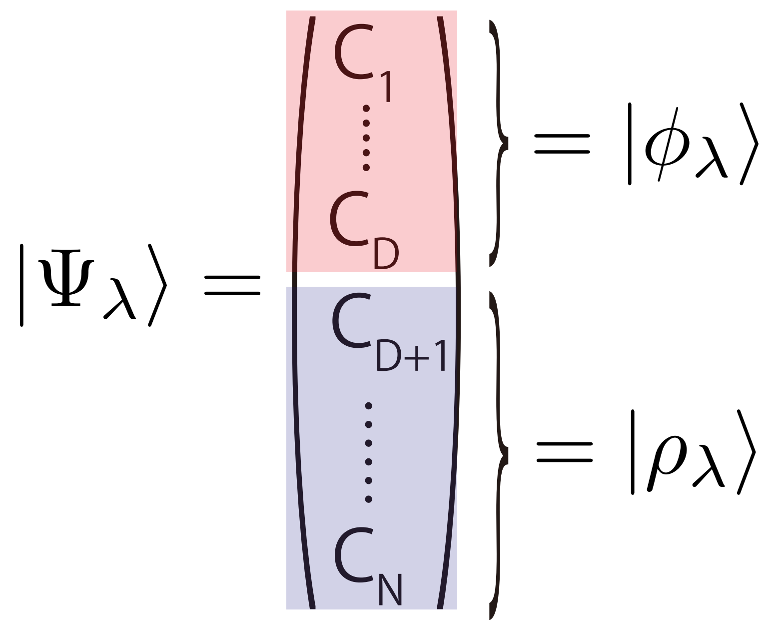
$$[P, Q] = 0.$$





Easy way to formulate

$$\begin{pmatrix} PHP & PVQ \\ QVP & QHQ \end{pmatrix}\begin{pmatrix} |\phi_\lambda\rangle \\ |\rho_\lambda\rangle \end{pmatrix} = E_\lambda \begin{pmatrix} |\phi_\lambda\rangle \\ |\rho_\lambda\rangle \end{pmatrix},$$

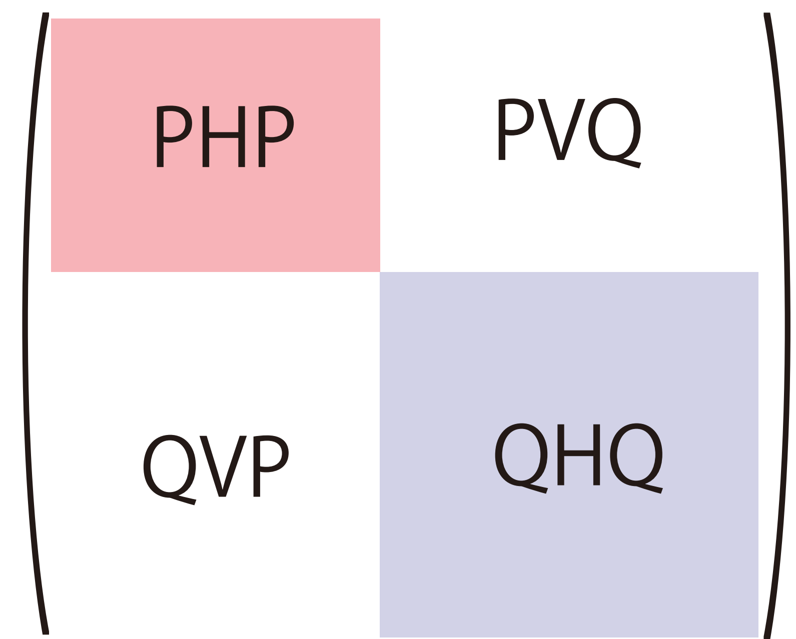


$$|\rho_\lambda\rangle = (E_\lambda - QHQ)^{-1} QVP|\phi_\lambda\rangle$$

$$|\phi_\lambda\rangle = (E_\lambda - PHP)^{-1} PVQ|\rho_\lambda\rangle.$$

$$\left(PHP - \frac{1}{E_\lambda - QHQ} QVP\right)|\phi_\lambda\rangle = E_\lambda|\phi_\lambda\rangle$$

$$\left(QHQ - \frac{1}{E_\lambda - PHP} PVQ\right)|\rho_\lambda\rangle = E_\lambda|\rho_\lambda\rangle.$$



Bloch-Horowitz Hamiltonian

$$H_{\rm BH}(E) = PHP + PVQ \frac{1}{E - QHQ} QVP.$$

$$H_{\rm BH}(E_\lambda)|\phi_\lambda\rangle = E_\lambda|\phi_\lambda\rangle, \quad \lambda = 1, \cdots, D.$$

<u>Unsatisfactory because H_{BH} depends on E.</u>

Energy independent Heff

What we want to know is energy independent Hamiltonian which satisfies

$$H_{\text{eff}}|\phi_i\rangle = E_i|\phi_i\rangle, \quad i = 1, \cdots, d.$$

IF we know true eigenstates and eigenenegies, we can formulate Heff immediately as follows,

$$H_{\text{eff}} = \sum_{i=1}^{d} |\phi_i\rangle E_i \langle\tilde{\phi}_i|, \qquad \langle\tilde{\phi}_i|\phi_j\rangle = \delta_{ij}$$

Bi-orthogonal basis is used because $|\phi>$ does not span whole P-space in general.

-> non Hermitian Heff (more later)

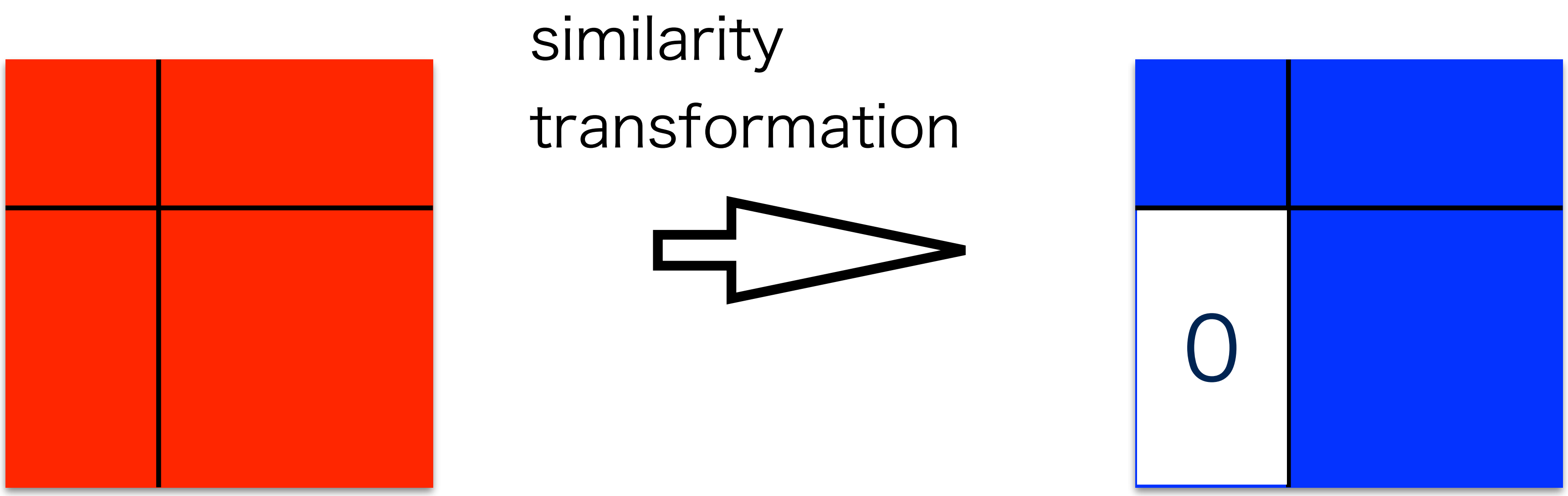
How to find the Heff : decoupling equation

Similarity transformation

$$\mathcal{H} = e^{-\omega} H e^{\omega}, \quad Q\omega P = \omega.$$

Decoupling condition

$$0 = Q\mathcal{H}P = QVP - \omega PHP + QHQ\omega - \omega PVQ\omega,$$



$$H_{\text{eff}} = P\mathcal{H}P$$

$$V_{\text{eff}} = PVP + PVQ\omega.$$

Next: Solve non-linear equation.

Formal solution of decoupling equation (KK method)

Assumption: the model space is degenerate

$$PH_0P = \epsilon_0 P.$$

Decoupling equation

$$0 = Q\mathcal{H}P = QVP - \omega PHP + QHQ\omega - \omega PVQ\omega,$$

A solution for this equation

$$(\epsilon_0 - QHQ)\omega = QVP - \omega PVP - \omega PVQ\omega.$$

$$\omega = \frac{1}{\epsilon_0 - QHQ}\left(QVP - \omega\left(PVP + PVQ\omega\right)\right)$$
$$= \frac{1}{\epsilon_0 - QHQ}\left(QVP - \omega V_{\mathrm{eff}}\right),$$

Solve this by iteration

Q-box:

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP,$$

$$\hat{Q}_k(E) = \frac{1}{k!}\frac{\mathrm{d}^k\hat{Q}(E)}{\mathrm{d}E^k}.$$

→

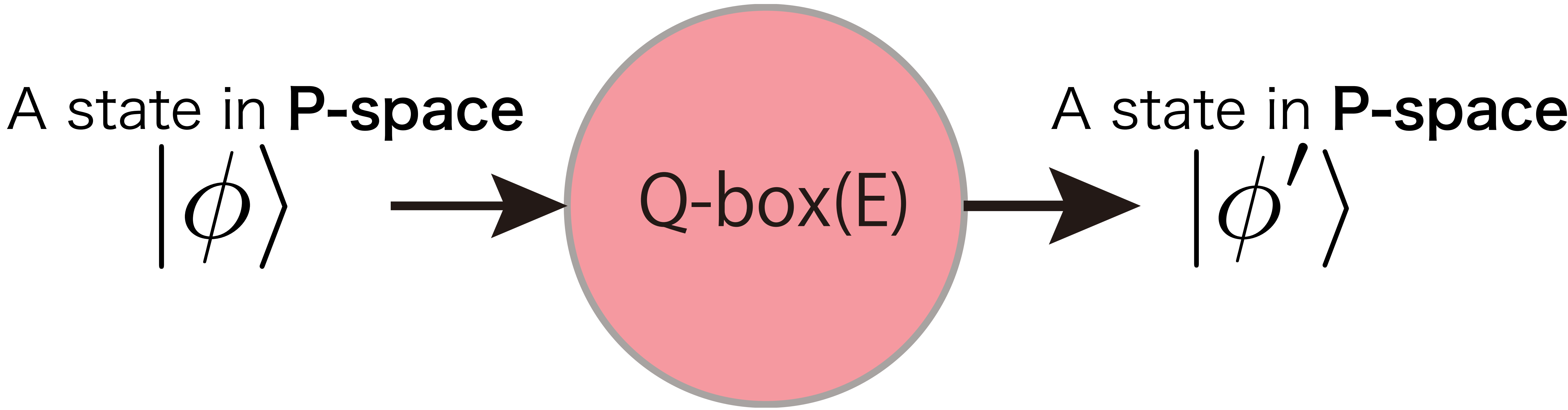
$$V_{\mathrm{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty}\hat{Q}_k(\epsilon_0)\{V_{\mathrm{eff}}^{(n-1)}\}^k.$$

Iterative equation for deriving the Effective interaction for degenerate model space

What is Q-box ?

$$V_{\text{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\text{eff}}^{(n-1)}\}^k.$$

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP,$$



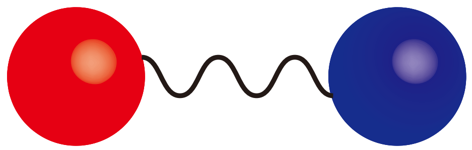
Something complicated happening

in **Q-space** with energy E

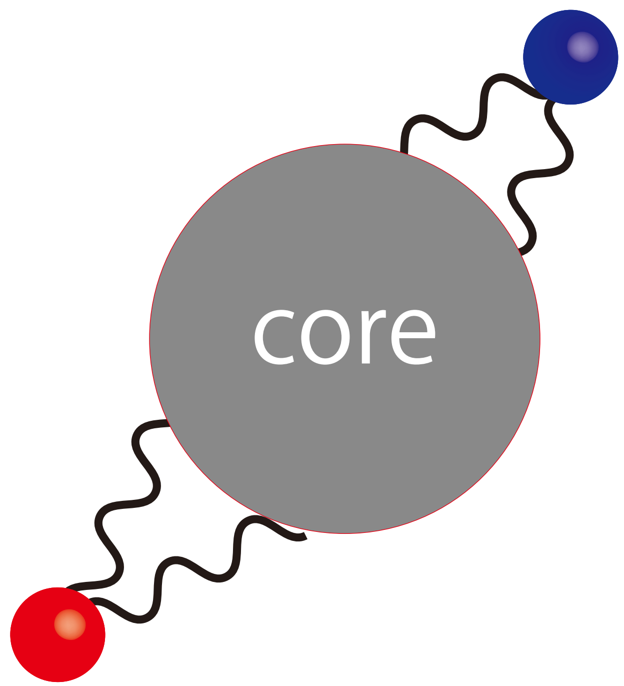
Next: Then, what is the "complicated" stuff and its derivatives ??

Conceptual drawing of Heff

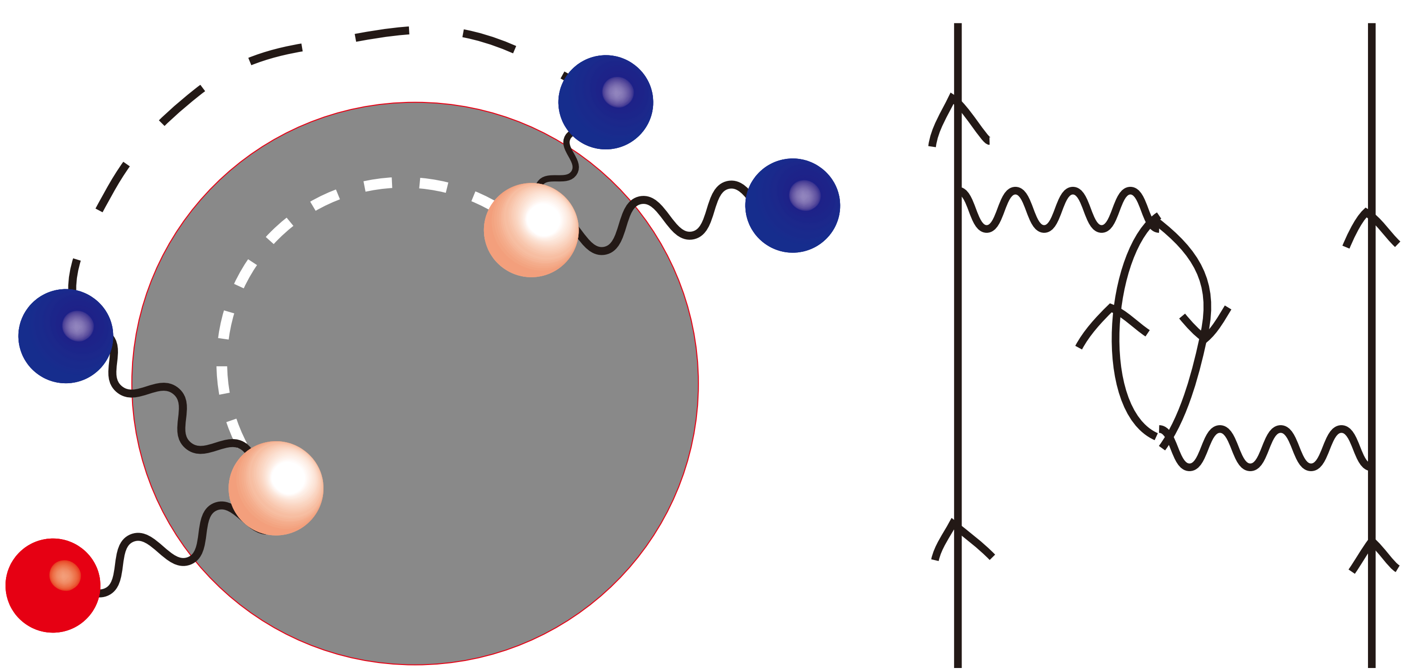
Nuclear force in vacuum



in-medium correction

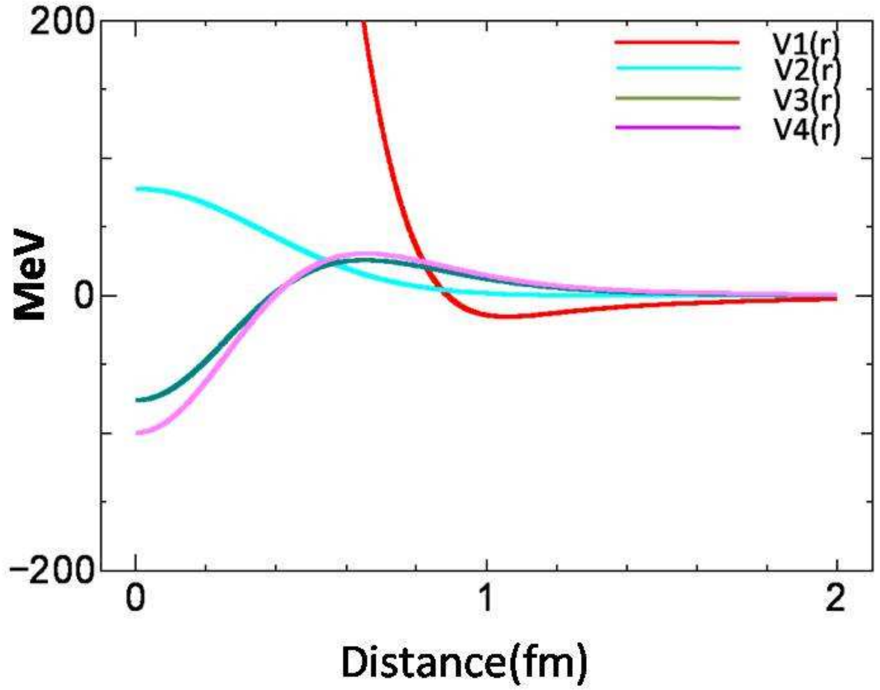


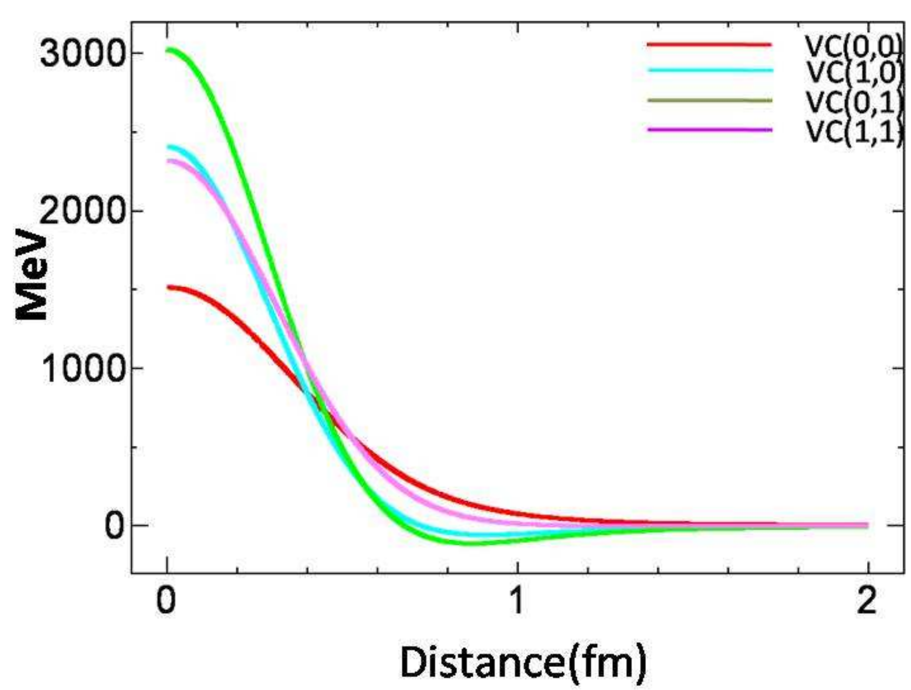
core polarization (3p1h)



Nuclear force

Central force of AV8' potential





$$V_C = v_1(r) + (\sigma_1 \cdot \sigma_2)v_2(r) + (\tau_1 \cdot \tau_2)v_3(r) + (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)v_4(r).$$

- Strong short range repulsion.
- High-momentum component.
- Can it be renormalized to low momentum int.?

