New dynamic critical phenomena in nuclear and quark superfluids

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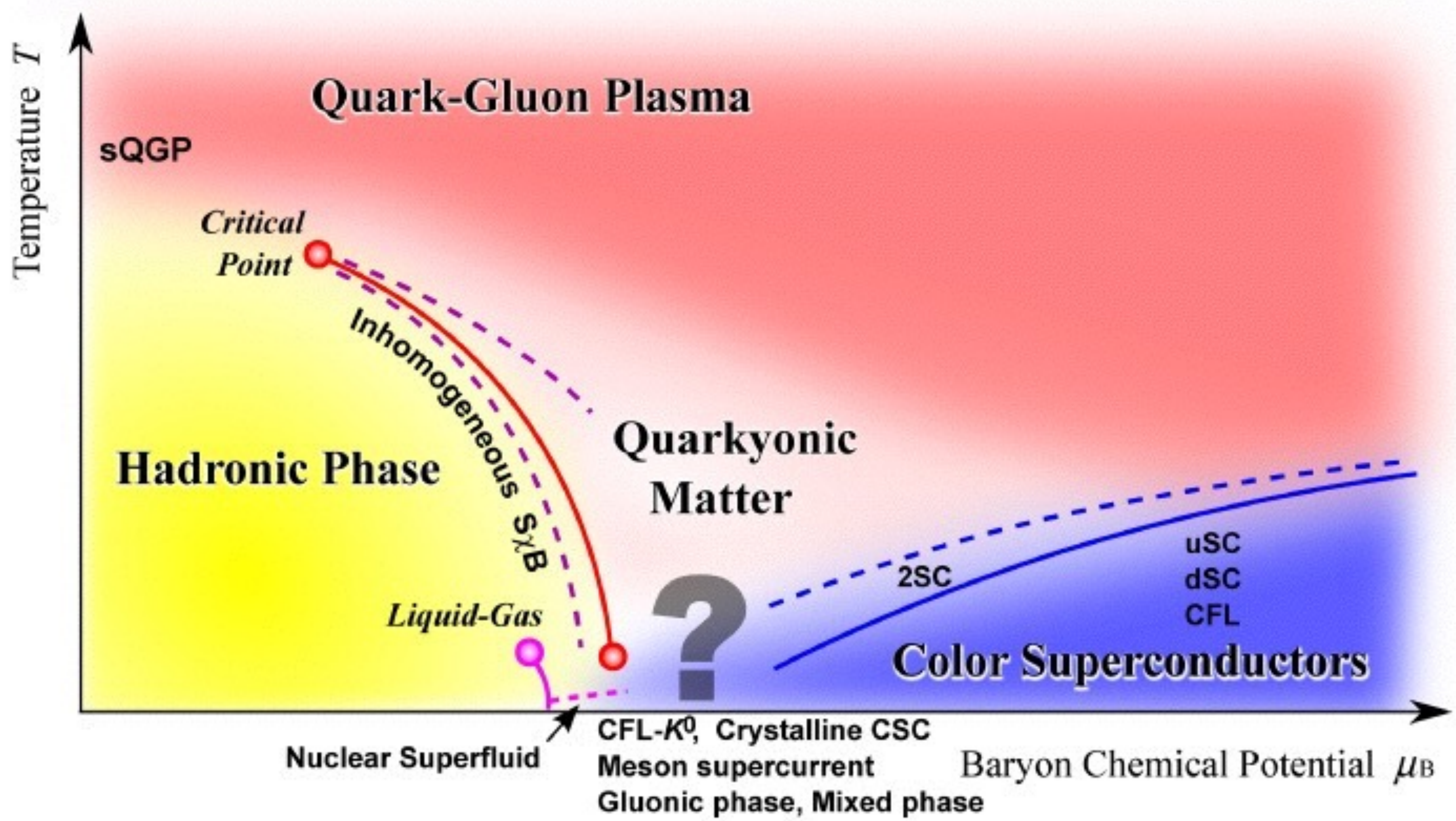
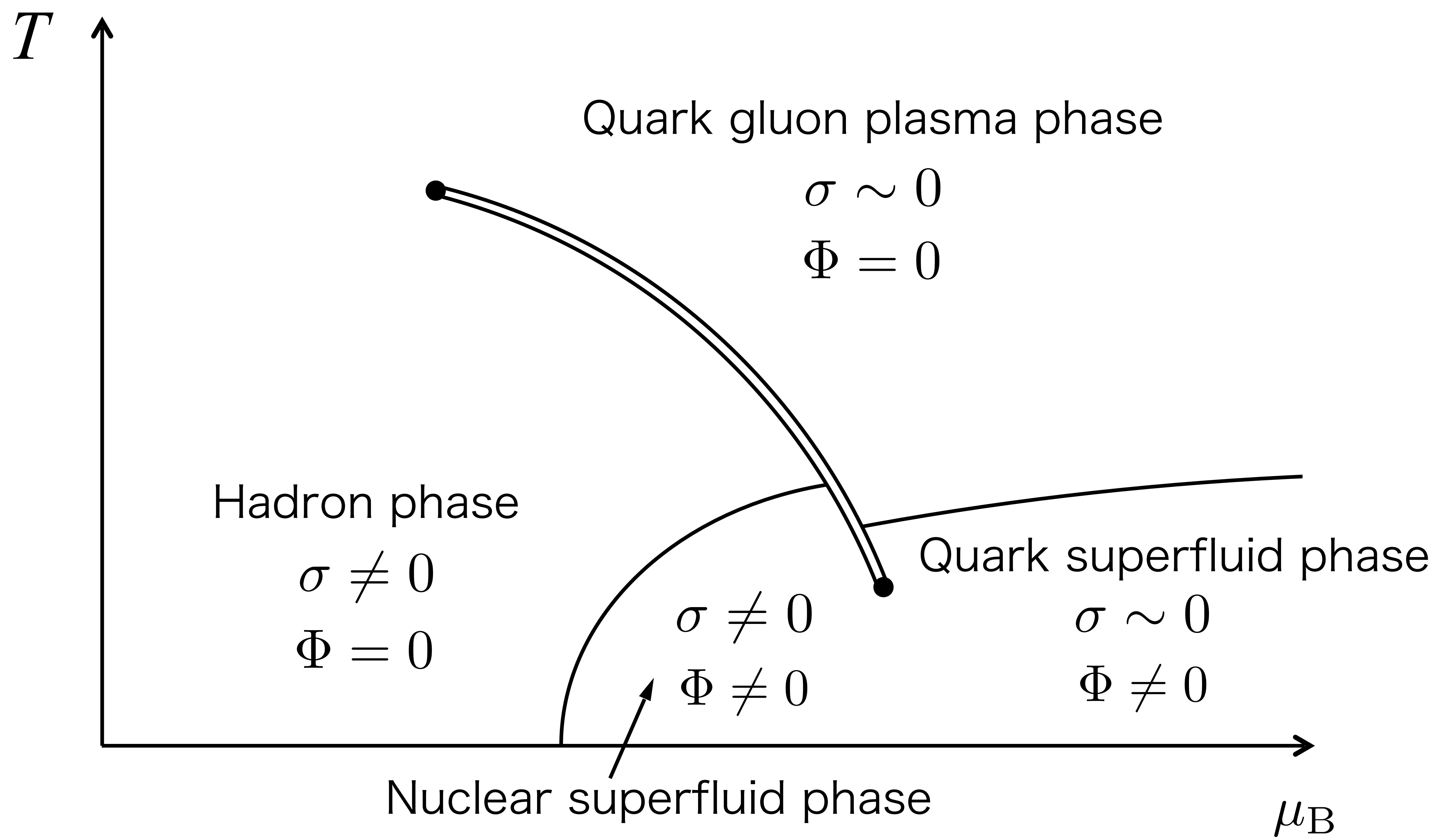
In collaboration with Naoki Yamamoto