Vlowk interaction

Lippmann-Schwinger equation

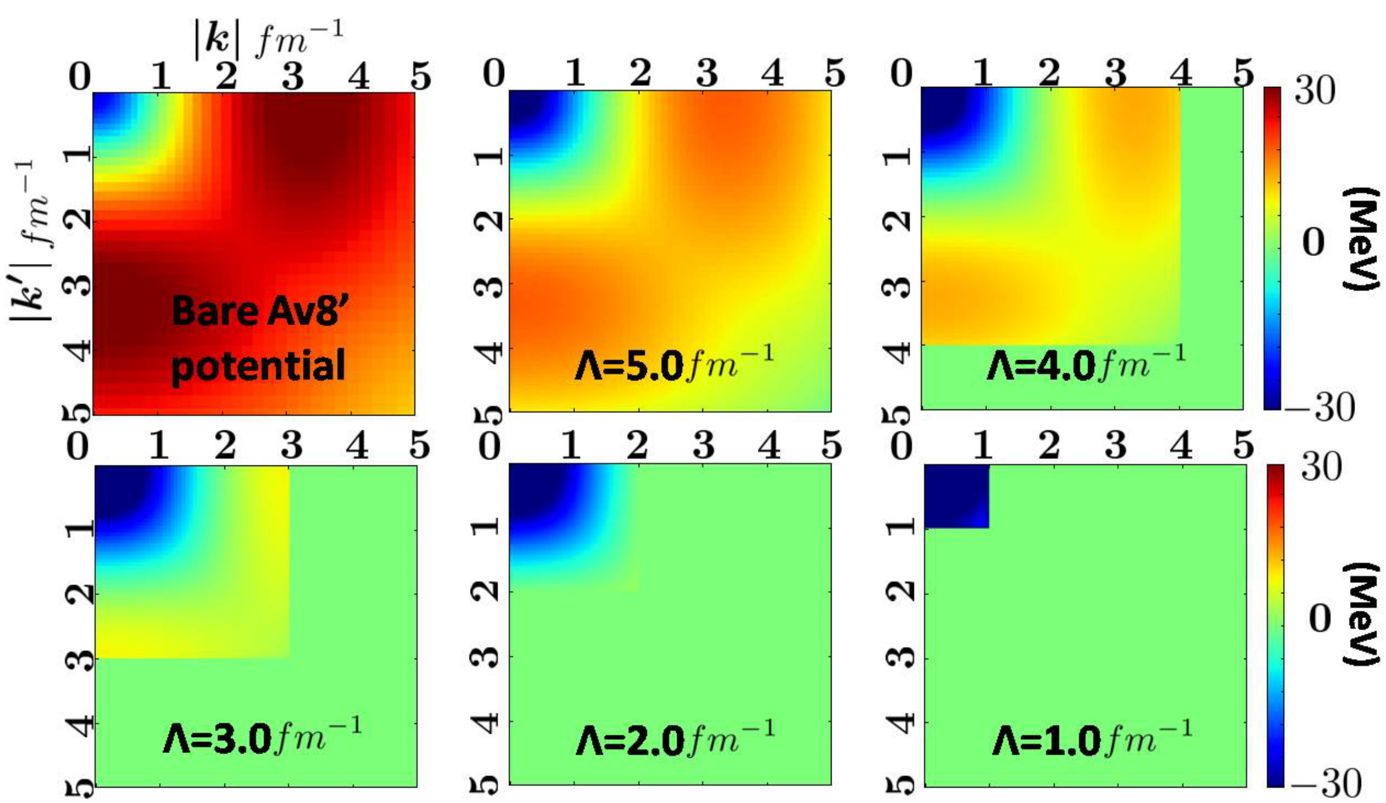
$$T(k',k;k^2) = V_{NN}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^\infty \frac{V_{NN}(k',p)T(p,k;k^2)}{k^2-p^2}p^2\mathrm{d}p$$

Low-momentum interaction V_{lowk} which preserves the HOS T-matrix

$$T(k',k;k^2) = V_{\mathrm{low}k}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^\Lambda \frac{V_{\mathrm{low}k}(k',p)T(p,k;k^2)}{k^2-p^2}p^2\mathrm{d}p.$$

RG equation of V_{lowk} equation with respect to cutoff parameter Λ

$$\longrightarrow \quad \frac{\mathrm{d}V_{\mathrm{low}k}(k',k)}{\mathrm{d}\Lambda} = \frac{2}{\pi}\frac{V_{\mathrm{low}k}(k',\Lambda)T(\Lambda,k;\Lambda^2)}{1-(k^2/\Lambda^2)}.$$



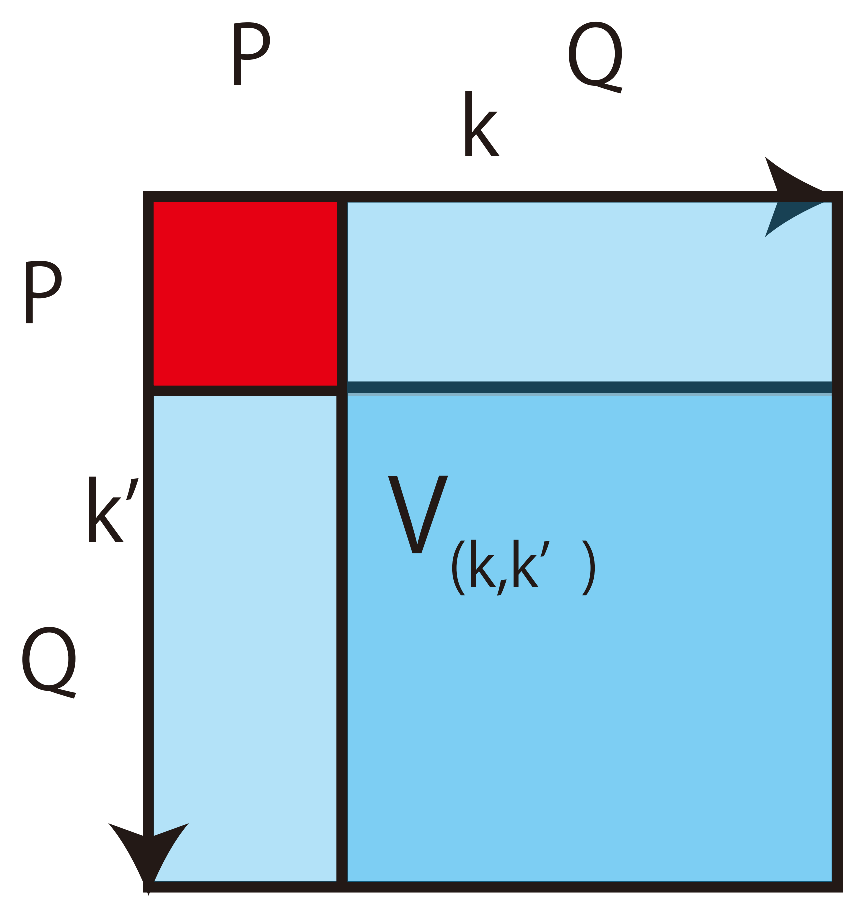
- Decouple low-momentum and high-momentum part
- Preserve physical observable and low-momentum wave function
- Remove repulsive core

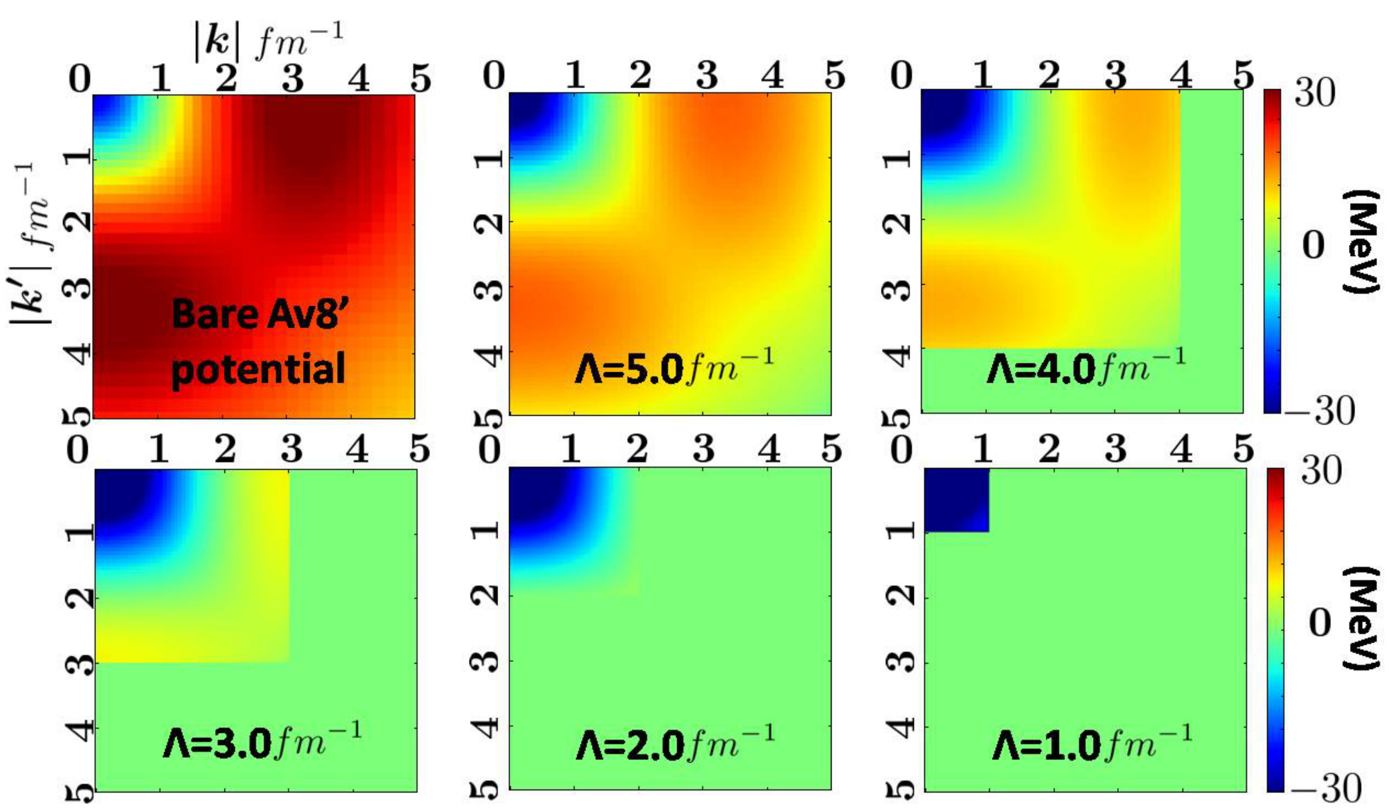
Implementation of Vlowk interaction

Usu this equation instead of RG equation

$$H_{\text{eff}} = \sum_{i=1}^{d} |\phi_i\rangle E_i \langle\tilde{\phi}_i|, \qquad \langle\tilde{\phi}_i|\phi_j\rangle = \delta_{ij}$$

$$V_{lowk} = \sum_{k<k_F} |\phi_k\rangle E_k \langle\tilde{\phi}_k| - T$$





Non hermiticity

$$V_{\text{eff}} = PVP + PVQ\omega.$$

$$(P + \omega^{\dagger}\omega)H_{\text{eff}} = H_{\text{eff}}^{\dagger}(P + \omega^{\dagger}\omega).$$

Cholesky decomposition: lower triangular matrix L

$$P + \omega^{\dagger}\omega = LL^{\dagger}.$$

$$L^{-1}H_{\text{eff}}^{\dagger}L = L^{\dagger}H_{\text{eff}}(L^{\dagger})^{-1} = \left(L^{-1}H_{\text{eff}}^{\dagger}L\right)^{\dagger}$$

$$H_{\text{eff}}^{her} = L^{\dagger}H_{\text{eff}}(L^{\dagger})^{-1}.$$

If ω is enough small, $H^{her}_{eff} \fallingdotseq H_{eff}$

Day 2

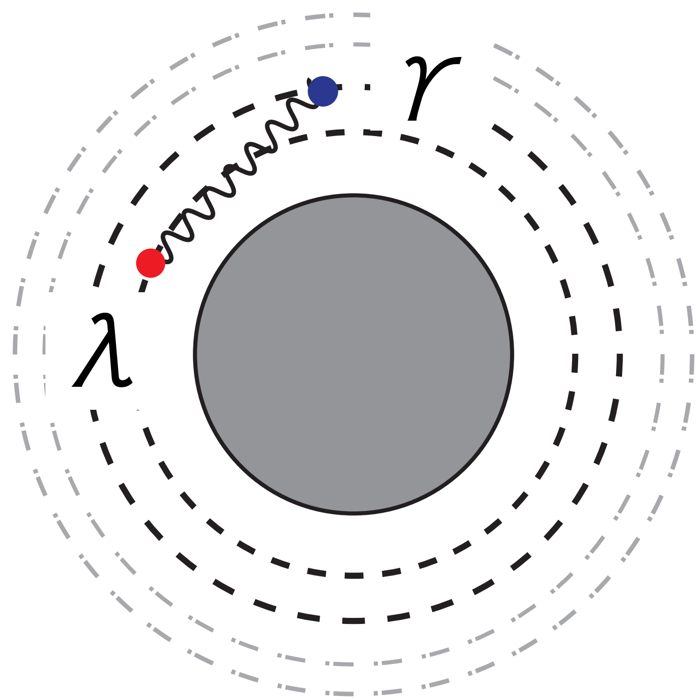
Many body problem

Hamiltonian in second quantized form

$$H = H_0 + V$$
$$= \sum \epsilon_\alpha a^\dagger_\alpha a_\alpha + \frac{1}{2}\sum V_{\alpha\beta,\gamma\delta} a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma,$$

$$P = \sum_{i}^{d} |\phi_i\rangle\langle\phi_i|$$

$$|\phi_i\rangle = \sum a^\dagger_\lambda a^\dagger_\gamma |c\rangle.$$



In Schrodinger picture

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

Interaction picture

Interaction picture is suitable for perturbation theory

$$H_1^I(t) = e^{-iH_0 t} H_1 e^{iH_0 t}$$

$$|\Psi^I(t)\rangle = e^{-iH_0 t}|\Psi^S(t)\rangle$$

$$\hat{O}^I(t) = e^{-iH_0 t} \hat{O} e^{iH_0 t},$$

Heisenberg formula

$$\frac{d}{dt}\hat{O}^I(t) = -i[H_0, O]$$

Creation and annihilation operators

$$\frac{d}{dt} a_i^+(t) = -i[\sum \epsilon_\alpha a_\alpha^+ a_\alpha, a_i^+]$$
$$= i\epsilon_i a_i^+$$

$$a_i(t) = e^{-i\epsilon_i t} a_i$$
$$a_i^\dagger(t) = e^{i\epsilon_i t} a_i^\dagger.$$

Dyson equation

Time-development of w.f. in interaction picture

$$i\frac{d}{dt}|\Psi(t)\rangle = H_1(t)|\Psi(t)\rangle$$

$$|\Psi(t)\rangle = |\Psi(t_0)\rangle + (-i)\int_{t_0}^{t} dt' H_1(t)|\Psi(t')\rangle$$

Defining time-development operator

$$|\Psi(t)\rangle = U(t,t')|\Psi(t')\rangle$$

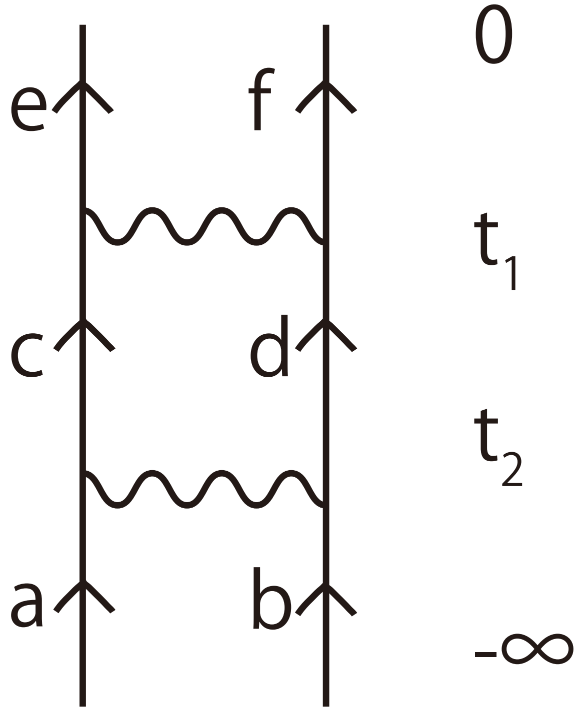
Iterative solution: Dyson equation

$$U(t,t') = \lim_{\epsilon\to 0}\lim_{t'\to-\infty(1-i\epsilon)}\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{t'}^{t}\mathrm{d}t_1\int_{t'}^{t}\mathrm{d}t_2\cdots\int_{t'}^{t}\mathrm{d}t_n T[H_1(t_1)H_1(t_2)\cdots H_1(t_n)].$$

Diagrammatic expression

$$U(0,-\infty)a_a^+ a_b^+|c\rangle$$

$=$

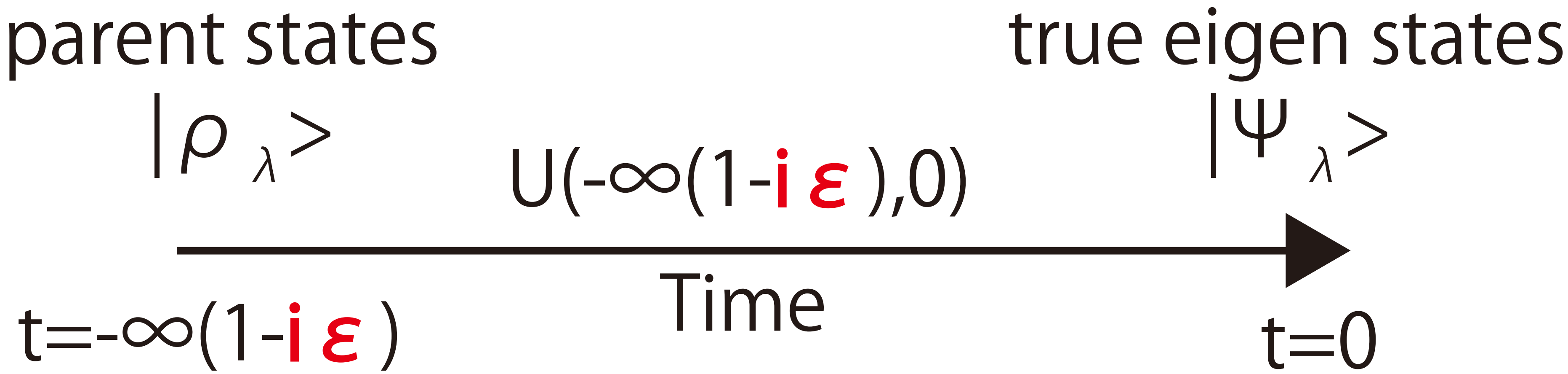


$$= a_e^+ a_f^+|c\rangle \times \lim_{t'\to-\infty(1+i\epsilon)} (-i)^2 \int_{t'}^{0} dt_1 \int_{t'}^{t_1} dt_2 \; e^{-i(\epsilon_c+\epsilon_d-\epsilon_e-\epsilon_f)t_1} \, e^{-i(\epsilon_a+\epsilon_b-\epsilon_c-\epsilon_d)t_2} \times V_{abcd}V_{cdef}$$

$$= a_e^+ a_f^+|c\rangle \times \lim_{t'\to-\infty(1+i\epsilon)} (-i) \int_{t'}^{0} dt_1 e^{-i(\epsilon_c+\epsilon_d-\epsilon_e-\epsilon_f)t_1} \frac{e^{-i(\epsilon_a+\epsilon_b-\epsilon_c-\epsilon_d)t_1}}{-i(\epsilon_a+\epsilon_b-\epsilon_c-\epsilon_d)} \times V_{abcd}V_{cdef}$$

$$= a_e^+ a_f^+|c\rangle \times \frac{1}{\epsilon_a+\epsilon_b-\epsilon_e-\epsilon_f} \frac{1}{\epsilon_a+\epsilon_b-\epsilon_c-\epsilon_d} \times V_{abcd}V_{cdef}$$

Time-dependent perturbation theory



Parent states: “projection” of true eigen states to P-space

$$\langle \rho_\lambda | P\Psi_\mu \rangle = 0 \qquad (\lambda \neq \mu = 1, 2, \cdots, D).$$

-> Because of i ε term the **lowest** eigenstates survive.

$$\frac{|\Psi_\lambda\rangle}{\langle \rho_\lambda | \Psi_\lambda \rangle} = \lim_{\epsilon \to 0} \lim_{t' \to -\infty(1-i\epsilon)} \frac{U(0,t')|\rho_\lambda\rangle}{\langle \rho_\lambda | U(0,t') | \rho_\lambda \rangle} \longrightarrow H \frac{U(0,-\infty)|\rho_\lambda\rangle}{\langle \rho_\lambda | U(0,-\infty) | \rho_\lambda \rangle} = E_\lambda \frac{U(0,-\infty)|\rho_\lambda\rangle}{\langle \rho_\lambda | U(0,-\infty) | \rho_\lambda \rangle}.$$

Time-dependent perturbation theory

Of course we do not know true eigenstates |Ψ> and therefore its projection | ρ >.

Being | ψ > known P-space basis vectors, we can formally write down as

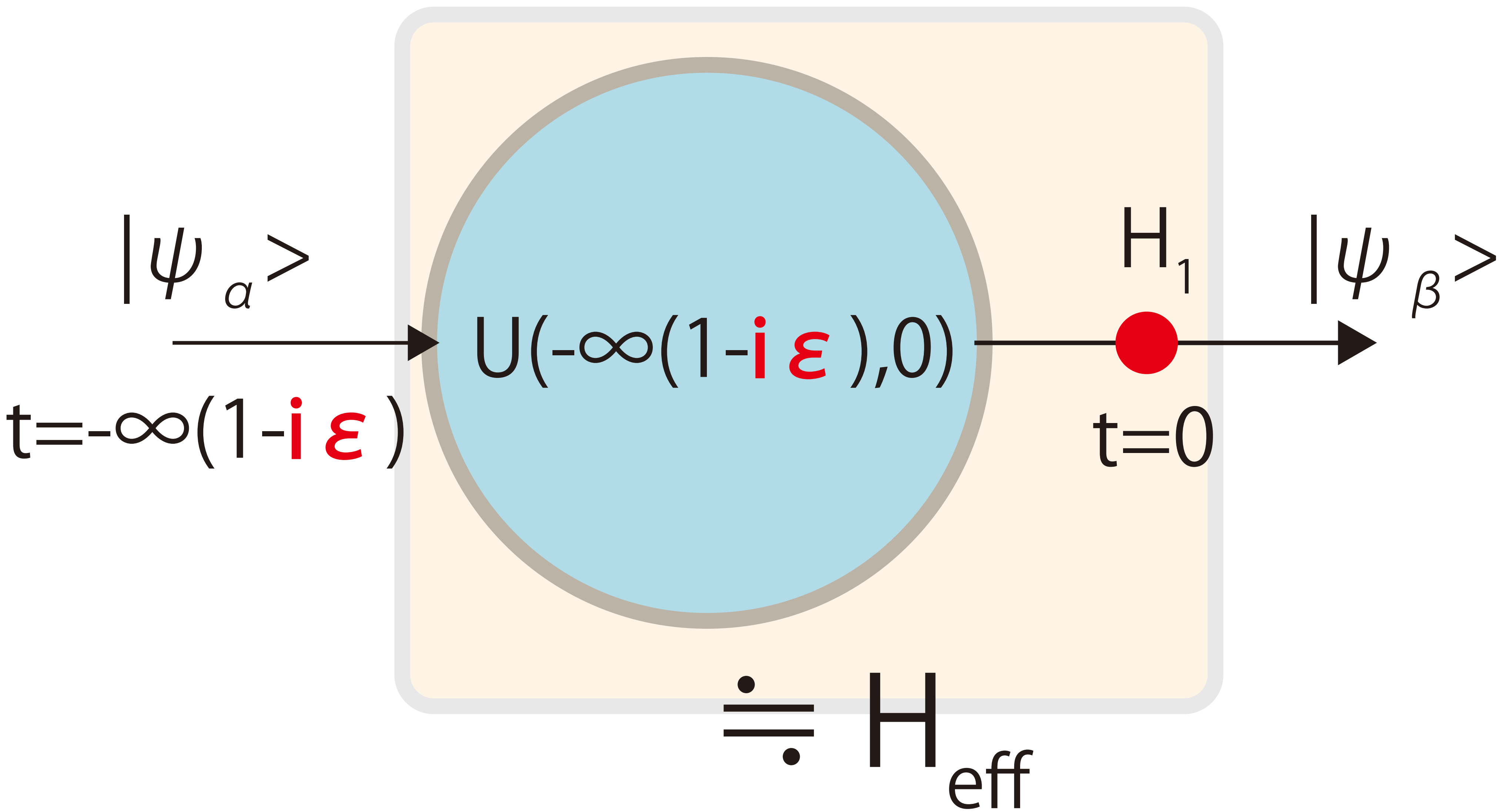
$$|\rho_\lambda\rangle = \sum_{\alpha=1}^{d} C_\alpha^{(\lambda)} |\psi_\alpha\rangle.$$

Then we obtain,

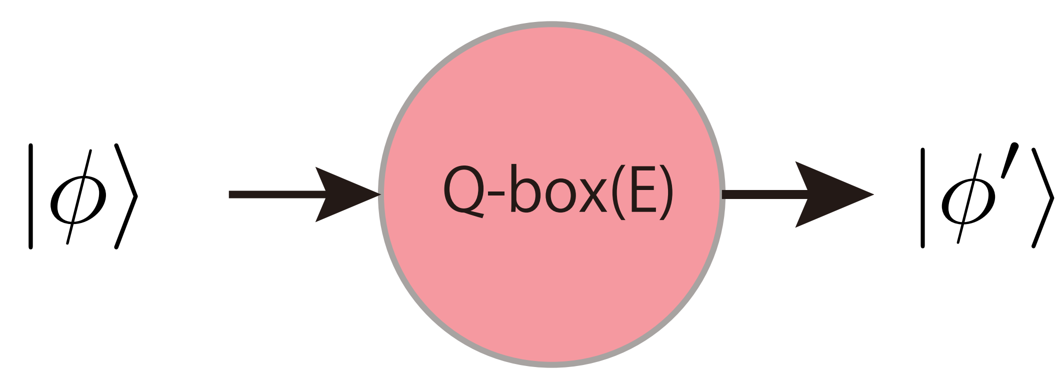
$$\sum_{\alpha=1}^{D} C_\alpha^{(\lambda)} H \frac{U(0,-\infty)|\psi_\alpha\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle} = \sum_{\beta=1}^{D} C_\beta^{(\lambda)} E_\lambda \frac{U(0,-\infty)|\psi_\beta\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle}.$$

$HU(0,-\infty) \fallingdotseq$ Heff

What we know so far



Still different from Q-box,
how come?



Factorization

U|ψ> can be factorized to several pieces

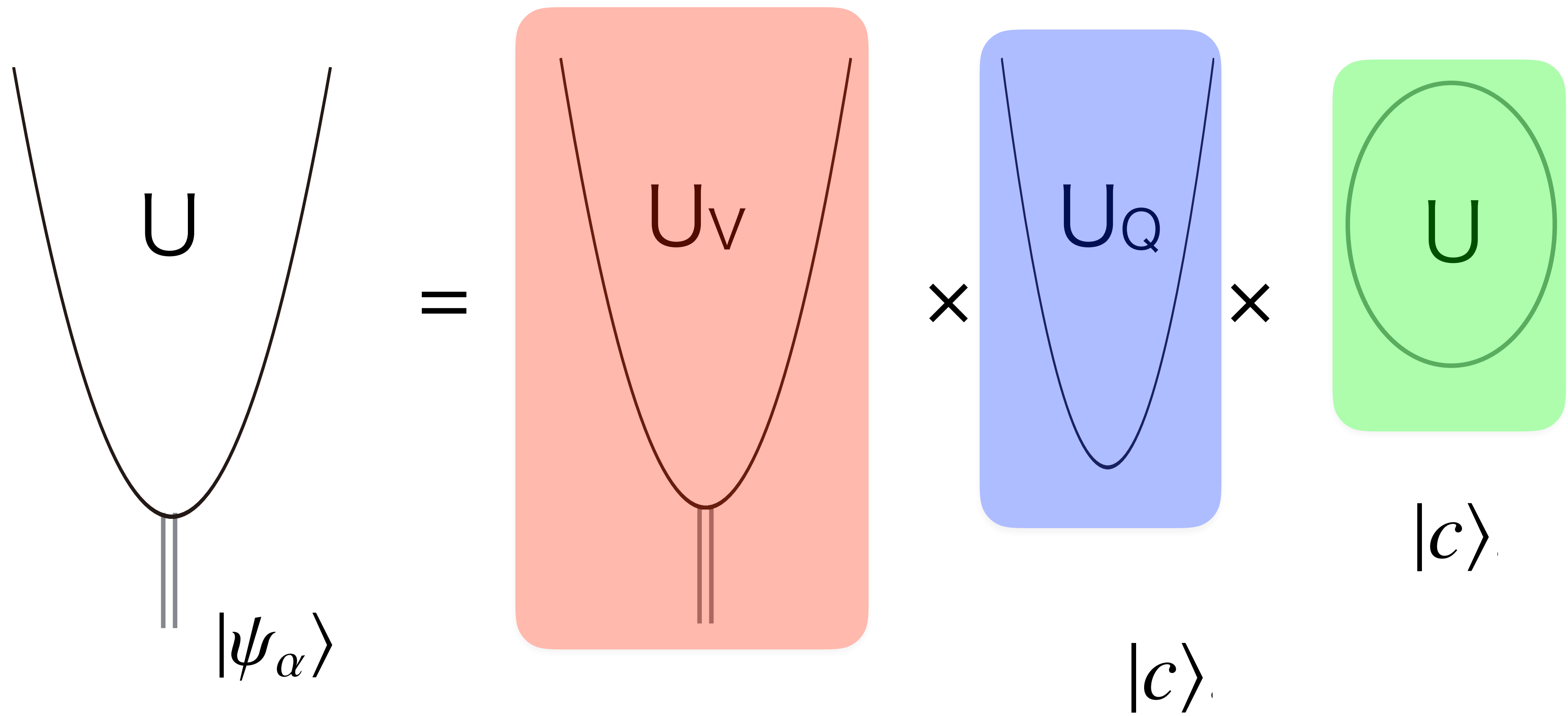
$$U(0,-\infty)|\psi_\alpha\rangle = U_V(0,-\infty)a_i^\dagger a_j^\dagger|c\rangle \times U(0,-\infty)|c\rangle,$$

$$U(0,-\infty)|c\rangle = U_Q(0,-\infty)|c\rangle \times \langle c|U(0,-\infty)|c\rangle,$$

V: Valence linked

Q: terminate as Q-space state

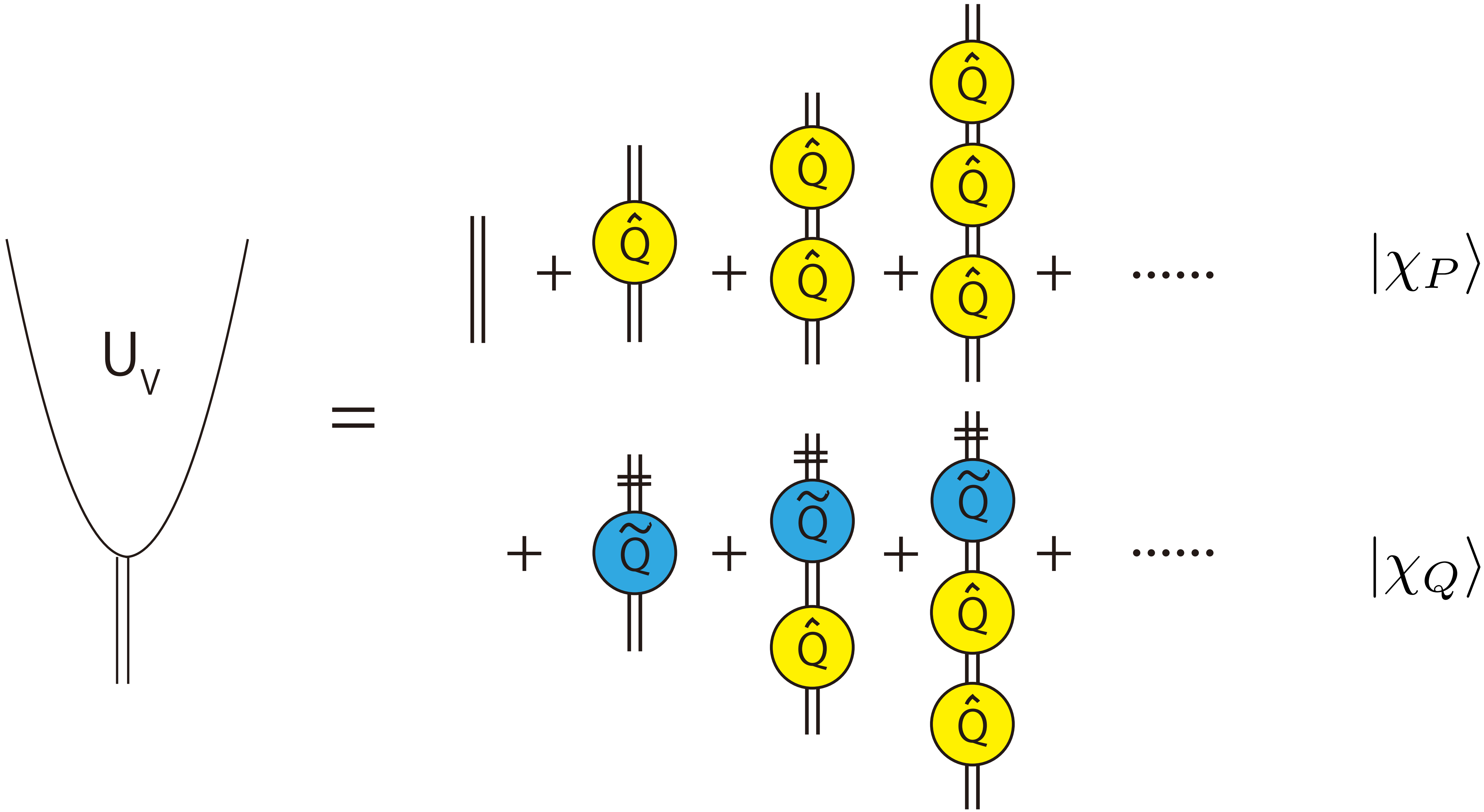
C: core state



Factorization

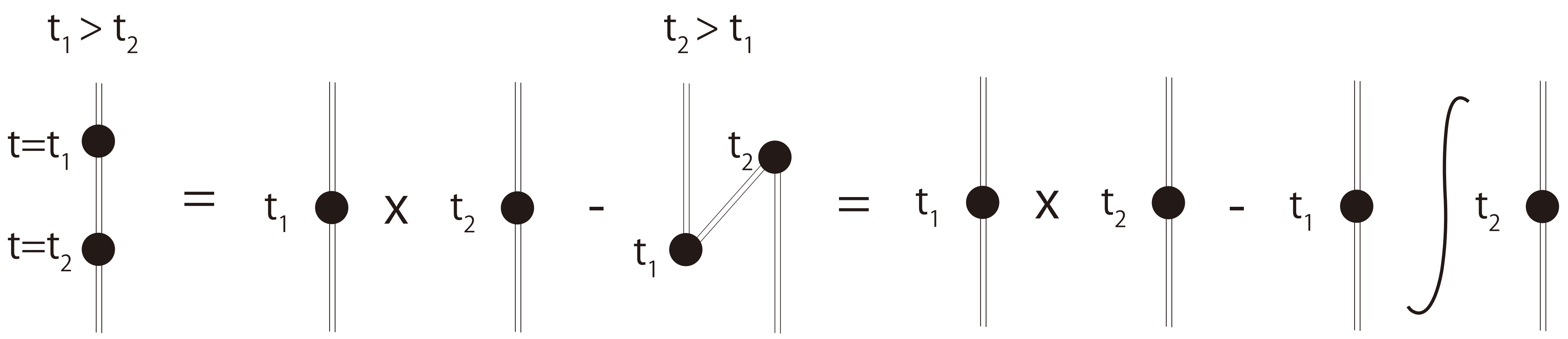
$$U_V(0, -\infty)|\psi_\alpha\rangle = |\chi_P\rangle + |\chi_Q\rangle.$$

P: terminate as P-space state
Q: terminate as Q-space state

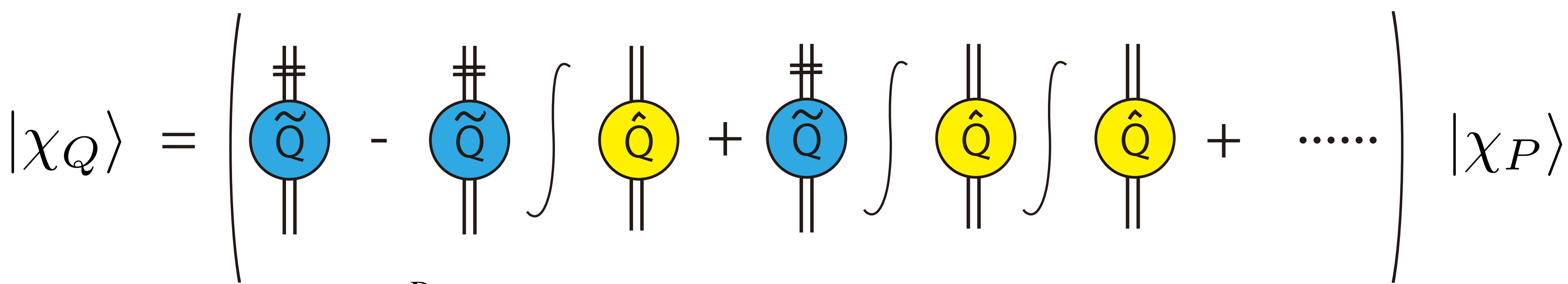


Folded diagrams

To factorize further, define folded diagrams as follows:



Then, we can factorize as follows,



$$U_V(0,-\infty)|\psi_\alpha\rangle = \sum_{\beta=1}^{D} U_{VQ}(0,-\infty)|\psi_\beta\rangle\langle\psi_\beta|U_V(0,-\infty)|\psi_\alpha\rangle.$$

Finally Heff

Remember

$$\sum_{\alpha=1}^{D} C_\alpha^{(\lambda)} H \frac{U(0,-\infty)|\psi_\alpha\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle} = \sum_{\beta=1}^{D} C_\beta^{(\lambda)} E_\lambda \frac{U(0,-\infty)|\psi_\beta\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle}.$$

Combining obtained knowledge,

$$U(0,-\infty)|\psi_\alpha\rangle = U_Q(0,-\infty)|c\rangle\langle c|U(0,-\infty)|c\rangle \times \sum_{\beta=1}^{d} U_{VQ}(0,-\infty)|\psi_\beta\rangle\langle\psi_\beta|U_V(0,-\infty)|\psi_\alpha\rangle$$

$$\longrightarrow \sum_{\gamma=1}^{d} b_\gamma^\lambda H U_Q(0,-\infty)|c\rangle U_{VQ}(0,-\infty)|\psi_\gamma\rangle = \sum_{\delta=1}^{d} b_\delta^\lambda E_\lambda U_Q(0,-\infty)|c\rangle U_{VQ}(0,-\infty)|\psi_\gamma\rangle$$

$$b_\gamma^{(\lambda)} = \sum_{\alpha=1}^{d} C_\alpha^{(\lambda)} \frac{\langle\psi_\gamma|U_V(0,-\infty)|\psi_\alpha\rangle\langle c|U(0,-\infty)|c\rangle}{\langle\rho_\lambda|U(0,-\infty)|\rho_\lambda\rangle}$$

Define linked pieces as follows,

$$U_L(0,-\infty)|\psi_\alpha\rangle \equiv U_{VQ}(0,-\infty)|\psi_\alpha\rangle U_Q(0,-\infty)|c\rangle,$$

$$\longrightarrow \sum_{\gamma=1}^{d} b_\gamma^{(\lambda)} \langle\psi_\sigma|H U_L(0,-\infty)|\psi_\lambda\rangle = E_\lambda|\psi_\sigma\rangle.$$

Heff_total

Factorization and folded diagram method

$$\sum_{\gamma=1}^{d} b_\gamma^{(\lambda)} \langle \psi_\sigma | H U_L(0, -\infty) | \psi_\lambda \rangle = E_\lambda | \psi_\sigma \rangle.$$

Heff_total

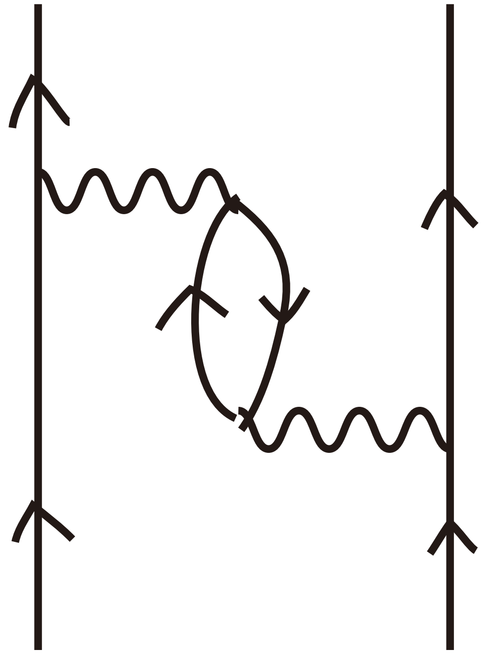
Extract contribution of the core

$$P H_{\text{eff}} P | \Psi_\alpha \rangle = (E_\alpha - E_C) P | \Psi_\alpha \rangle$$

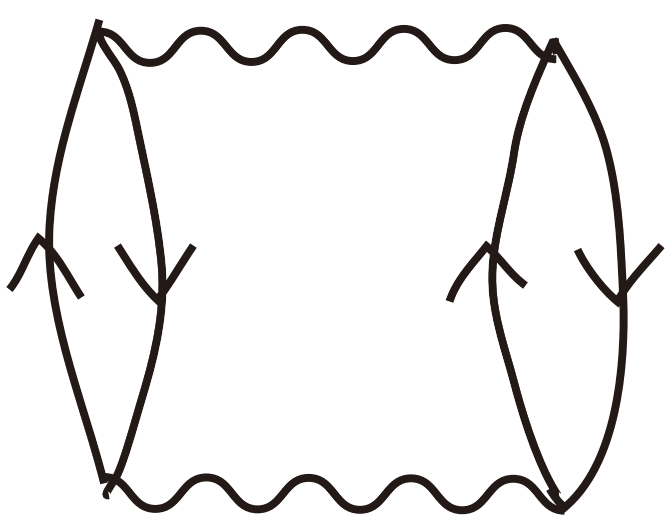
$$H_{\text{eff}} = \langle \psi_\sigma | (H_0(V) + H_1(V)) U_L(0, -\infty) | \psi_\lambda \rangle.$$

V: valence linked

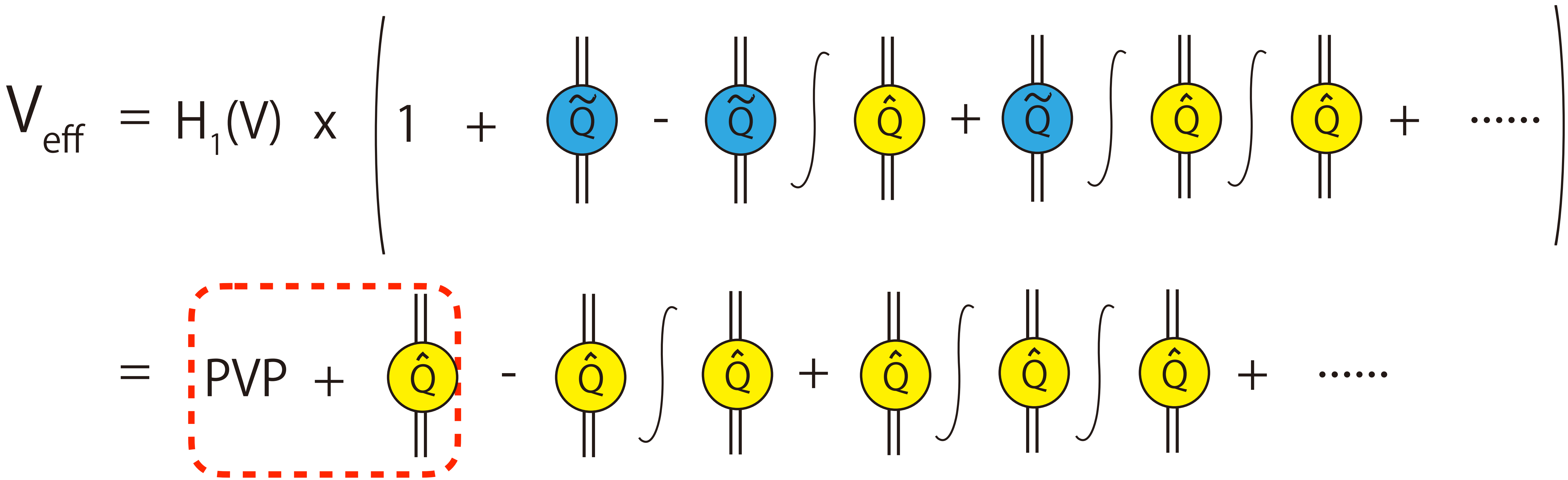
valence linked



core



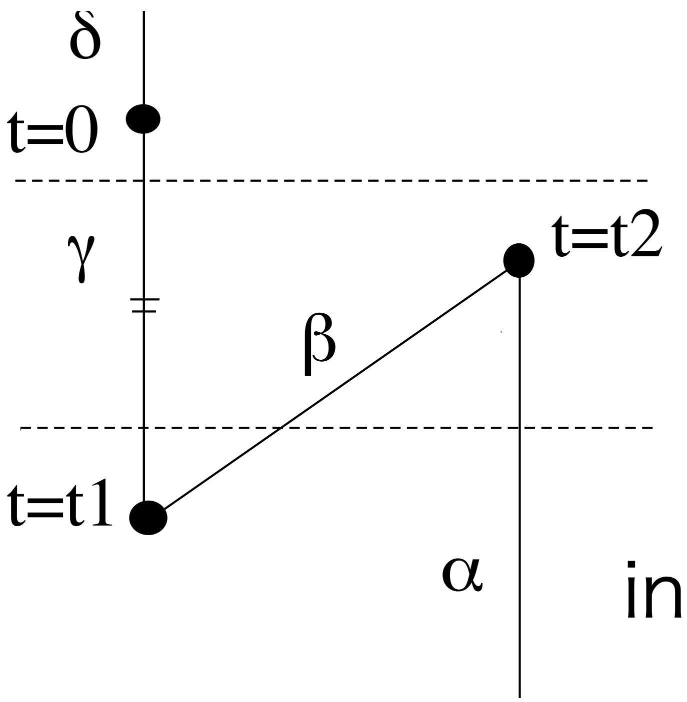
Heff



This is Q-box !