Phase diagram of QCD



K. Fukushima, T. Hatsuda, Rept. Prog. Phys. (2010)

phases of QCD



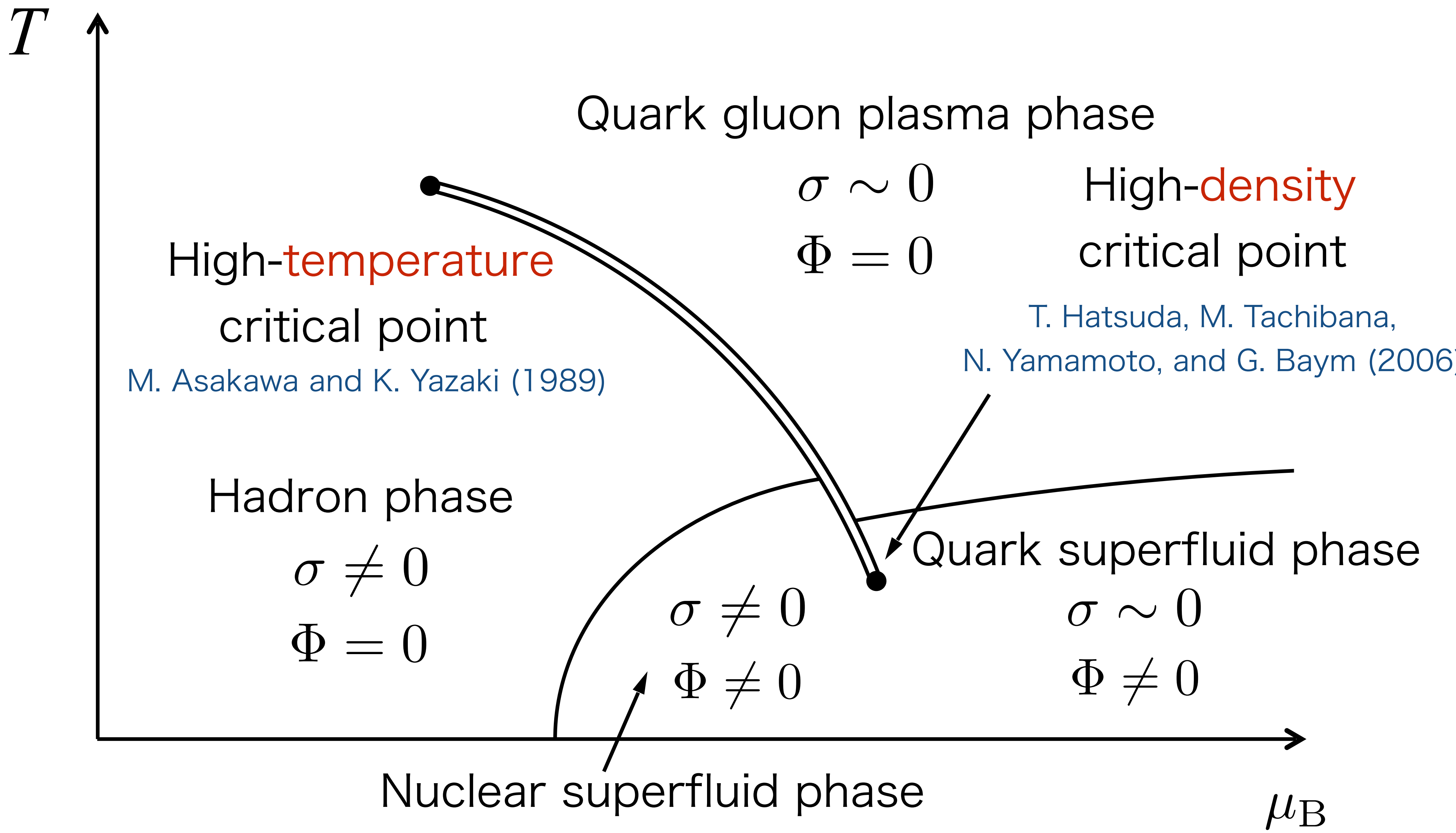
Chiral condensate: $\sigma \equiv \langle \bar{q} q \rangle$

~~Chiral symmetry~~

Diquark condensate: $\Phi \equiv \langle q q \rangle$

~~$U(1)_B$ symmetry~~

QCD Critical points



chiral condensate: $\sigma \equiv \langle \bar{q} q \rangle$

~~Chiral symmetry~~

diquark condensate: $\Phi \equiv \langle qq \rangle$

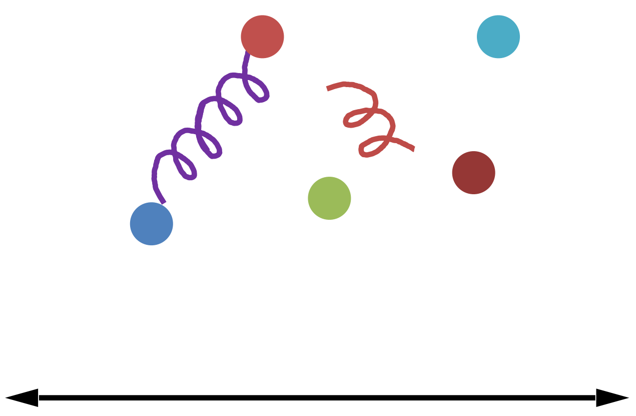
~~$U(1)_B$ symmetry~~

Universality classes of QCD critical points

Universality class	High-temperature	High-density
Static	3D Ising	?
Dynamic	Model H H. Fujii (2003), D. T. Son and M. A. Stephanov (2004)	?

Dynamic universality class

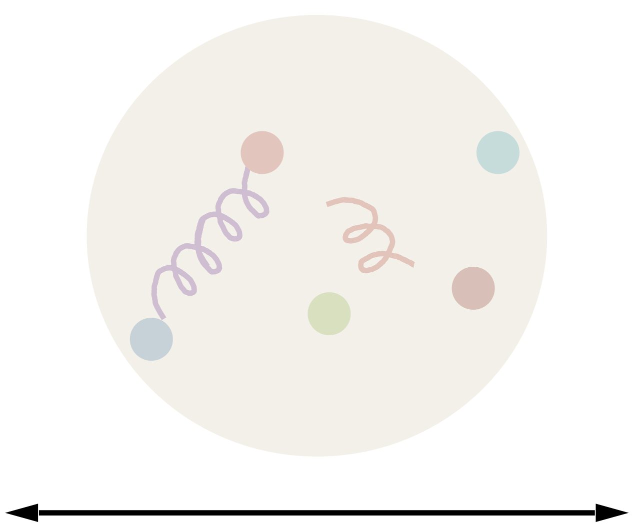
Microscopic theory



$$\xi \gg \Lambda_{\mathrm{QCD}}^{-1}$$

Dynamic universality class

Microscopic theory



$\xi \gg \Lambda_{\mathrm{QCD}}^{-1}$

coarse-graining $\longrightarrow$

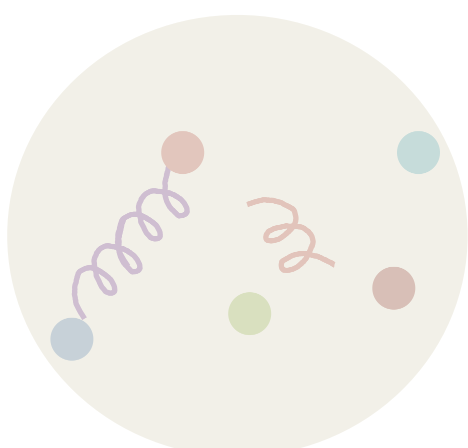
Effective theory

Hydrodynamic variables:

- Order parameters
- Conserved charge densities
- Nambu-Goldstone modes

Dynamic universality class

Microscopic theory



$\xi \gg \Lambda_{\rm QCD}^{-1}$

coarse-graining →

Effective theory

Hydrodynamic variables:

- Order parameters
- Conserved charge densities
- Nambu-Goldstone modes

Same symmetries

Conventional classification

Hohenberg and Halperin: Theory of dynamic critical phenomena

TABLE I. Some dynamical models treated by renormalization-group methods.