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP,$$

Folded diagrams and energy derivative



$$= \frac{V_{\alpha\beta}V_{\beta\gamma}V_{\gamma\delta}}{(\epsilon_\alpha - \epsilon_\gamma - (\epsilon_\alpha - \epsilon_\beta))(\epsilon_\alpha - \epsilon_\gamma)}$$

$$= V_{\alpha\beta}V_{\beta\gamma}V_{\gamma\delta}\frac{\Big((\epsilon_\alpha - \epsilon_\gamma) - (\epsilon_\alpha - \epsilon_\beta)\Big)^{-1} - (\epsilon_\alpha - \epsilon_\gamma)^{-1}}{\epsilon_\alpha - \epsilon_\beta}$$

in the limit of $\epsilon_\beta \to \epsilon_\alpha$

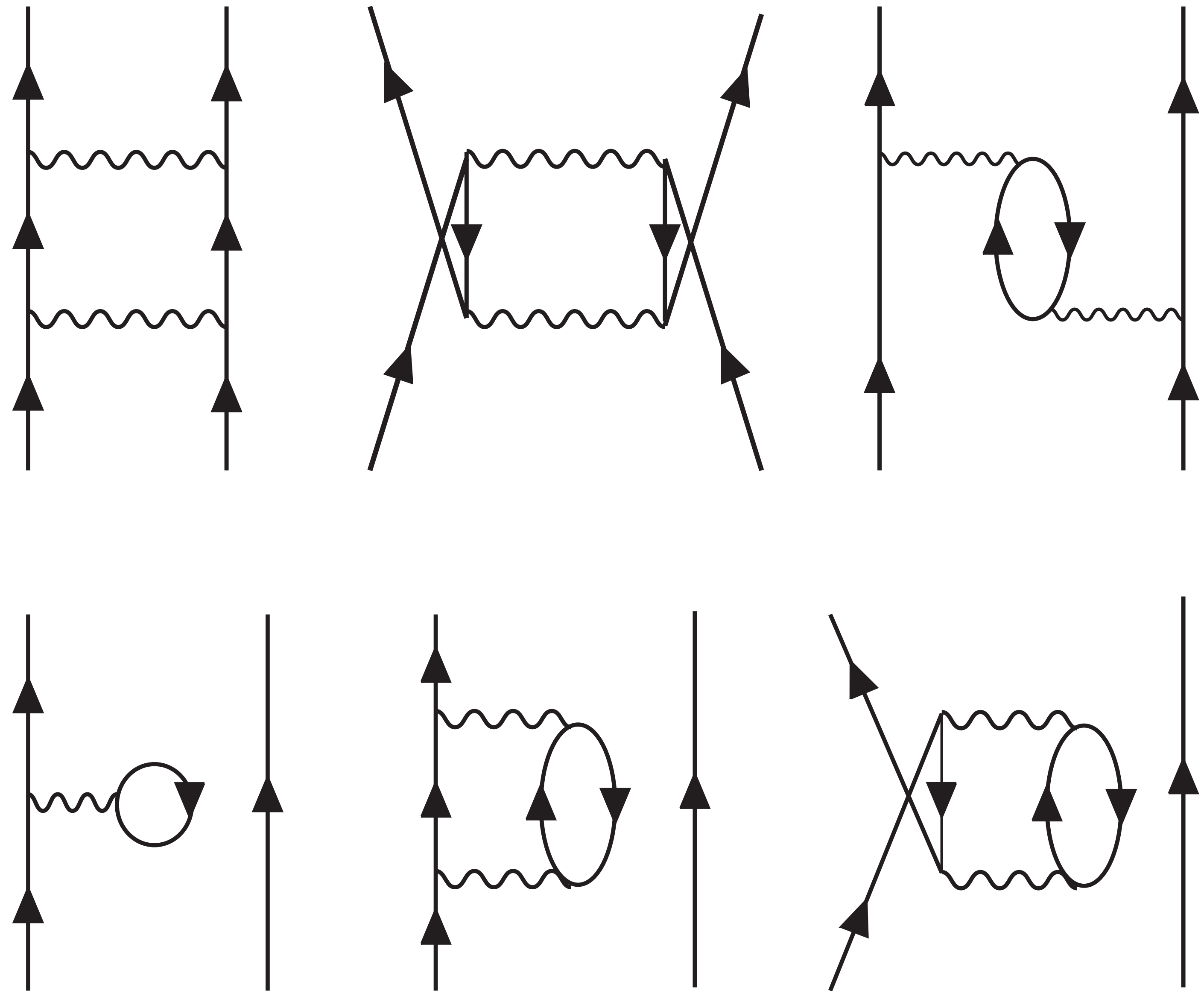
$$= \frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{V_{\beta\gamma}V_{\gamma\delta}}{\omega - \epsilon_\gamma}\right)_{\omega=\alpha} \times V_{\alpha\beta}$$

→ Folded diagrams can be calculated by energy derivative if the model space is degenerate

→ $$V_{\mathrm{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\mathrm{eff}}^{(n-1)}\}^k.$$

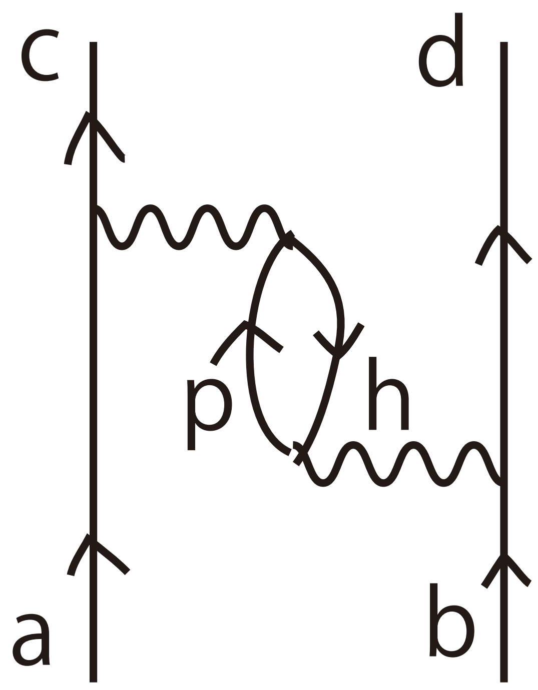
Final expression

Q-box expansion

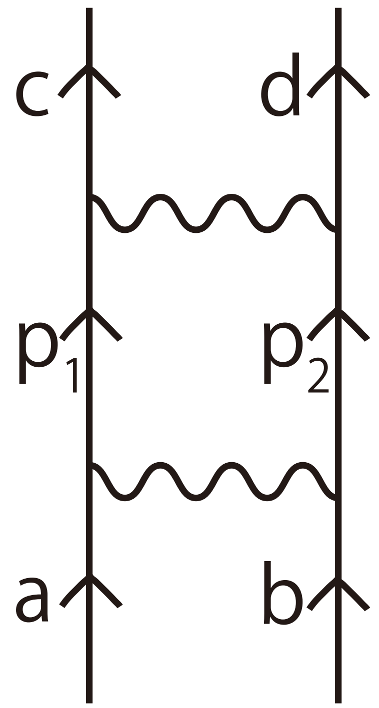


Diagrams appearing in 2nd order

Example of Q-box calculation

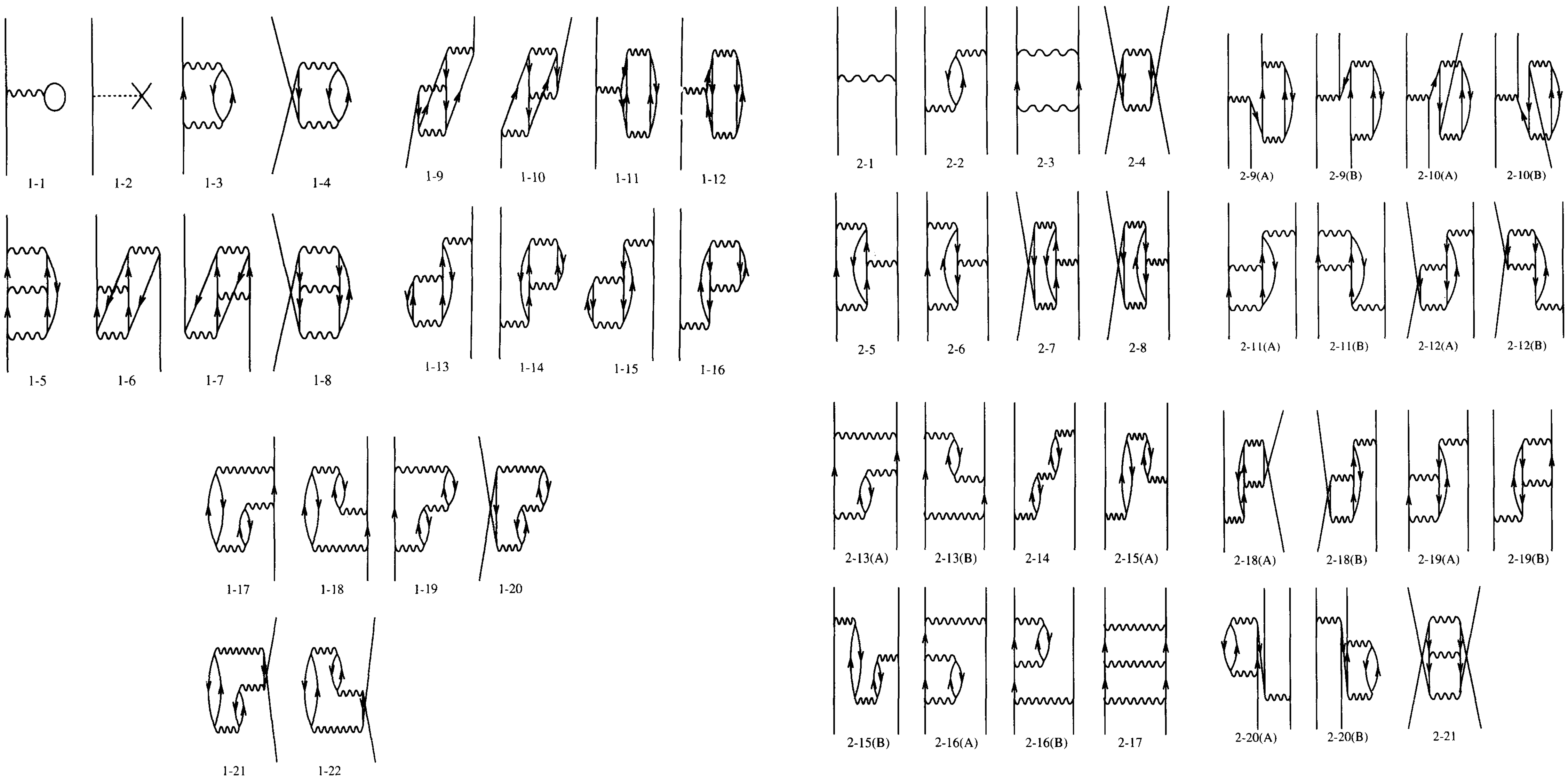


$$= \sum_{p,h} \frac{V_{ch,ap} V_{pd,hb}}{\epsilon_b - \epsilon_d - \epsilon_p + \epsilon_h}$$



$$= \sum_{p_1,p_2} \frac{V_{cd,p_1 p_2} V_{p_1 p_2,ab}}{\epsilon_a + \epsilon_b - \epsilon_{p_1} - \epsilon_{p_2}}$$

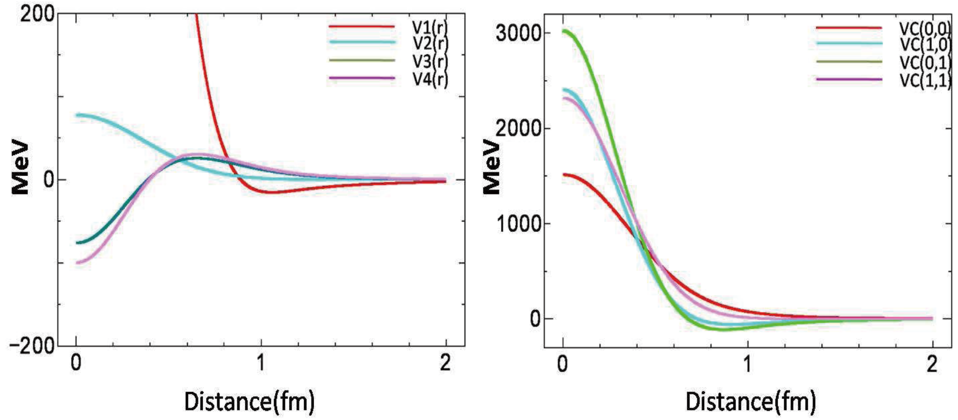
EKK code algorithm



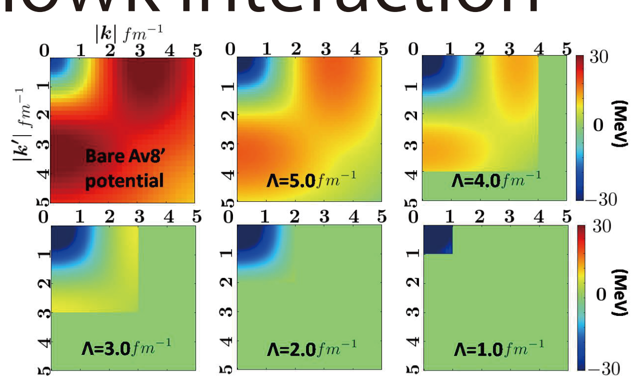
Diagrams appearing in 3rd order

Summary of folded diagram method

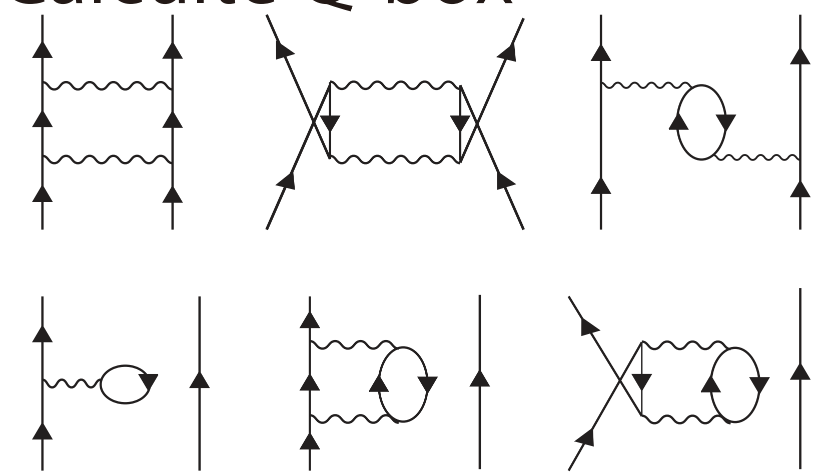
Bare nuclear interaction

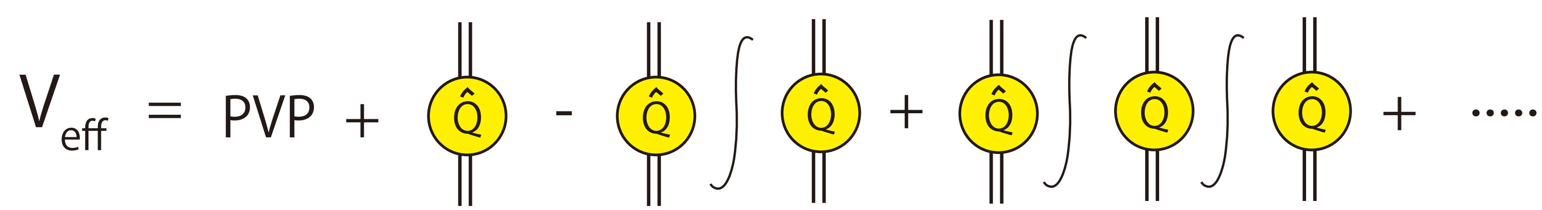


Vlowk interaction



Calculte Q-box





$$V^{(n)}_{\mathrm{eff}} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V^{(n-1)}_{\mathrm{eff}}\}^k.$$

EKK code algorithm

SPEs TBMEs

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k.$$

Time required (single node)

Step	Time required (single node)	
read bare interaction from file	~10 s	
calculate SPEs ~ o(10) elements	~5 s	calculate ϵ_i
calculate TBMEs ~ o(10^2~10^4) elements	10 h~100 days	calculate $\hat{Q}_k(E)$
calculate folded diagrams	~5 s	iteration

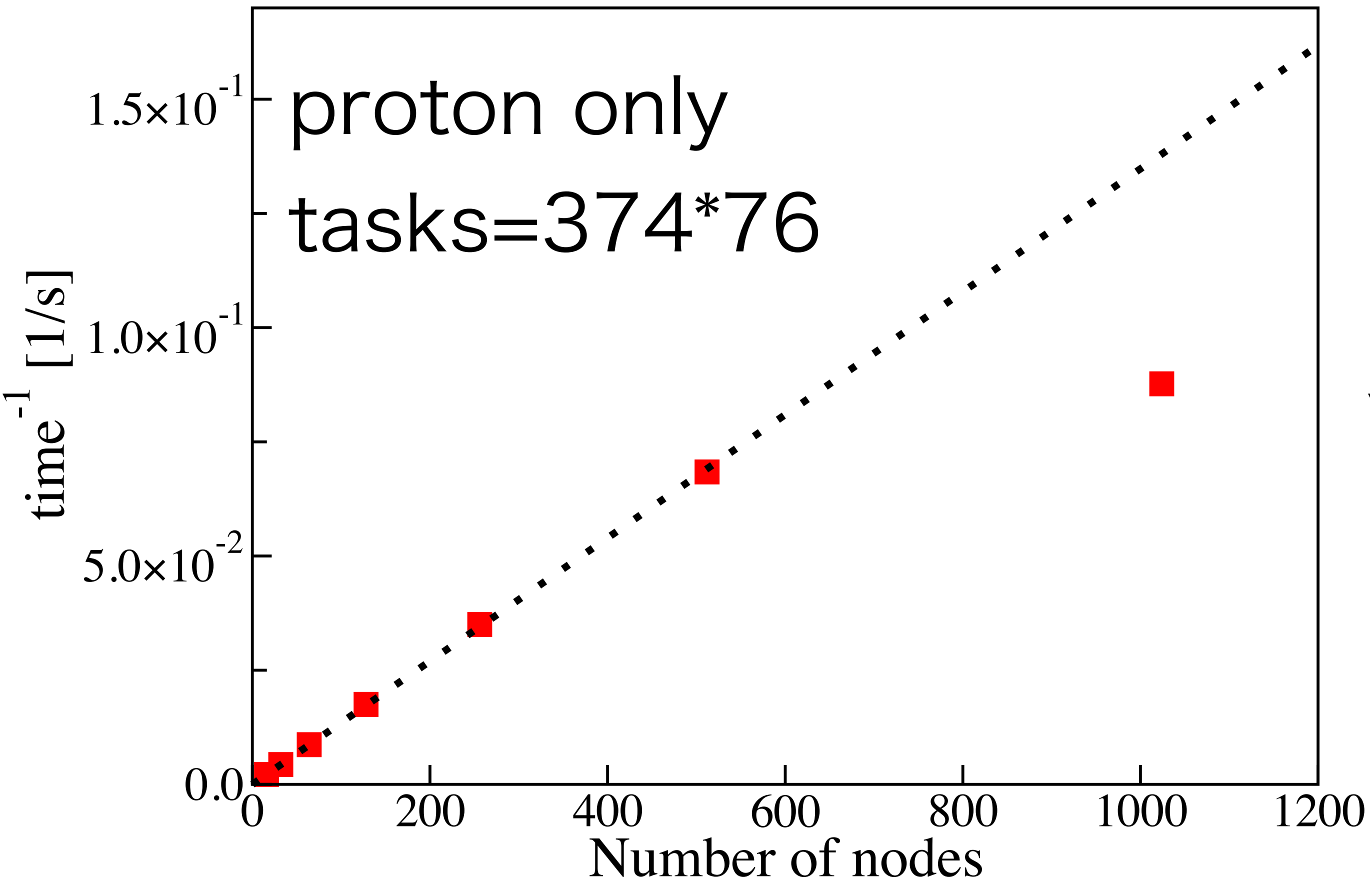
$$\tilde{H}_{\rm eff}^{(n)} = \tilde{H}_{\rm BH}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E) \{\tilde{H}_{\rm eff}^{(n-1)}\}^k$$

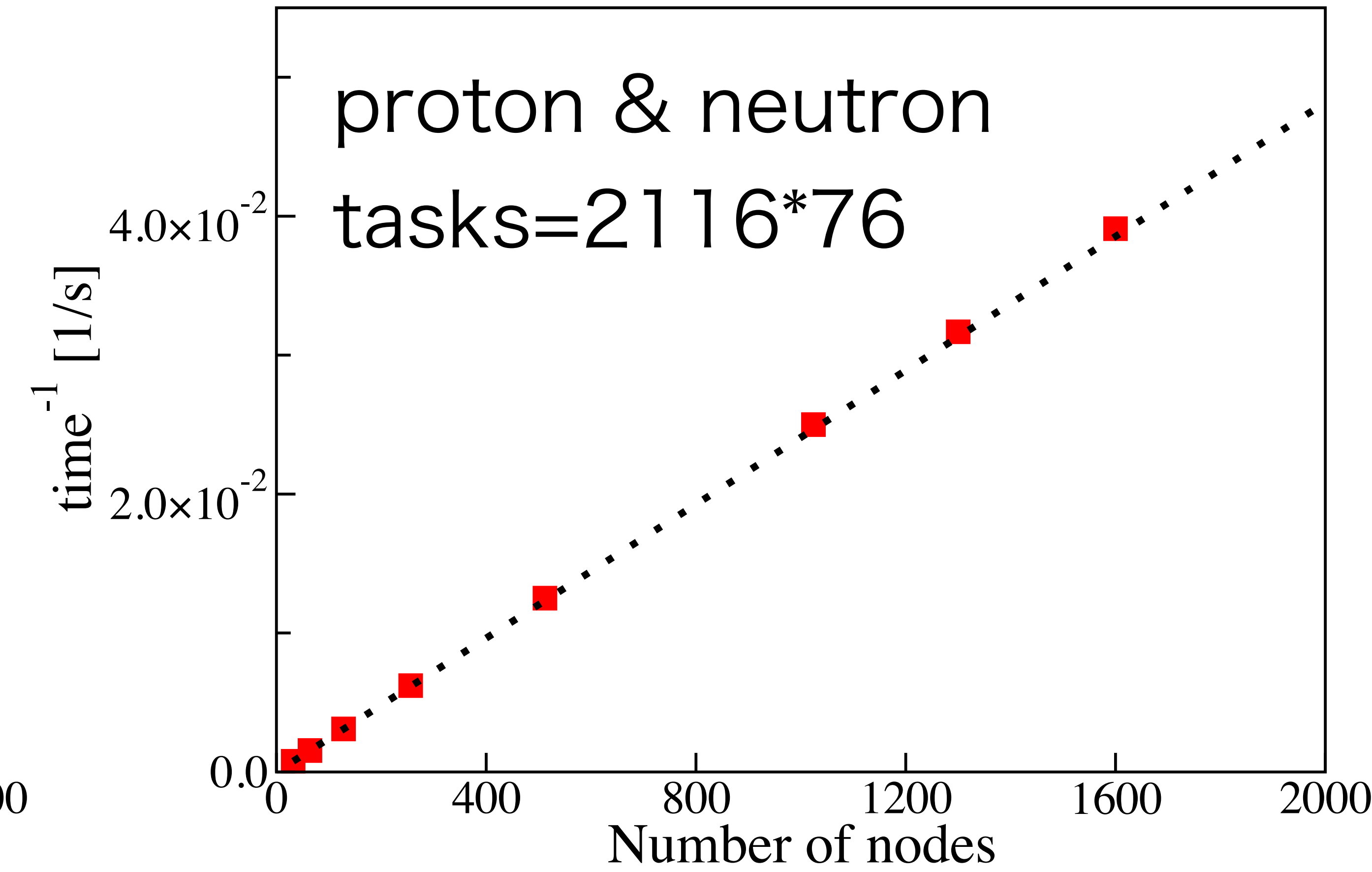
MPI+openMP scaling

Test case: sdpf-shell

13major shells

oakforest (64 core, max 2048 nodes)





# of nodes ~ # of TBMEs

nice scaling

Day 3

Model space and shell model Hamiltonian

	Lower shell	Upper shell
Lower shell	Plenty of exp. data	Few exp. data
Upper shell	Few exp. data	Plenty of exp. data

ex) Phenom. int.

- USD int.
- gxpf1
- KB3
- Cohen-Kurath
- sdpf-m
- sdpf-U-mix

etc…

Cross-shell mtx: difficult to fit to exp. data

EKK method

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Effective interaction in non-degenerate model space

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EKK method in a schematic model

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Multi-shell effective interactions

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Background: Effective interactions, either derived from microscopic theories or based on fitting selected properties of nuclei in specific mass regions, are widely used inputs to shell-model studies of nuclei. The commonly used unperturbed basis functions are given by the harmonic oscillator. Until recently, most shell-model calculations have been confined to a single oscillator shell like the sd shell or the pf shell. Recent interest in nuclei away from the stability line requires, however, larger shell-model spaces. Because the derivation of microscopic

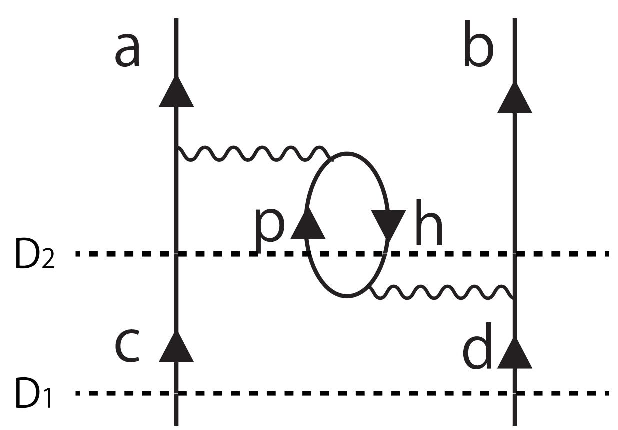
EKK method in nuclei

Divergent problem of Q-box in non-degenerate model space

(A)KK method requires assumption that the model space is **degenerate**

(B)Naive perturbation theory leads a **divergence** in non-degenerate model space

Example



$$\frac{V_{ah,cp}V_{pb,hd}}{(\epsilon_c + \epsilon_d) - \epsilon_c - \epsilon_p + \epsilon_h - \epsilon_b}$$

→ **Energy denominator is zero when $\varepsilon_d - \varepsilon_b = \varepsilon_p - \varepsilon_h$**

We need a theory which satisfies

(a)The assumption of degenerate model space is **removed**

(b)**Avoid** the divergence appearing in Q-box diagrams

→ **EKK method as a re-summation scheme of KK method**

Decoupling equation for the EKK method (formal solution)

Decoupling equation

$$0 = Q\mathcal{H}P = QVP - \omega PHP + QHQ\omega - \omega PVQ\omega,$$

Introduce energy parameter **E**

$$(E - QHQ)\omega = QVP - \omega P\tilde{H}P - \omega PVQ\omega,$$
$$\tilde{H} = H - E$$

EKK solution of parameter E

$$H_{\mathrm{BH}}(E) = PHP + PVQ\frac{1}{E - QHQ}QVP.$$

$$\longrightarrow \quad \boxed{\tilde{H}_{\mathrm{eff}}^{(n)} = \tilde{H}_{\mathrm{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\mathrm{eff}}^{(n-1)}\}^k,}$$

$$\tilde{H}_{\mathrm{eff}} = H_{\mathrm{eff}} - E, \quad \tilde{H}_{\mathrm{BH}}(E) = H_{\mathrm{BH}}(E) - E,$$

Points:

1. Arbitrary energy parameter **E** is introduced
   → results do **not** depend on the choice of **E**
2. Veff is substituted by Heff
3. Q-box and its derivatives are not changed, but evaluated at **E**

Extended KK method as a re-summation of the perturbative series

EKK method

New parameter E (arbitrary parameter)

$$\begin{aligned} H &= H_0' + V' \\ &= \begin{pmatrix} E & 0 \\ 0 & QH_0Q \end{pmatrix} + \begin{pmatrix} P\tilde{H}P & PVQ \\ QVP & QVQ \end{pmatrix}, \end{aligned}$$

$$H_{\mathrm{BH}}(E) = PHP + PVQ\frac{1}{E - QHQ}QVP.$$

$$\tilde{H}_{\mathrm{eff}}^{(n)} = \tilde{H}_{\mathrm{BH}}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\mathrm{eff}}^{(n-1)}\}^k$$

KK method (conventional)

$$\begin{aligned} H &= H_0 + V \\ &= \begin{pmatrix} PH_0P & 0 \\ 0 & QH_0Q \end{pmatrix} + \begin{pmatrix} PVP & PVQ \\ QVP & QVQ \end{pmatrix} \end{aligned}$$

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP$$

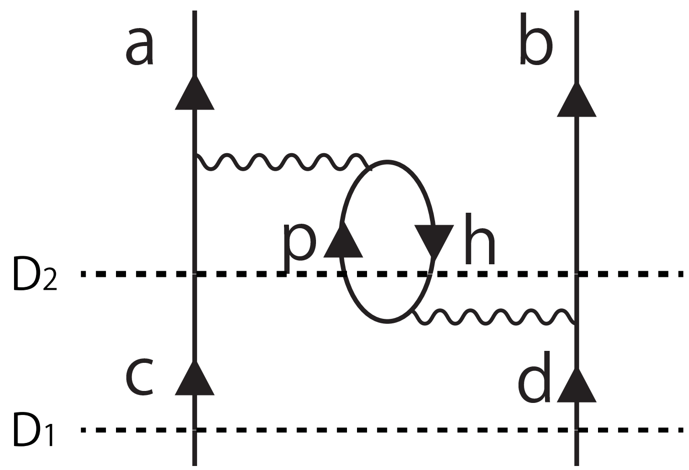
$$V_{\mathrm{eff}}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\mathrm{eff}}^{(n-1)}\}^k.$$

- EKK method can be interpreted as a re-summation of KK method
- All the arguments are kept unchanged with the new division of the Hamiltonian

N. Tsunoda, K. Takayanagi, M. Hjorth-Jensen, and T. Otsuka, Phys. Rev. C 89, 024313 (2014).

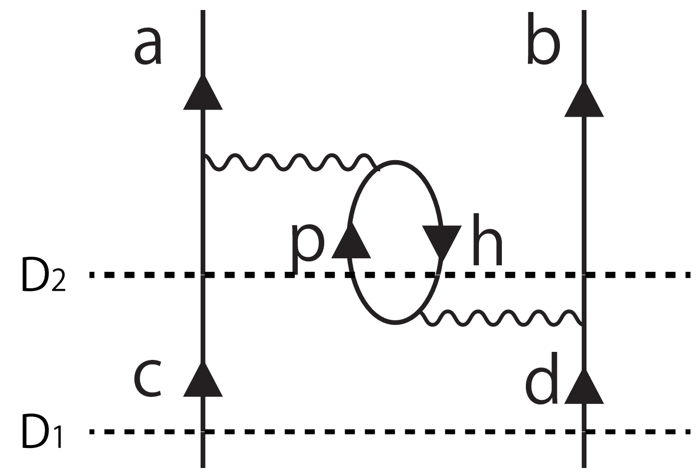
Example: EKK method avoids the divergences

EKK method



$$\frac{V_{ah,cp}V_{pb,hd}}{E - \epsilon_c - \epsilon_b - \epsilon_p + \epsilon_h}$$

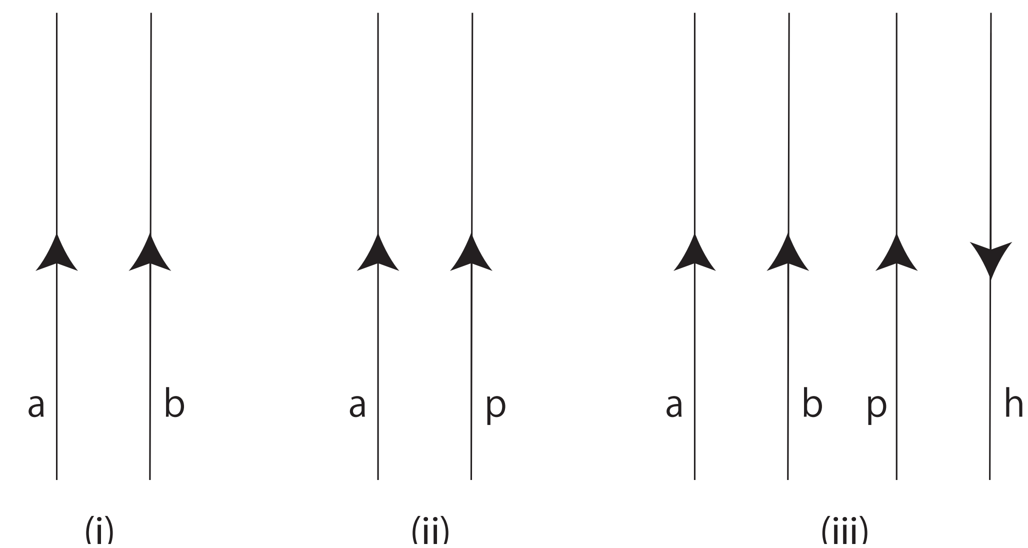
KK method



$$\frac{V_{ah,cp}V_{pb,hd}}{(\epsilon_c + \epsilon_d) - \epsilon_c - \epsilon_p + \epsilon_h - \epsilon_b}$$

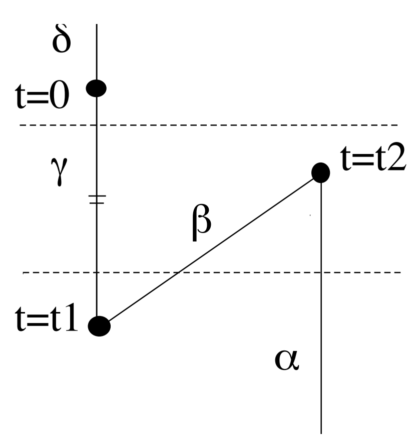
- We can choose E to avoid divergence !
- Note that the choice of E is arbitrary and should give the same result if the Q-box is calculated without any approximation.
- Inversely, E-dependence is a **measure of error** coming from the approximation

Diagrams appearing in EKK method



(i)	$\|\psi_i(t)\rangle$	$=e^{-iH_0't}\|\psi_i\rangle$	$=e^{-iEt}\|\psi_i\rangle$	P-space
(ii)	$\{a_a^\dagger a_p^\dagger\|c\rangle\}(t)$	$=e^{-iH_0't}\{a_a^\dagger a_p^\dagger\|c\rangle\}$	$=e^{-i(\epsilon_a+\epsilon_p)t}a_a^\dagger a_p^\dagger\|c\rangle,$	Q-space
(iii)	$\{a_a^\dagger a_b^\dagger a_p^\dagger a_h\|c\rangle\}(t)$	$=e^{-iH_0't}\{a_a^\dagger a_b^\dagger a_p^\dagger a_h\|c\rangle\}$	$=e^{-i(\epsilon_a+\epsilon_b+\epsilon_p-\epsilon_h)t}a_a^\dagger a_b^\dagger a_p^\dagger a_h\|c\rangle,$	Q-space

The argument of folded diagram is the same
→ derivatives indicate the folded diagram contribution



$$=\frac{V_{\alpha\beta}V_{\beta\gamma}V_{\gamma\delta}}{(\epsilon_\alpha-\epsilon_\gamma-(\epsilon_\alpha-\epsilon_\beta))(\epsilon_\alpha-\epsilon_\gamma)}$$

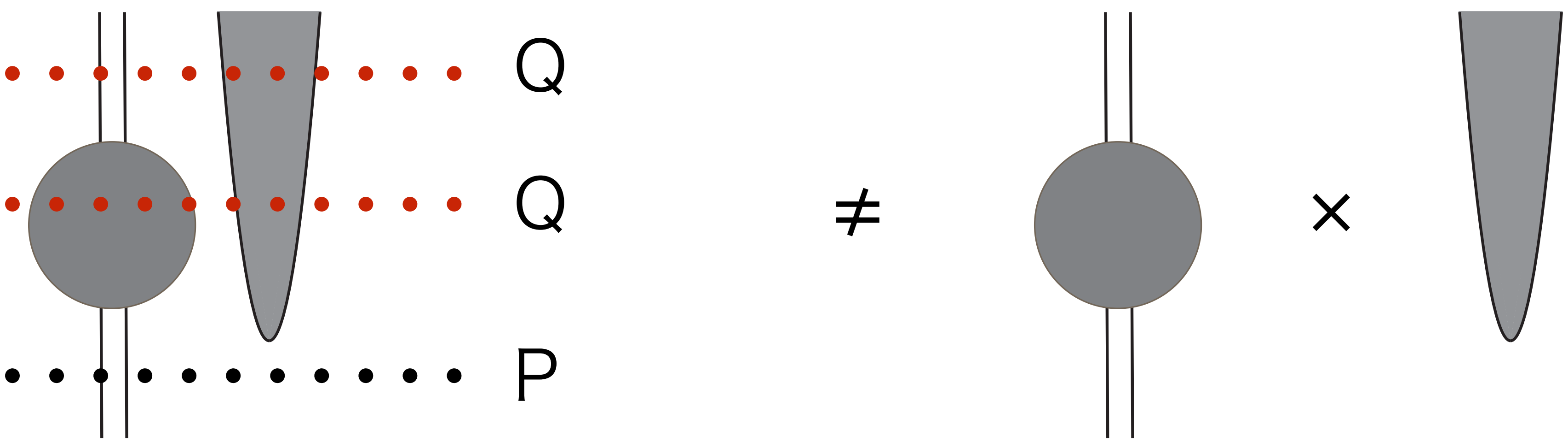
$$=V_{\alpha\beta}V_{\beta\gamma}V_{\gamma\delta}\frac{\left((\epsilon_\alpha-\epsilon_\gamma)-(\epsilon_\alpha-\epsilon_\beta)\right)^{-1}-(\epsilon_\alpha-\epsilon_\gamma)^{-1}}{\epsilon_\alpha-\epsilon_\beta}$$

in the limit of $\epsilon_\beta \to \epsilon_\alpha$

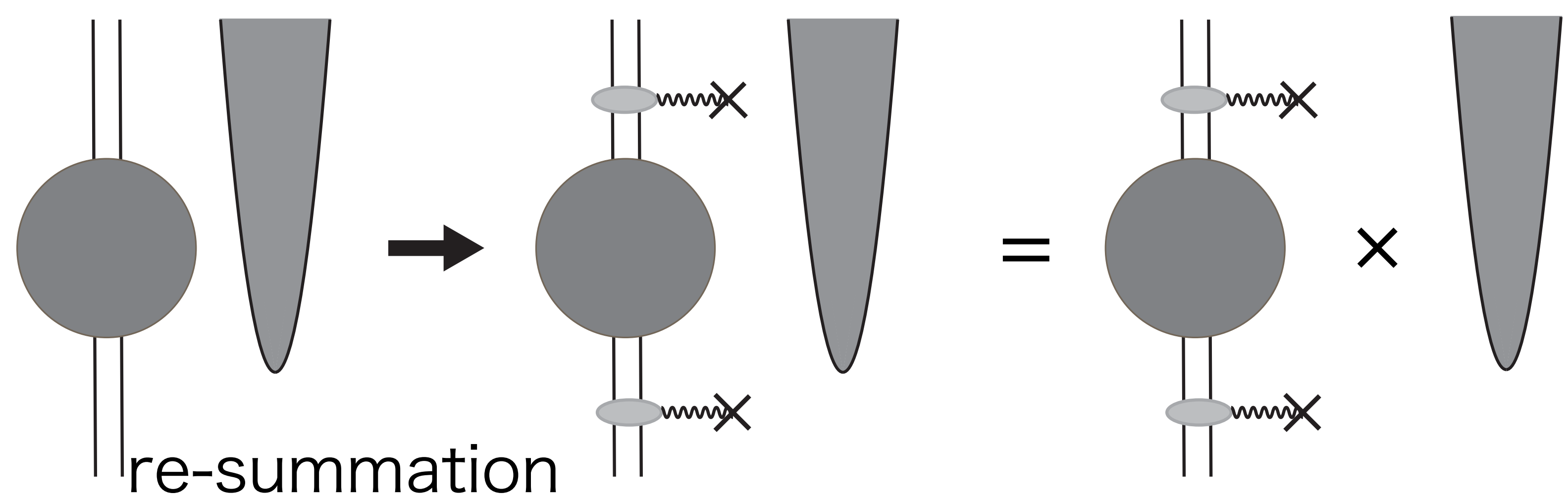
$$=\frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{V_{\beta\gamma}V_{\gamma\delta}}{\omega-\epsilon_\gamma}\right)_{\omega=\alpha}\times V_{\alpha\beta}$$

Factorization theorem in EKK method

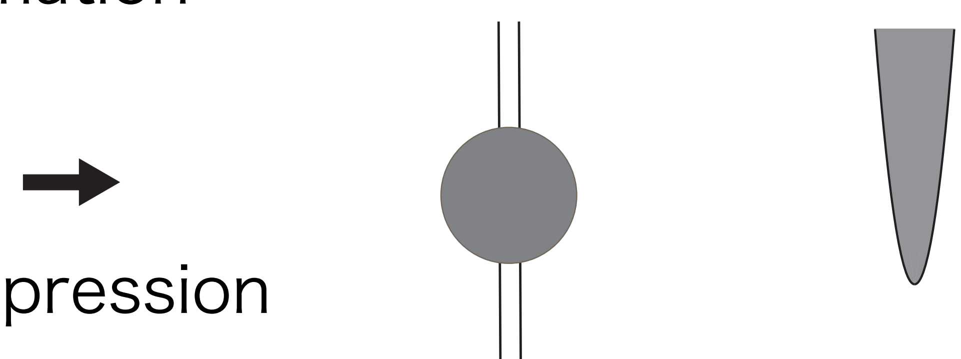
Factorization theorem does not hold in EKK method naively



$$
\begin{aligned}
H &= H_0 + V \\
&= H_0' + V' \\
&= H_0' - P(E - H_0)P + V \\
&= H_0' + V_1 + V,
\end{aligned}
$$



Insert V_1 vertex up to infinite order

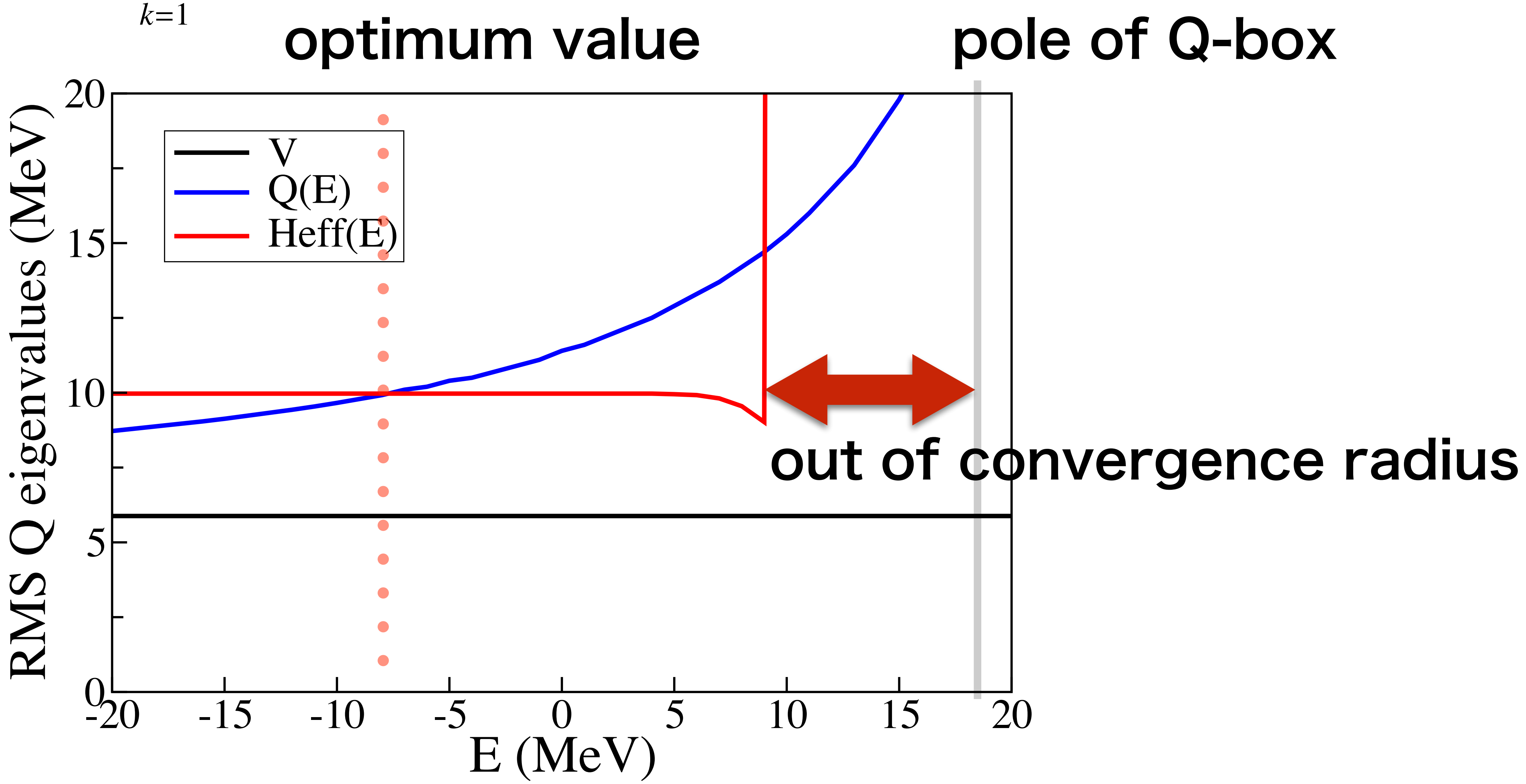


Final expression

valence linked piece core part

E-depdence w and w/o non-perturbative correction

$$\tilde{H}_{\rm eff}^{(n)} = \tilde{H}_{\rm BH}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\rm eff}^{(n-1)}\}^k$$