Model	Designation	System	Dimension order of parameter	Non-conserved fields	Conserved fields	Non-vanishing Poisson bracket
Relaxational	A	Kinetic Ising anisotropic magnets	n	ψ	None	None
	B	Kinetic Ising uniaxial ferromagnet	n	None	ψ	None
	C	Anisotropic magnets structural transition	n	ψ	m	None
Fluid	H	Gas–liquid binary fluid	1	None	ψ $\mathbf{j}$	$\{\psi, \mathbf{j}\}$
Symmetric planar magnet	E	Easy-plane magnet, $h_z = 0$	2	ψ	m	$\{\psi, m\}$
Asymmetric planar magnet	F	Easy-plane magnet, $h_z \neq 0$ superfluid helium	2	ψ	m	$\{\psi, m\}$
Isotropic antiferromagnet	G	Heisenberg antiferromagnet	3	ψ	$\mathbf{m}$	$\{\psi, \mathbf{m}\}$
Isotropic ferromagnet	J	Heisenberg ferromagnet	3	None	ψ	$\{\psi, \psi\}$

P. C. Hohenberg and B. I. Halperin (1977)

Universality classes of QCD critical points

Universality class	High-temperature	High-density
Static	3D Ising	3D Ising
Dynamic	Model H H. Fujii (2003), D. T. Son and M. A. Stephanov (2004)	New class

New dynamic universality class beyond the conventional classification

Outline

1. Hydrodynamic variables
2. Static critical phenomena
3. Dynamic critical phenomena

Hydrodynamic variables

- Chiral condensate: $\sigma \equiv \bar{q}q - \langle \bar{q}q \rangle$
- Baryon number density: $n \equiv \bar{q}\gamma^0 q - \langle \bar{q}\gamma^0 q \rangle$
- Superfluid phonon: $\langle qq \rangle \sim e^{i\theta}$
- Energy density: $\varepsilon \equiv T^{00} - \langle T^{00} \rangle$
- Momentum density: $\pi^i \equiv T^{0i}$

Ginzburg-Landau theory

$$F[\sigma, n, \theta] = \int \mathrm{d}\boldsymbol{r} \left[\frac{a}{2} (\boldsymbol{\nabla}\sigma)^2 + b \boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla} n + \frac{c}{2} (\boldsymbol{\nabla} n)^2 + \frac{d}{2} (\boldsymbol{\nabla}\theta)^2 + V(\sigma, n) + \cdots \right]$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2} n^2$$

- Near critical point ⟶ Small order parameter
- Interested in long-range behavior ⟶ Derivative expansion
- QCD symmetries ⟶ constrains on the expansion

chiral symmetry, baryon-number symmetry, CPT symmetries

Ginzburg-Landau theory

$$F[\sigma, n, \theta] = \int \mathrm{d}\boldsymbol{r} \left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b\boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla} n + \frac{c}{2}(\boldsymbol{\nabla} n)^2 + \frac{d}{2}(\boldsymbol{\nabla}\theta)^2 + V(\sigma, n) + \cdots \right]$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2$$

- Superfluid phonon θ is irrelevant to the statics.

$$F[\sigma, n, \theta] = F_{\mathrm{MF}}[\sigma, n] + F_{\mathrm{MF}}[\theta] + \gamma\sigma(\boldsymbol{\nabla}\theta)^2 + \cdots$$

decoupled from the time reversal symmetry

derivative coupling due to U(1) symmetry

Ginzburg-Landau theory

$$F[\sigma, n, \theta] = \int \mathrm{d}\boldsymbol{r} \left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b\boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla}n + \frac{c}{2}(\boldsymbol{\nabla}n)^2 + \frac{d}{2}(\boldsymbol{\nabla}\theta)^2 + V(\sigma, n) + \cdots \right]$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2$$