★ Non perturbative correction vanishes the E-dependecne

★ Optimum value of E

APPLICATION

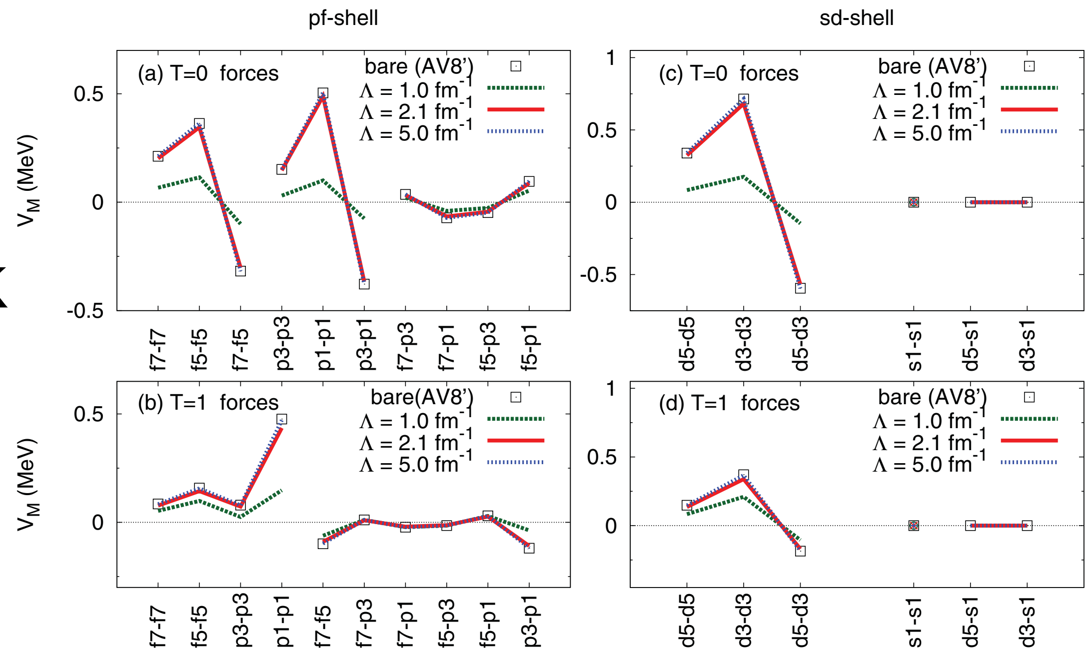
Renormalization persistency of tensor force

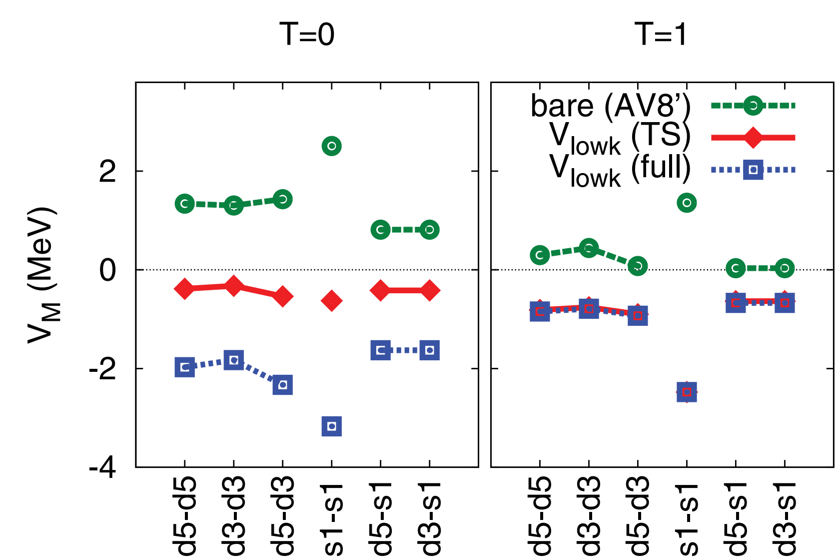
= tensor force **survives** renormalization treatment

tensor

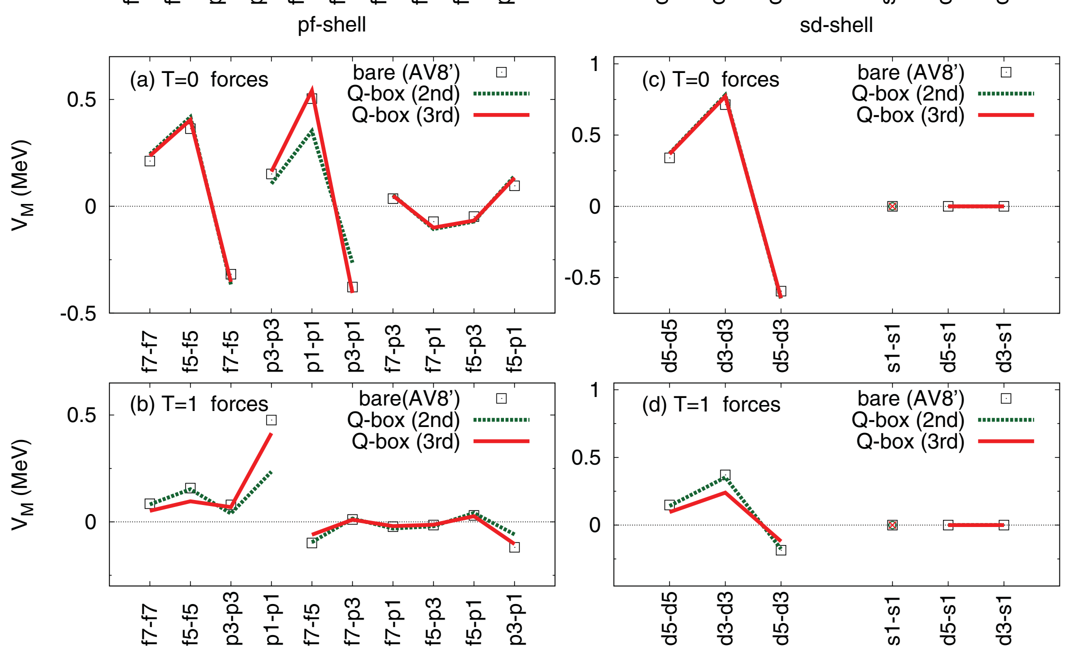
central

Vlowk





MBPT



N. Tsunoda et al., Phys. Rev. C 84, 044322 (2011).

VMU ~ renormalization persistency

T. Otsuka et al., Phys. Rev. Lett. 104, 012501 (2010).

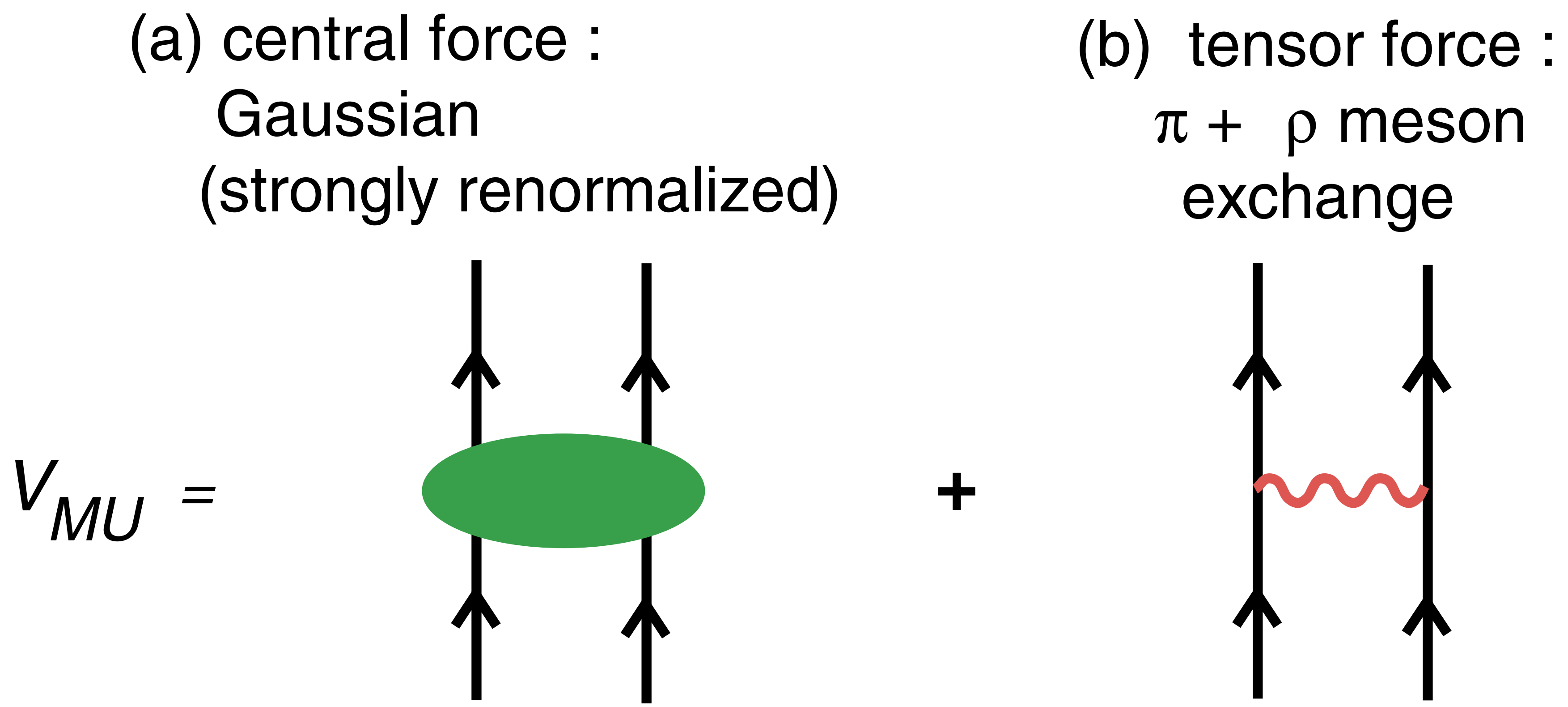


FIG. 2 (color online). Diagrams for the V_{MU} interaction.

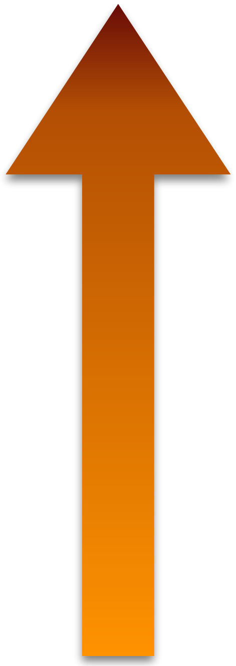
$$V_c = \sum_{S,T} f_{S,T} P_{S,T} \exp(-(r/\mu)^2),$$

- Central force -> modeled by gaussian
- Tensor force -> bare $\pi + \rho$ meson exchange
- widely used as effective int. for SM calc.

Neutron-rich nuclei~ island of inversion

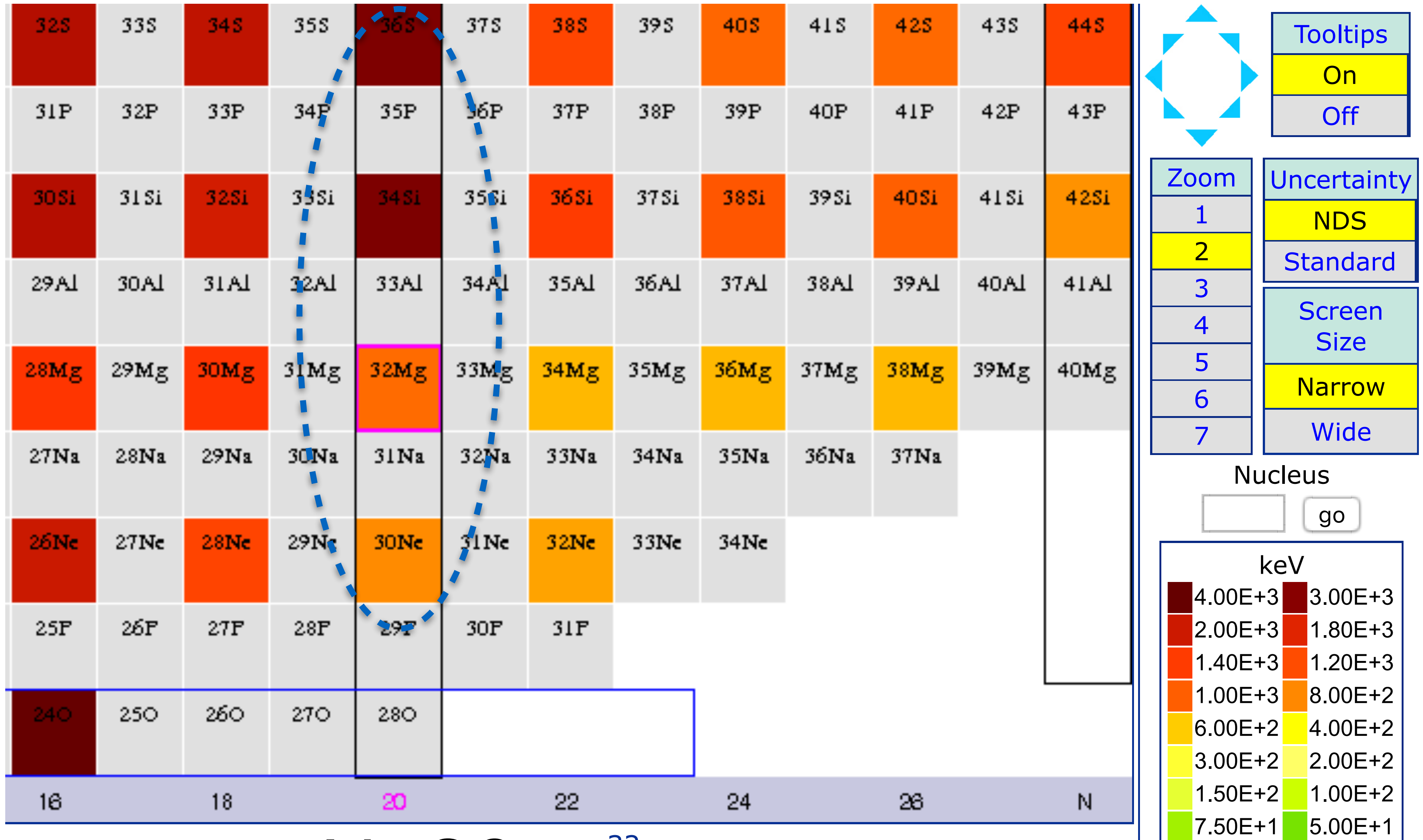
http://www.nndc.bnl.gov/nudat2/reCenter.jsp?z=12&n=20

shell gap



breaking of

shell gap



N=20

- E(2+)~1 MeV on N=20 indicate **breaking** of major shell gap
- Unified treatment of beyond and below the N=20 gap is necessary
- And this is one of many examples….

PHYSICAL REVIEW C **95**, 021304(R) (2017)

Exotic neutron-rich medium-mass nuclei with realistic nuclear forces

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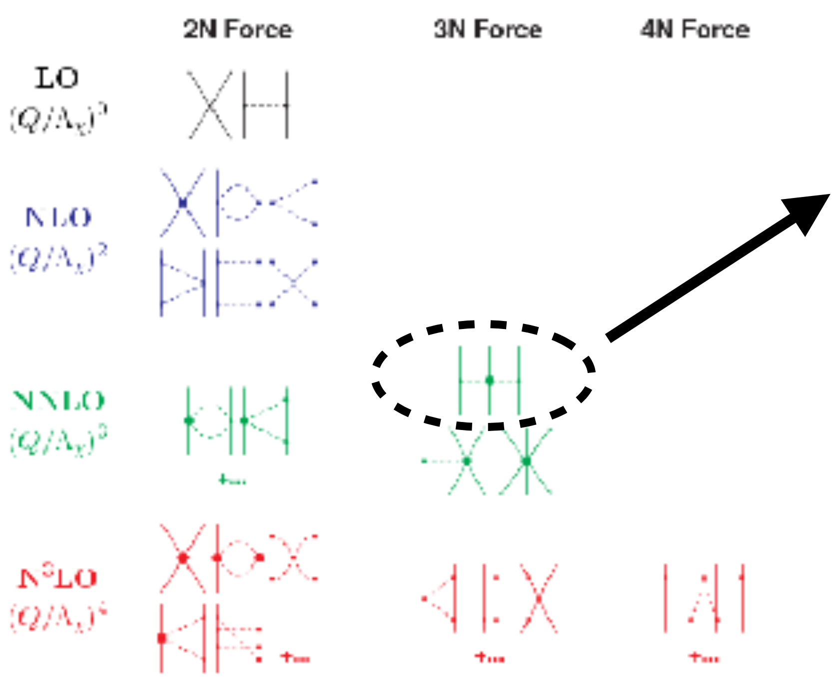
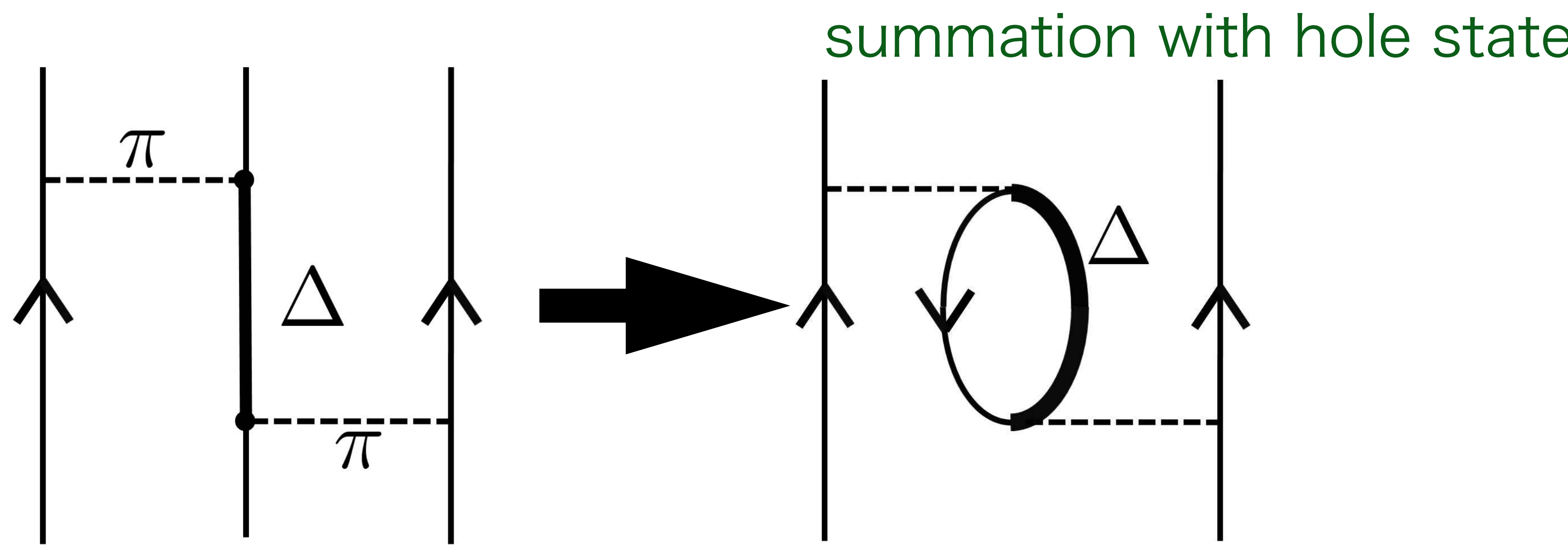
We present the first application of the newly developed extended Kuo-Krenciglowa (EKK) theory of the effective nucleon-nucleon interaction to shell-model studies of exotic nuclei, including those where conventional approaches with fitted interactions encounter difficulties. This EKK theory enables us to derive an interaction that is suitable for several major shells ($sd + pf$ in this work). By using such an effective interaction obtained from the Entem-Machleidt QCD-based χN^3LO interaction and the Fujita-Miyazawa three-body force, the energies, $E2$ properties, and spectroscopic factors of low-lying states of neutron-rich Ne, Mg, and Si isotopes are nicely described, as the first shell-model description of the "island of inversion" without fit of the interaction. The long-standing question as to how particle-hole excitations occur across the sd-pf magic gap is clarified with distinct differences from the conventional approaches. The shell evolution is shown to appear similarly to earlier studies.

DOI: 10.1103/PhysRevC.95.021304

3N interaction (Δ-hole interaction)

main contribution of 3N





Fujita-Miyazawa type
3N interaction

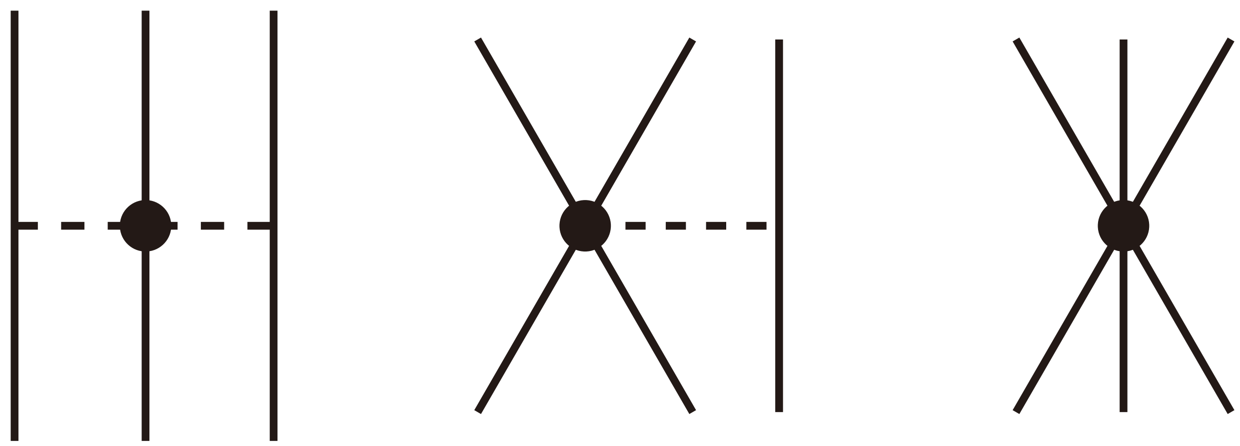
Effective
2N interaction

- Adding up effective 2N interaction derived from 3N interaction to EKK 2N effective interaction [1]
- This is one of the lowest order interaction from 3N force and for higher order we are working on…

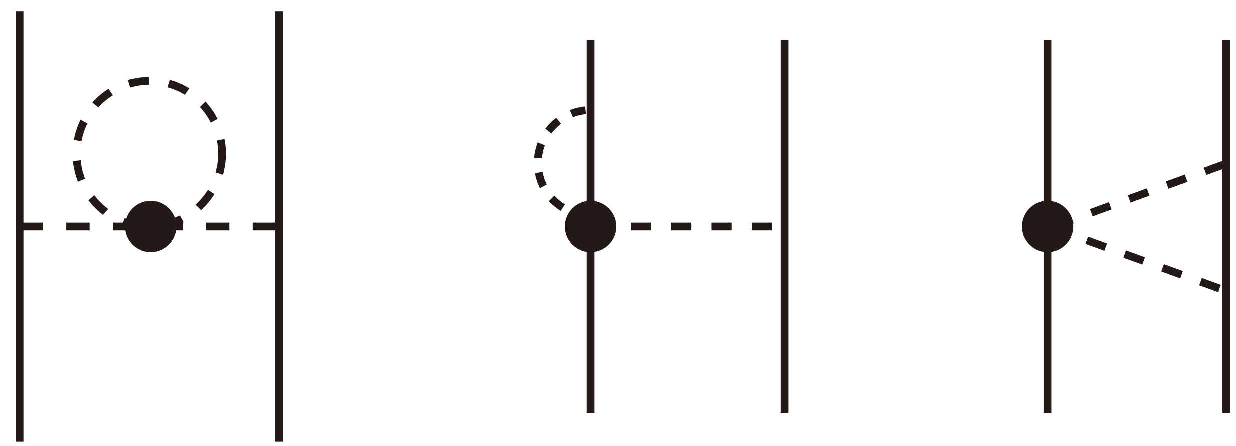
[1] T. Otsuka, T. Suzuki, J. D. Holt, A. Schwenk, and Y. Akaishi, Phys. Rev. Lett. 105, 032501 (2010).

3N interaction (momentum space integration)

More modern and sophisticated choice of 3N



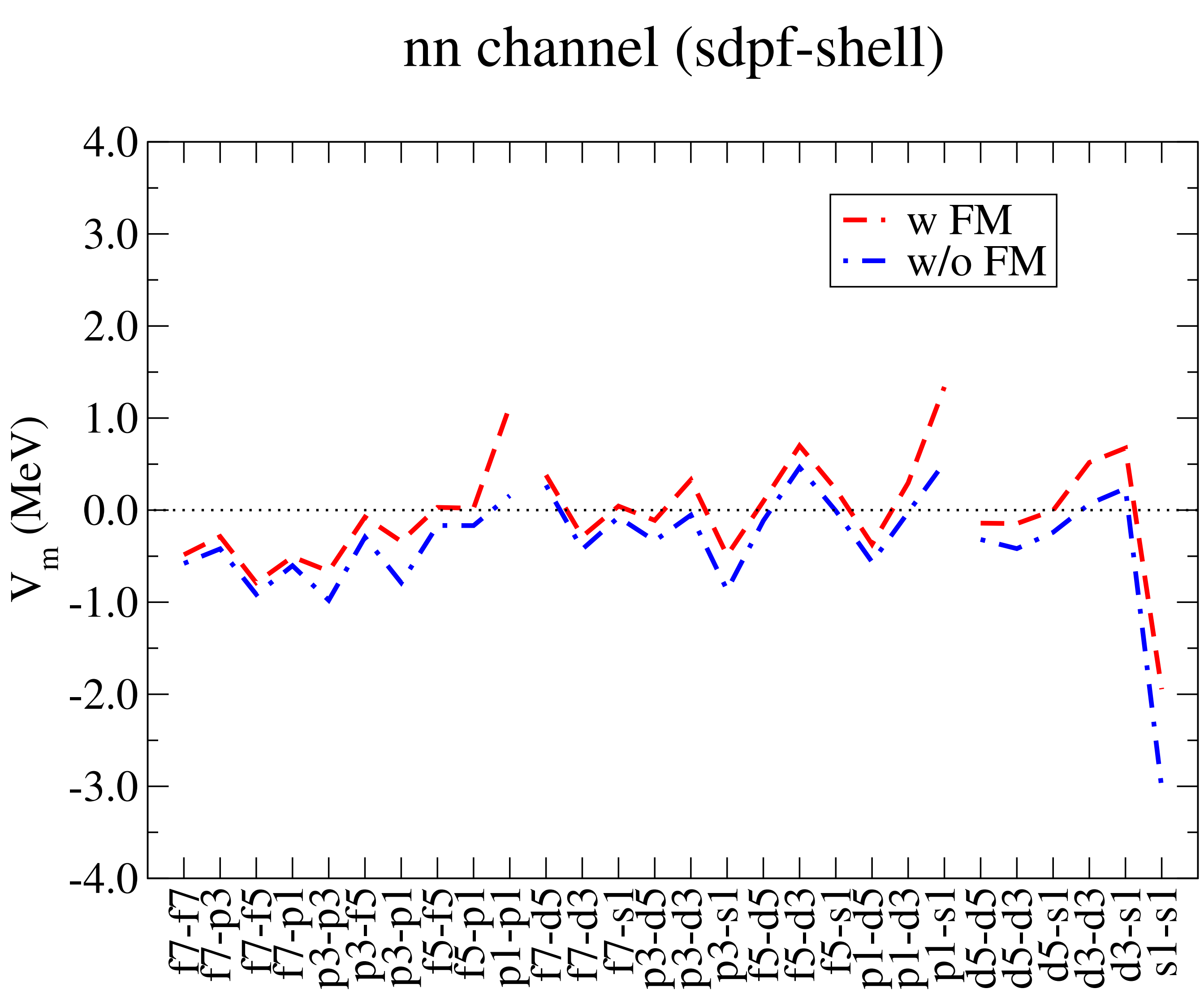
3N force from effective filed theory

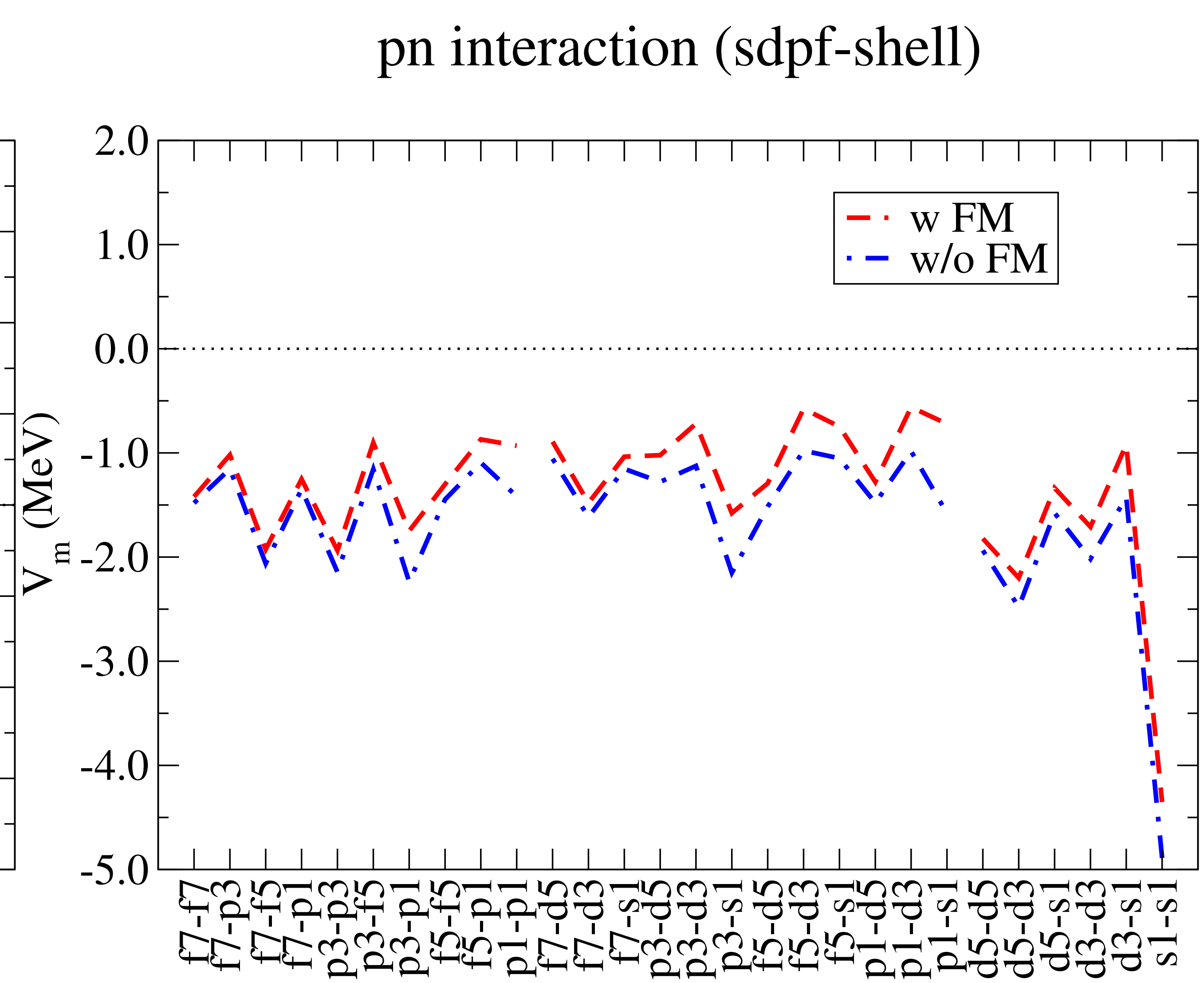


Effective 2N force from 3N force

-> $V_{lowk} + V_{3N}$ as starting point of MBPT

Monopole interactions





repulsive 3N force

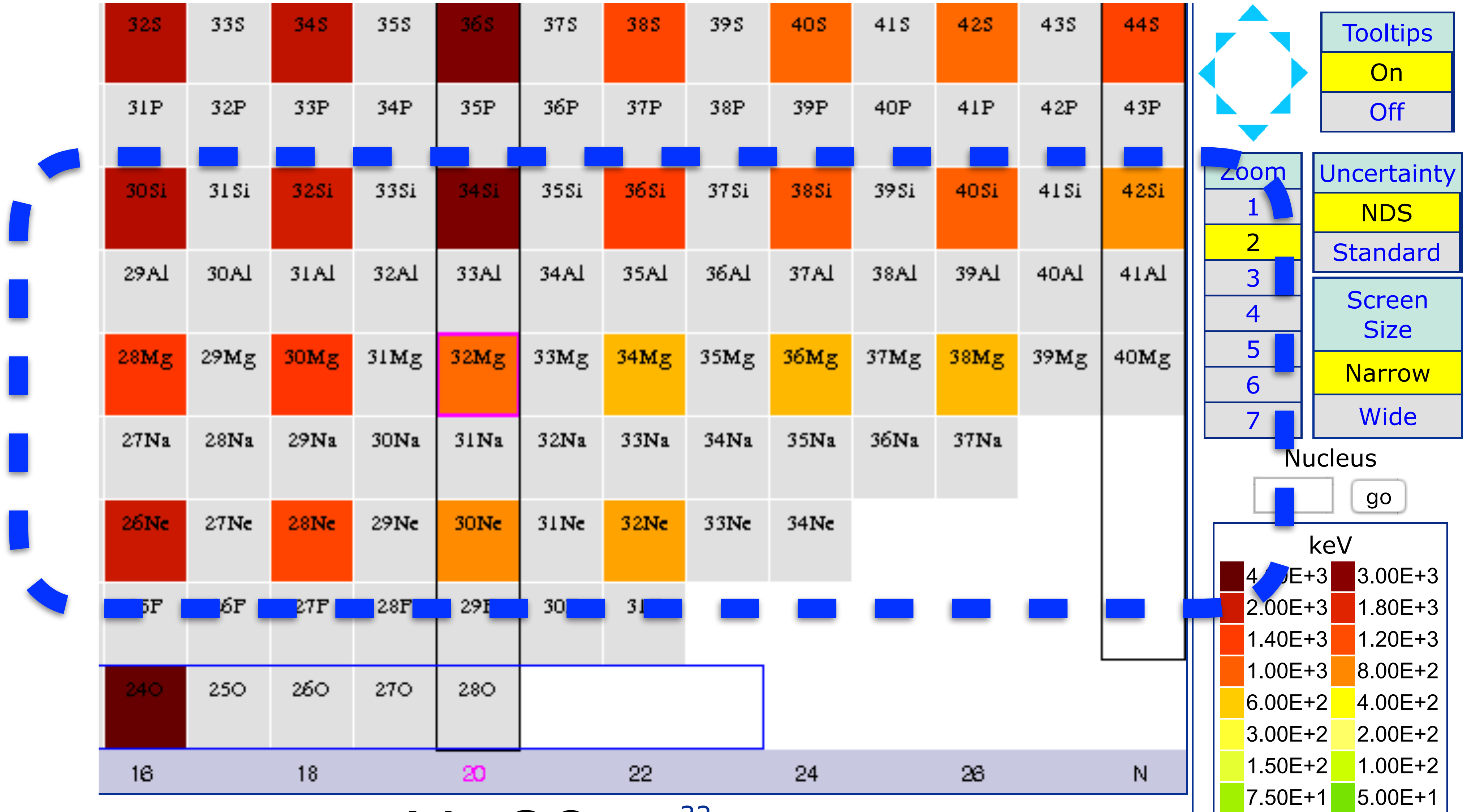
SPE fitted

SPE set (MeV)

d5/2	-5.7
s1/2	-3.0
d3/2	1.8
f7/2	2.9
p3/2	3.6
p1/2	5.4
f5/2	5.4

Island of inversion

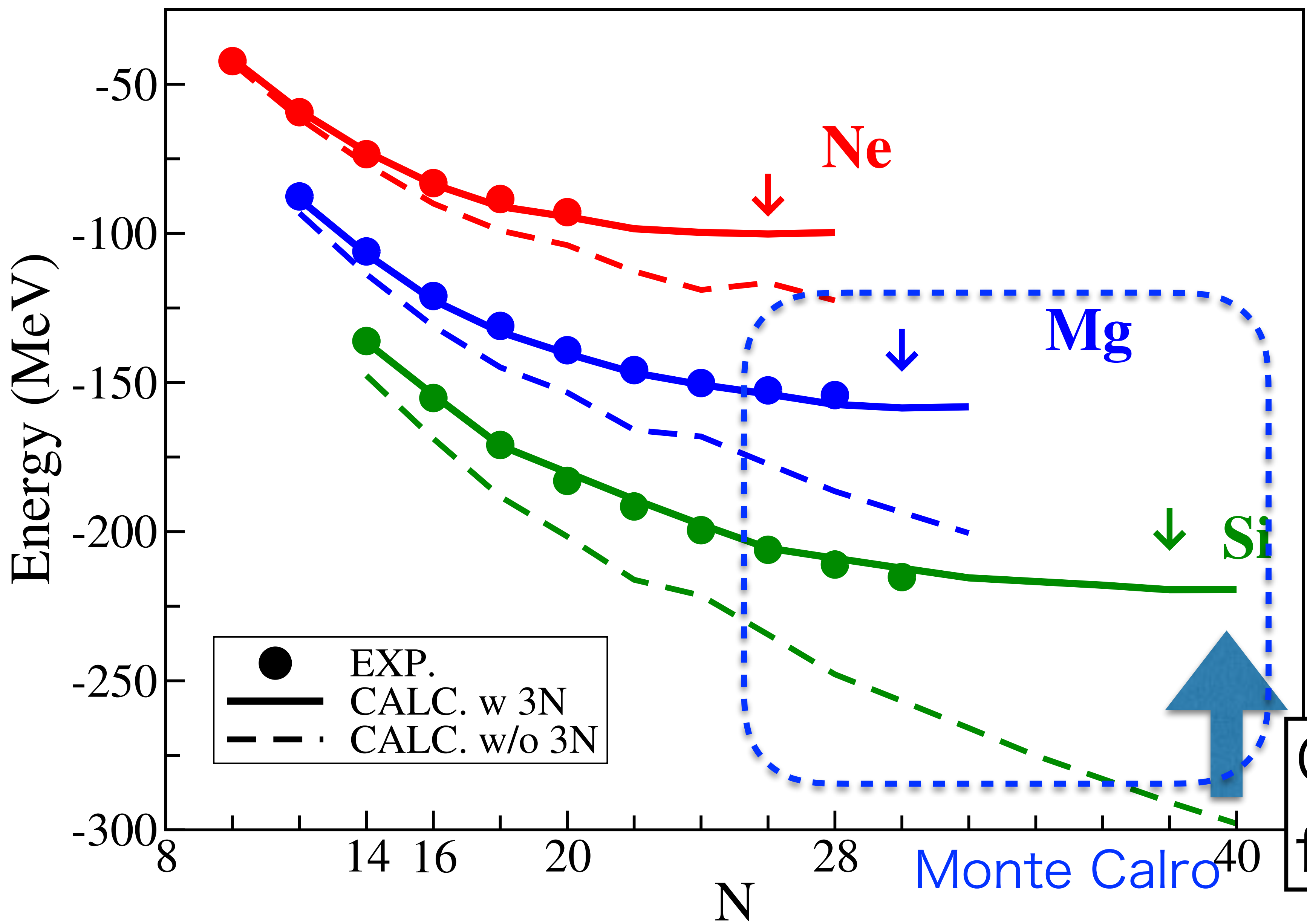
http://www.nndc.bnl.gov/nudat2/reCenter.jsp?z=12&n=20



N=20

- Around Ne and Mg region N=20 major gap disappears. (small 2+ energy for even-even nuclei, large deformation, etc…)
- Ground state is consist of “**inverse**” configuration, i.e. intruder configuration
- Can microscopic theory describe this disappearance of major magic number?

Ground state energies and dripline



Monte Calro
Shell model

Contribution
from 3N force

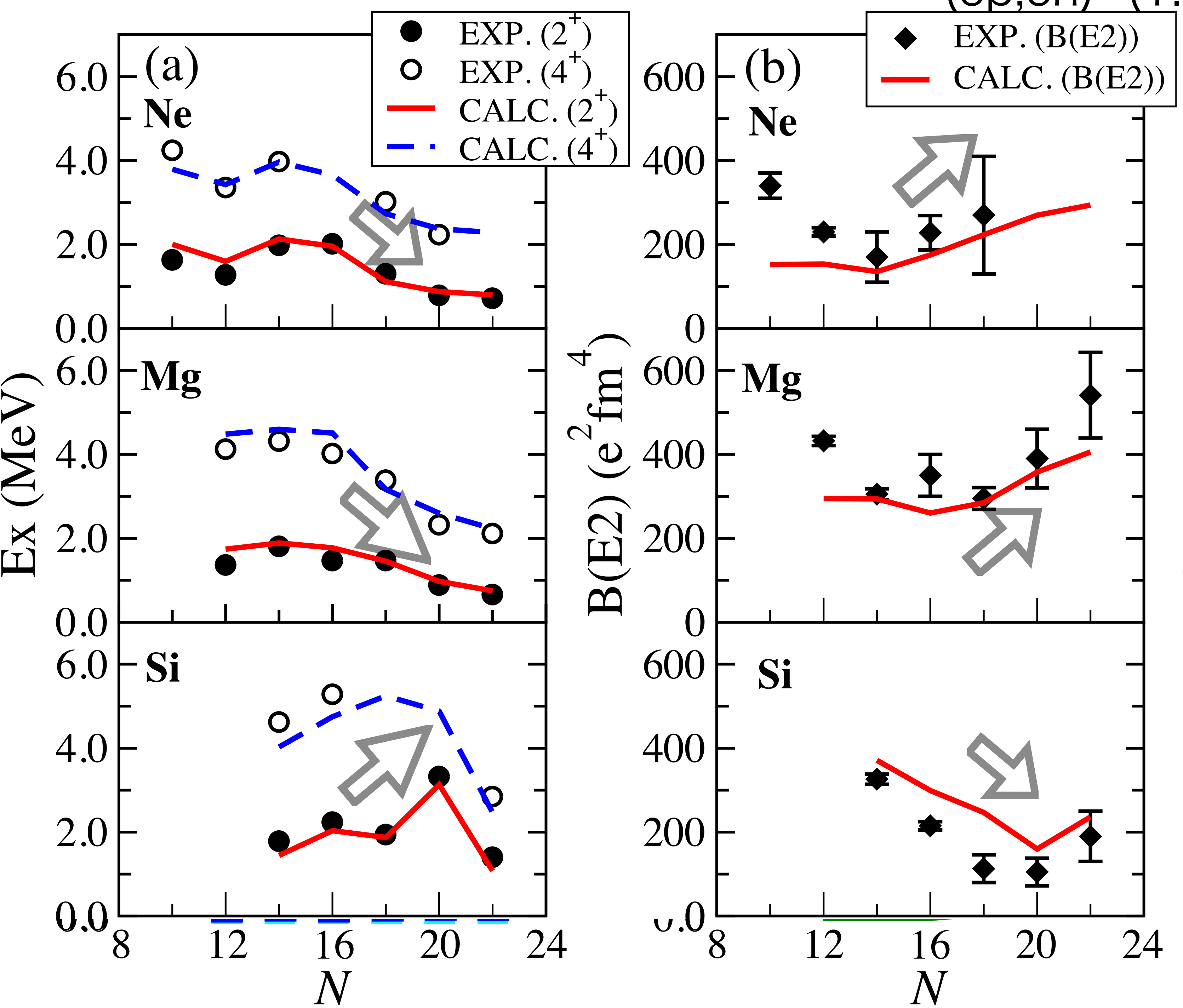
Many body perturbation
up to 3rd order
starting from N3LO
P+Q: 17hw (converged)
SPE fitted

Resource:
~10^6 node*hour
in K-computer for
around **100 states**

- Contribution of 3N force is significant in neutron-rich nuclei
- Predictions of dripline
- Combination of **Microscopic theory** and **Large scale calc.**

Shell structure in “island of inversion”

Effective charges
(ep,en)=(1.25, 0.25)

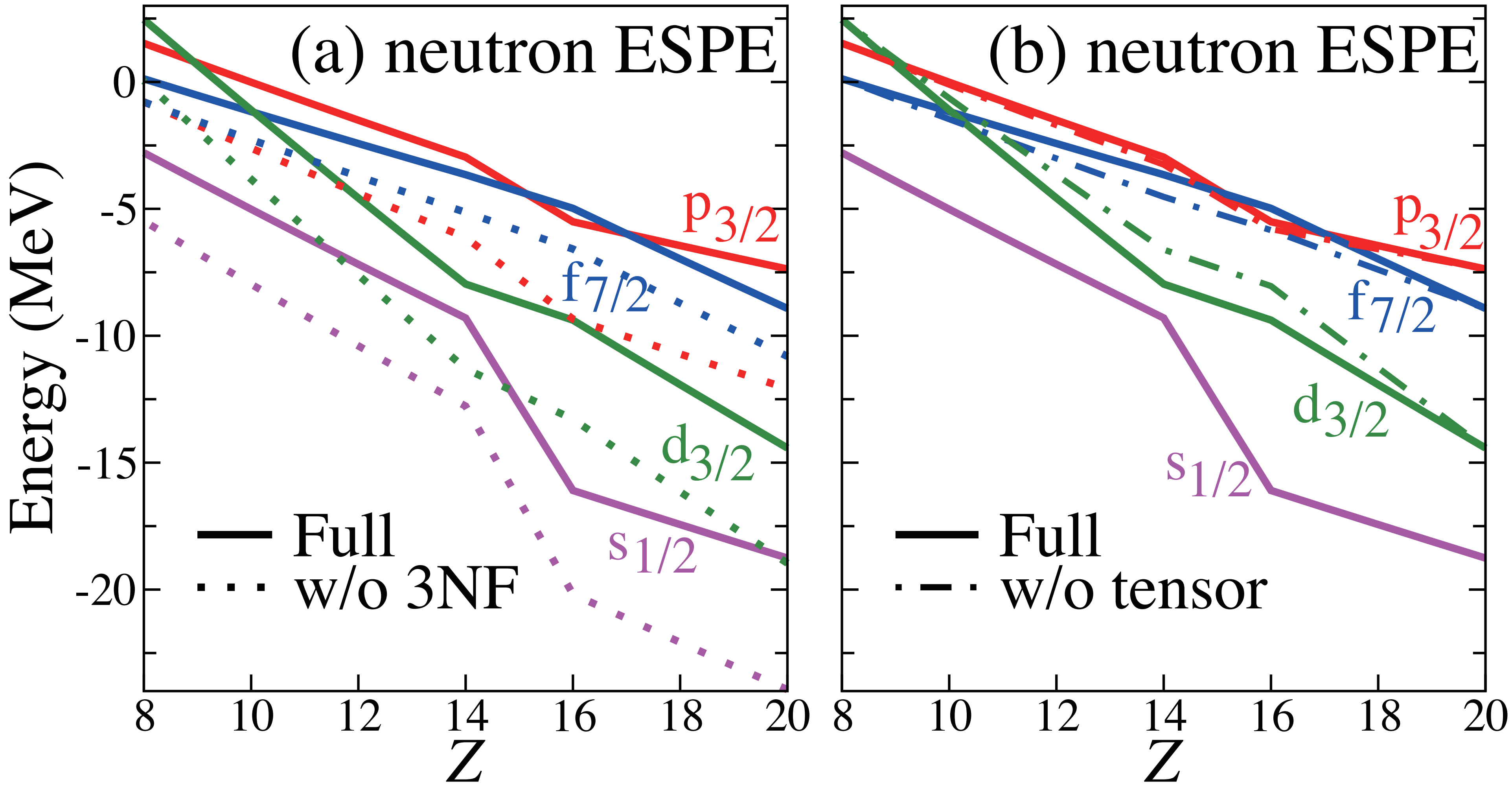


Clear indication of breaking of N=20 gap for Ne and Mg.