- Static critical phenomena

$$\langle \sigma(\boldsymbol{r})\sigma(0) \rangle = \frac{1}{4\pi r} e^{-r/\xi} \qquad \xi \sim \frac{1}{\sqrt{AC - B^2}}$$

$$\chi_{\mathrm{B}} \equiv \frac{\partial n}{\partial \mu} = T \left\langle n^2 \right\rangle_{\boldsymbol{q} \to \boldsymbol{0}} \sim \xi^{2-\eta} \qquad \eta = 0.04$$

same as high-temperature critical point

Langevin equation

$$\dot{x}_i(\boldsymbol{r},t) = -\gamma_{ij}\frac{\delta F}{\delta x_j} - \int \mathrm{d}\boldsymbol{r}' \left[x_i(\boldsymbol{r}), x_j(\boldsymbol{r}')\right] \frac{\delta F}{\delta x_j(\boldsymbol{r}')} + \text{noise term}$$

dissipative term reversible term

$$(x_i \equiv \sigma, n, \theta)$$

Langevin equation

$$\dot{\sigma}(\boldsymbol{r}) = -\Gamma_{\sigma\sigma}\frac{\delta F}{\delta\sigma(\boldsymbol{r})} + \Gamma_{\sigma n}\boldsymbol{\nabla}^2\frac{\delta F}{\delta n(\boldsymbol{r})}$$

$$\dot{n}(\boldsymbol{r}) = \Gamma_{\sigma n}\boldsymbol{\nabla}^2\frac{\delta F}{\delta\sigma(\boldsymbol{r})} + \Gamma_{nn}\boldsymbol{\nabla}^2\frac{\delta F}{\delta n(\boldsymbol{r})} - \int \mathrm{d}\boldsymbol{r}'\,[n(\boldsymbol{r}),\theta(\boldsymbol{r}')]\,\frac{\delta F}{\delta\theta(\boldsymbol{r}')}$$

$$\dot{\theta}(\boldsymbol{r}) = -\Gamma_{\theta\theta}\frac{\delta F}{\delta\theta(\boldsymbol{r})} - \int \mathrm{d}\boldsymbol{r}'\,[\theta(\boldsymbol{r}),n(\boldsymbol{r}')]\,\frac{\delta F}{\delta n(\boldsymbol{r}')}$$

$$[\theta(\boldsymbol{r}),n(\boldsymbol{r}')] = \delta(\boldsymbol{r}-\boldsymbol{r}')$$

Hydrodynamic modes

$$\begin{pmatrix} i\omega - \Gamma_{\sigma\sigma}A - (\Gamma_{\sigma\sigma}a + \Gamma_{\sigma n}B)\boldsymbol{q}^2 & -\Gamma_{\sigma\sigma}B - (\Gamma_{\sigma\sigma}b + \Gamma_{\sigma n}C)\boldsymbol{q}^2 & 0 \\ -(\Gamma_{\sigma n}A + \Gamma_{nn}B)\boldsymbol{q}^2 & i\omega - (\Gamma_{\sigma n}B + \Gamma_{nn}C)\boldsymbol{q}^2 & d\boldsymbol{q}^2 \\ -B - b\boldsymbol{q}^2 & -C - c\boldsymbol{q}^2 & i\omega - \Gamma_{\theta\theta}d\boldsymbol{q}^2 \end{pmatrix} \begin{pmatrix} \sigma \\ n \\ \theta \end{pmatrix} = \mathbf{0}$$

$$\omega = -i\Gamma_{\sigma\sigma}A \qquad \omega^2 = c_{\mathrm{s}}^2\boldsymbol{q}^2$$

relaxation mode of σ | superfluid phonon

Dynamic critical phenomena

- Speed of superfluid phonon

$$c_{\mathrm{s}} \equiv \sqrt{\frac{d}{\chi_{\mathrm{B}}}} \to 0 \qquad \chi_{\mathrm{B}} \equiv \frac{\partial n}{\partial \mu} \sim \xi^{2-\eta}$$

critical slowing down

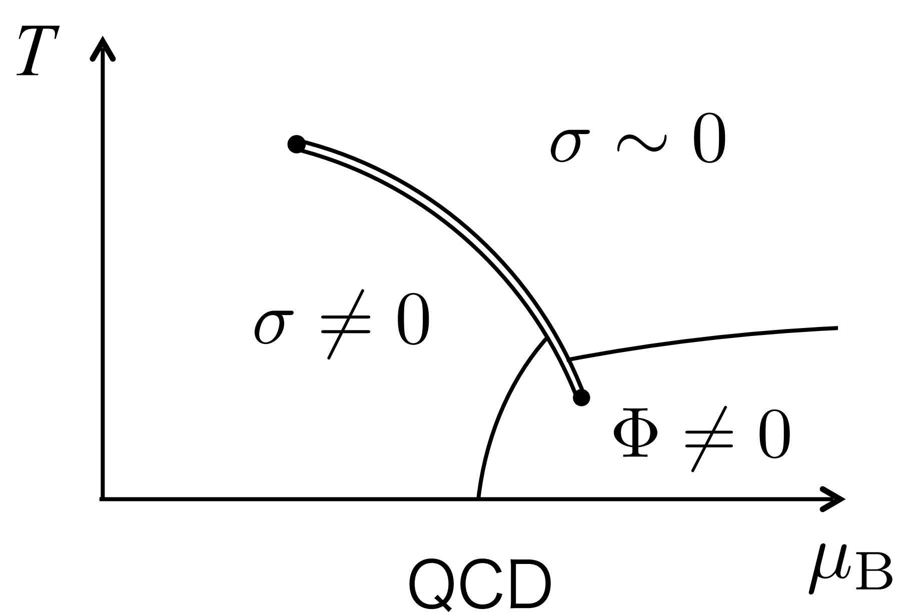
- Dynamic critical exponent $\xi^{-z} \sim c_{\mathrm{s}}|\boldsymbol{q}|$

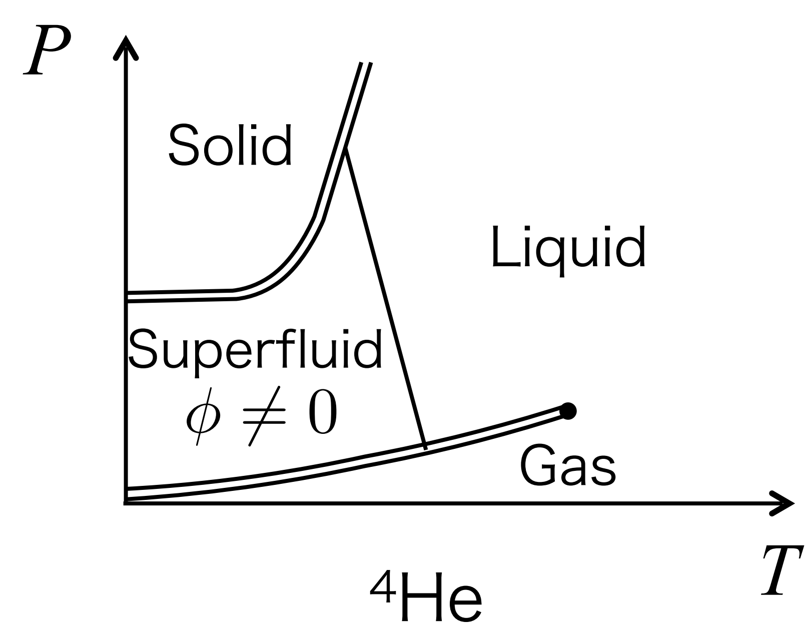
$$z = 2 - \frac{\eta}{2}$$

New dynamic universality class beyond Hohenberg and Halperin's classification

Why the universality class is new?

	Superfluidity	Interplay with chiral condensate
High-μ_B QCD critical point	✔	✔
High-T QCD critical point (Model H)		
Superfluid transition of ^{4}He (Model F)	✔	





ϕ: Superfluid gap

Future heavy-ion collisions

- Dynamic critical phenomena **can** distinguish the QCD critical points. (static critical phenomena **can not**.)
- Observation of the high-density critical point

$\longrightarrow$ indirect evidence of the superfluidity in QCD.

Comparison between QCD critical points

	high-temperature	high-density
dynamical exponent	$z \approx 3$	$z \approx 2$
critical modes	diffusion	superfluid phonon

consequence on observables?

Conclusion

- We found the new dynamic universality class beyond the conventional Hohenberg and Halperin's classification.

Universality class	High-temperature	High-density
Static	3D Ising	3D Ising NS, N. Yamamoto (2017)
Dynamic	Model H H. Fujii (2003), D. T. Son and M. A. Stephanov (2004)	New class NS, N. Yamamoto (2017)