N=20 gap remains in Si case.

Evolution of single particle states

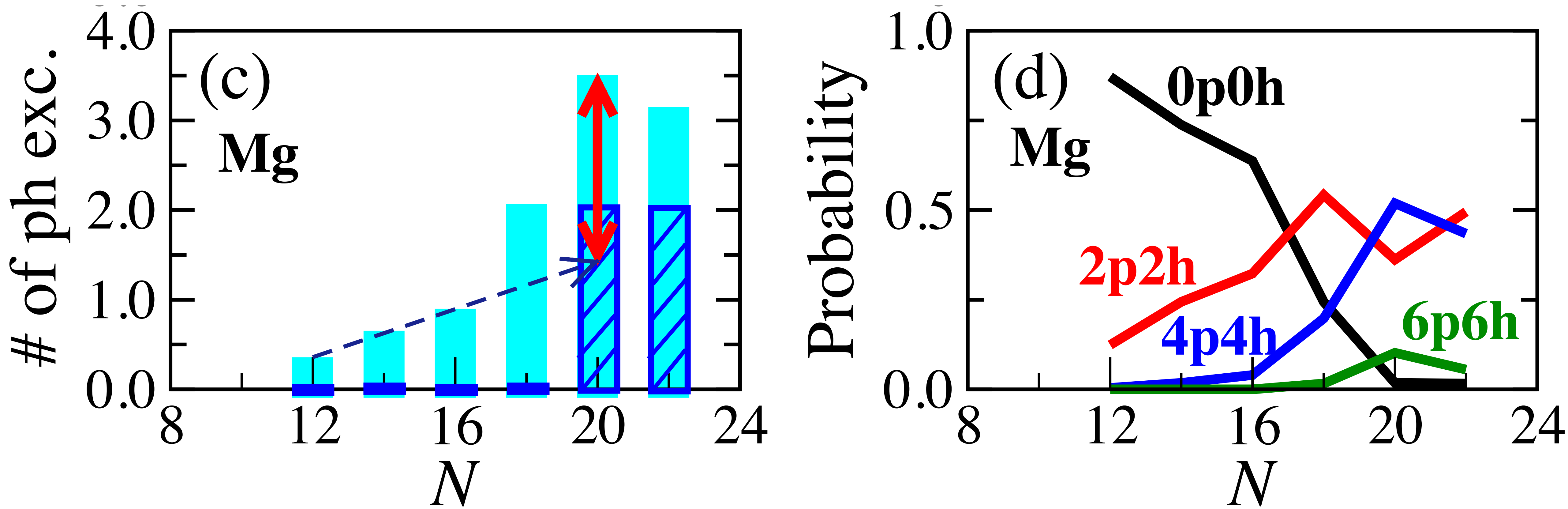
Effective single particle energies at N=20 isotones



3NF: general shift

Tensor force: drive sd to pf gap

Wave function of Mg isotopes



<u>"modest" and "abrupt" excitation</u>

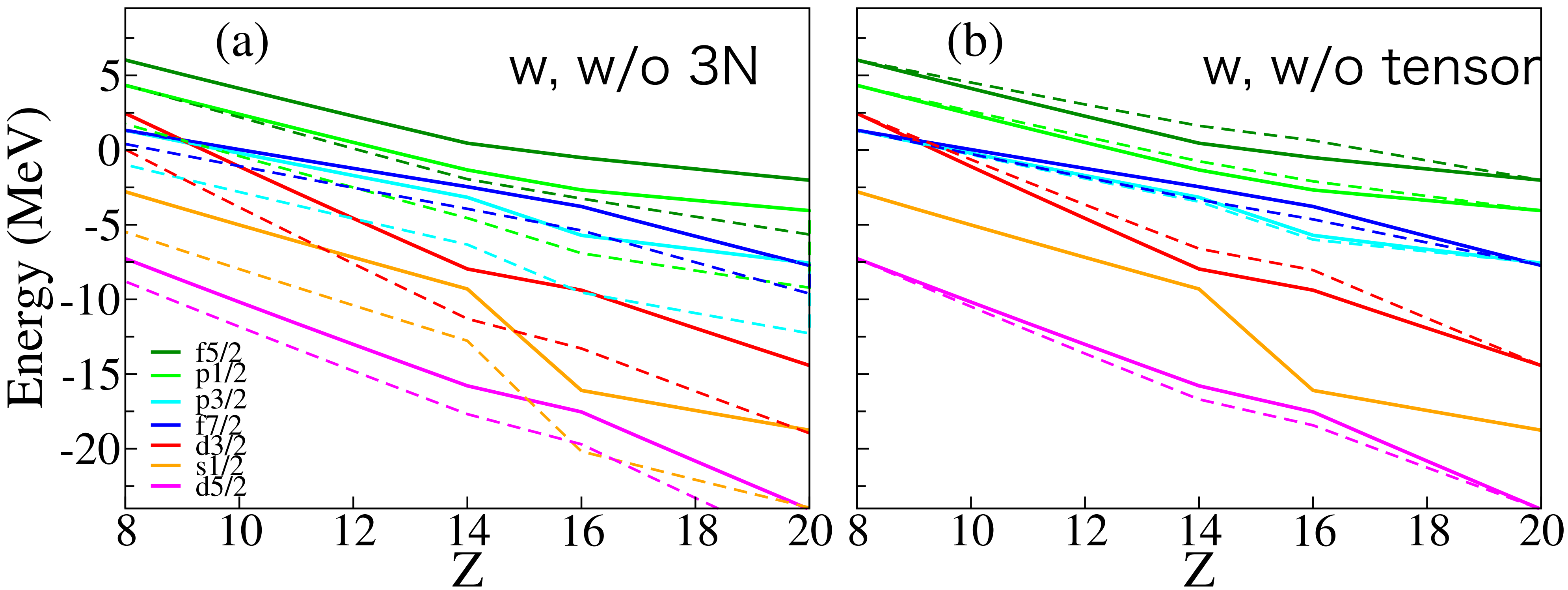
modest: shifting between two shells (e.g. pairing)

abrupt : strong deformation

Abrupt excitation roughly corresponds to conventional 2p2h excitation model

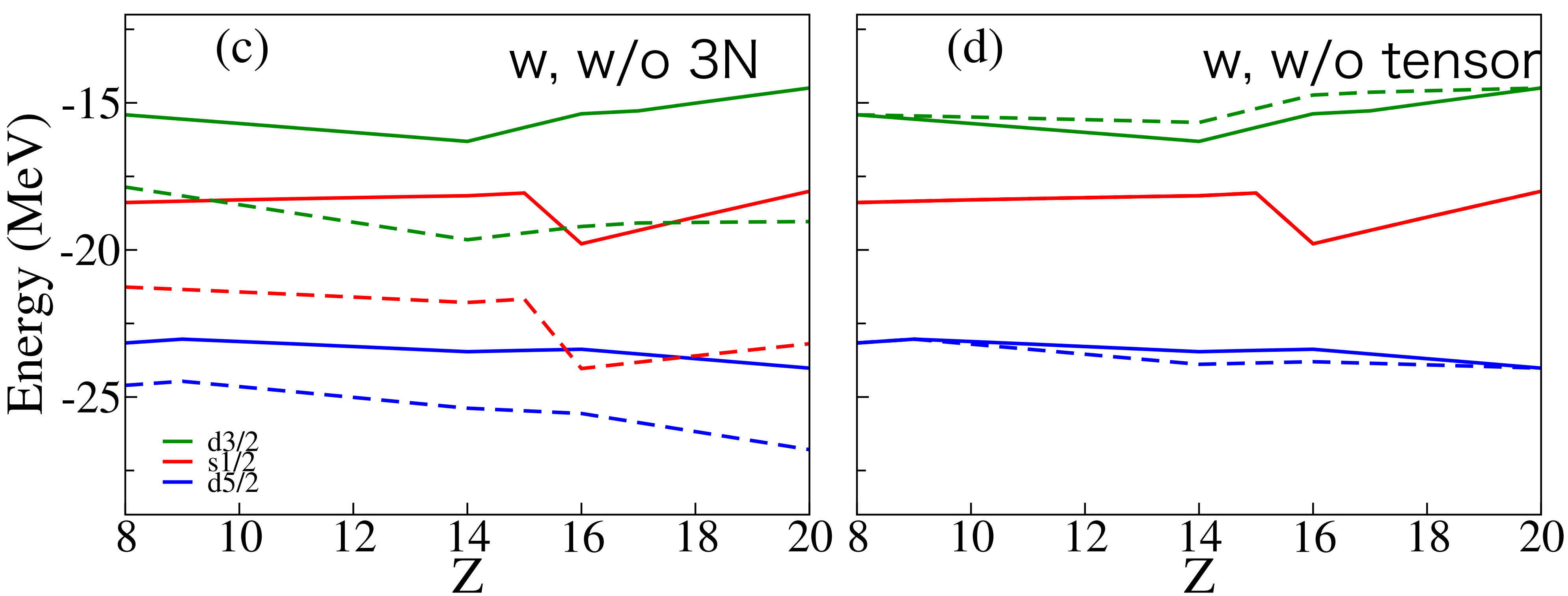
Effective single particle energies

Neutron(N=20)



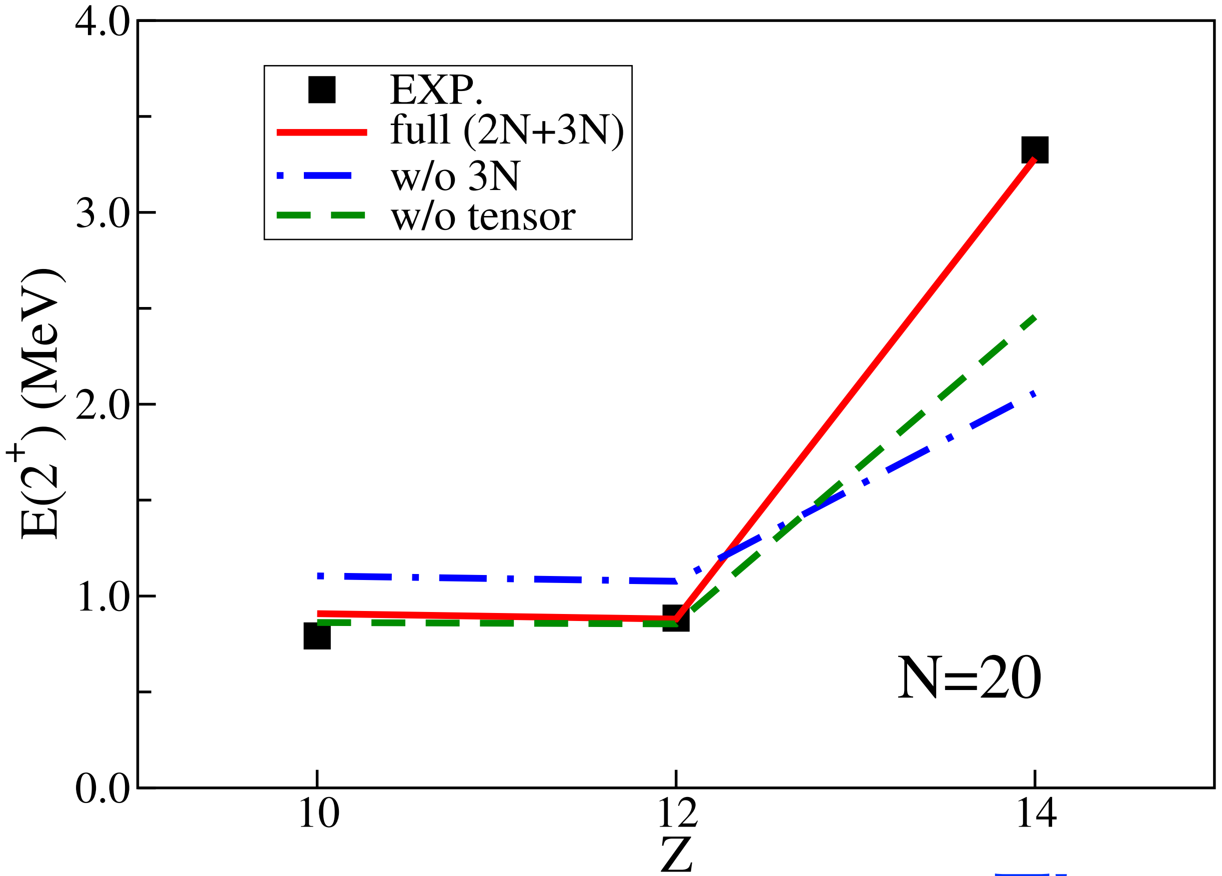
- N=20 gap enhanced by **tensor force**

Proton(N=20)



- Z=14 gap enhanced by **3N force**

Tensor force and 3N force



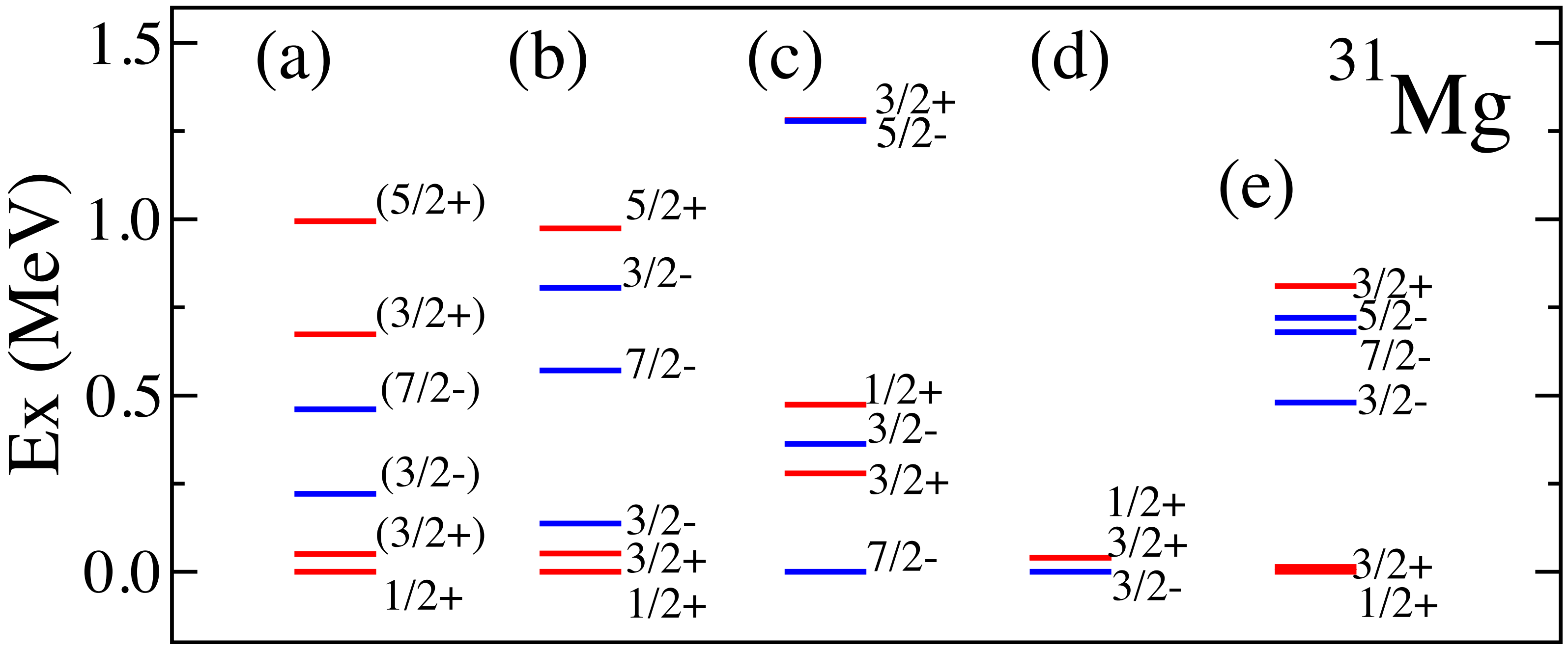
Tensor force

- Enlarge the neutron **N=20 gap**
- π d5/2 - ν f7/2(p3/2) = **repulsive**
- π d5/2 - ν d3/2 = **attractive**
- Needed for doubly magic structure of Si34

Three-body force

- Enlarge the proton **Z=14 gap**
- Different physics, but needed for **doubly magic** structure of Si34 as well as tensor force

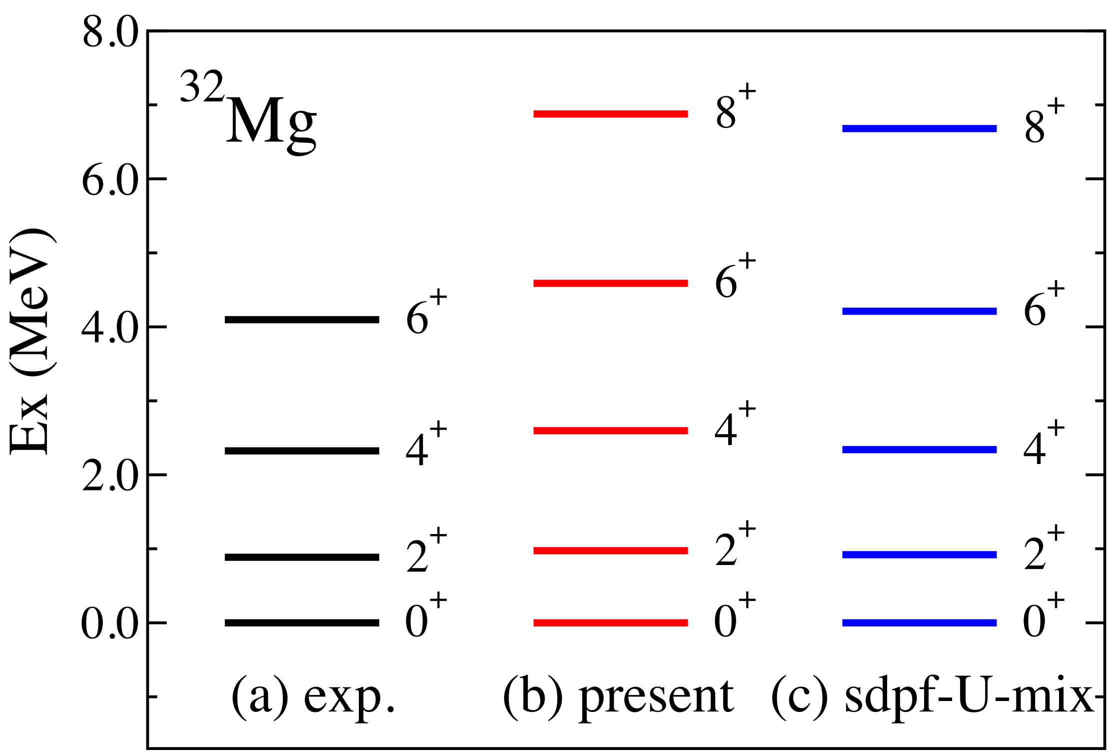
31Mg



(a) EXP.
(b) EKK+3N
(c) sdpf-m (phenom.)
(d) sdpf-U-mix (phenom.)
(e) AMD+GCM

- onset of island of inversion
- ordering of levels reproduced
- positive=2hw dominanted
- negative=1hw dominated

<u>Deformation of 32Mg</u>



Summary and conclusion

- MBPT is the theory to construct the effective Hamiltonian starting from nuclear force.
- KK method or folded diagram method is introduced with formal theory and time-dependent perturbation theory
- EKK method is introduced to derive the effective interaction for the shell model which is applicable to **multi-shell** system.
- As an application of EKK method, the physics in the **"island of inversion"** is discussed in K-computer.
- **EKK** and **3N** combination is the powerful tool to explore the wide area of the nuclear chart