Relativistic quantum molecular dynamics With scalar and vector interactions

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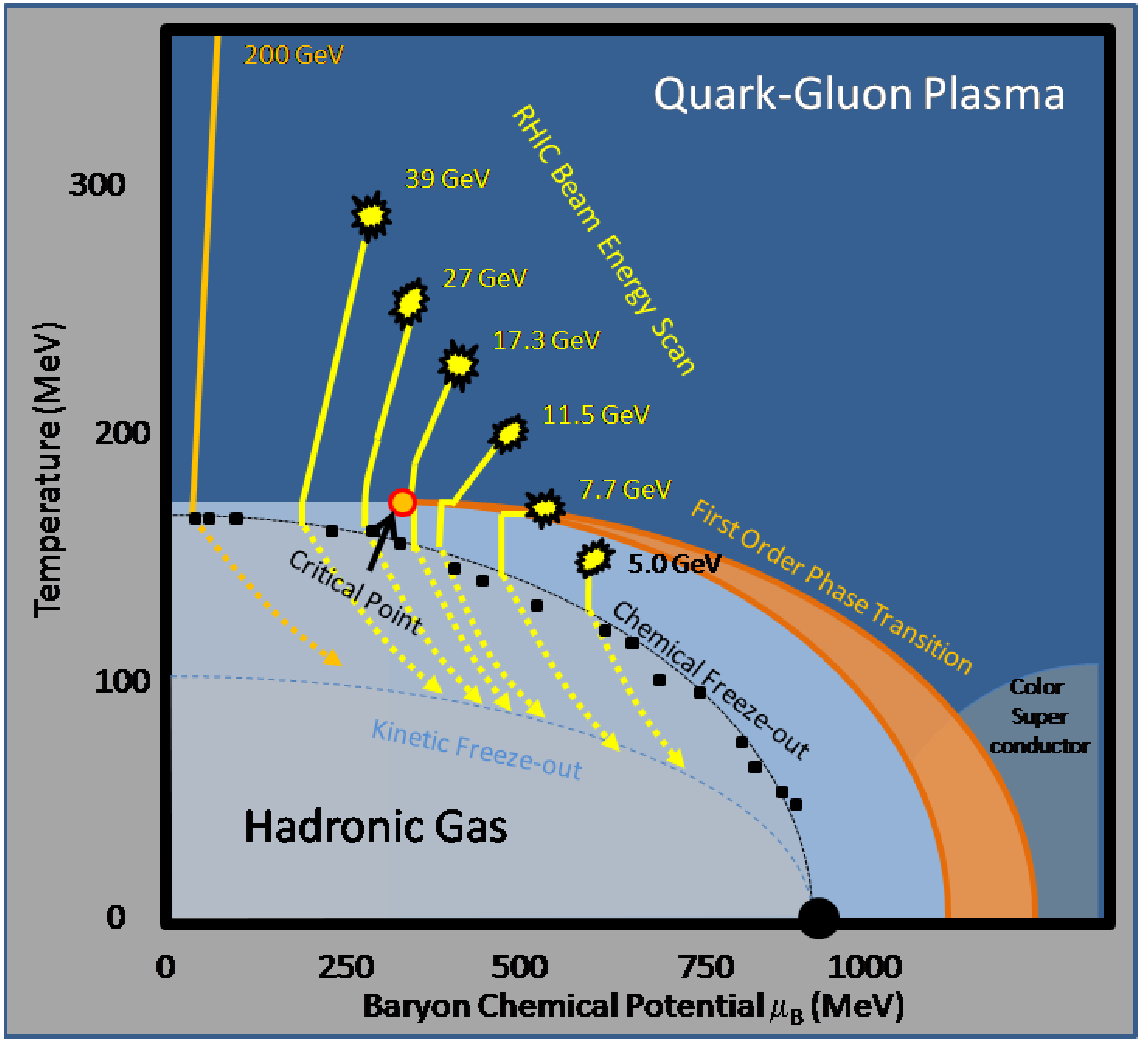
Y. Nara and H. Stoecker, arXiv:1906.03537 [nucl-th]

- Introduction
- RQMD with relativistic mean-field theory (RMF)
  effects of delta matter transition
- Flow, cluster formation, and Baryon number fluctuations results

Heavy Ion Cafe at Sophia University 2019/6/22-23

Search for the QCD equation of state (EoS) by the beam energy scan



RHIC BES and NA61/SHINEprovides valuable information for the QCD phase structure at high baryon densities. New experiments such as FAIR, J-PARC, NICA is planning.

How do we construct dynamical models which can simulate heavy-ion collisions at high baryon density?

Lattice QCD has not covered the J-PARC, FAIR, NICA energy regions.

Modeling at RHIC/LHC

Initial conditions — CGC, Glauber, Event generators

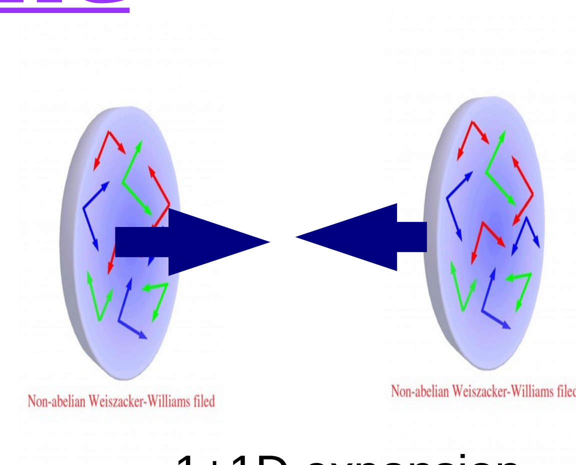
Single thermalization time at tau = 0.6 fm/c

EoS: crossover from lQCD

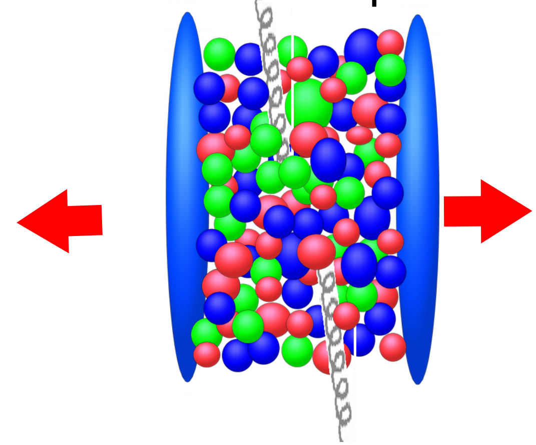
hydrodynamics ⟷ Jet propagation

Jet energy loss

Hadron transport



1+1D expansion



At high energies, factorization in time and energy works:

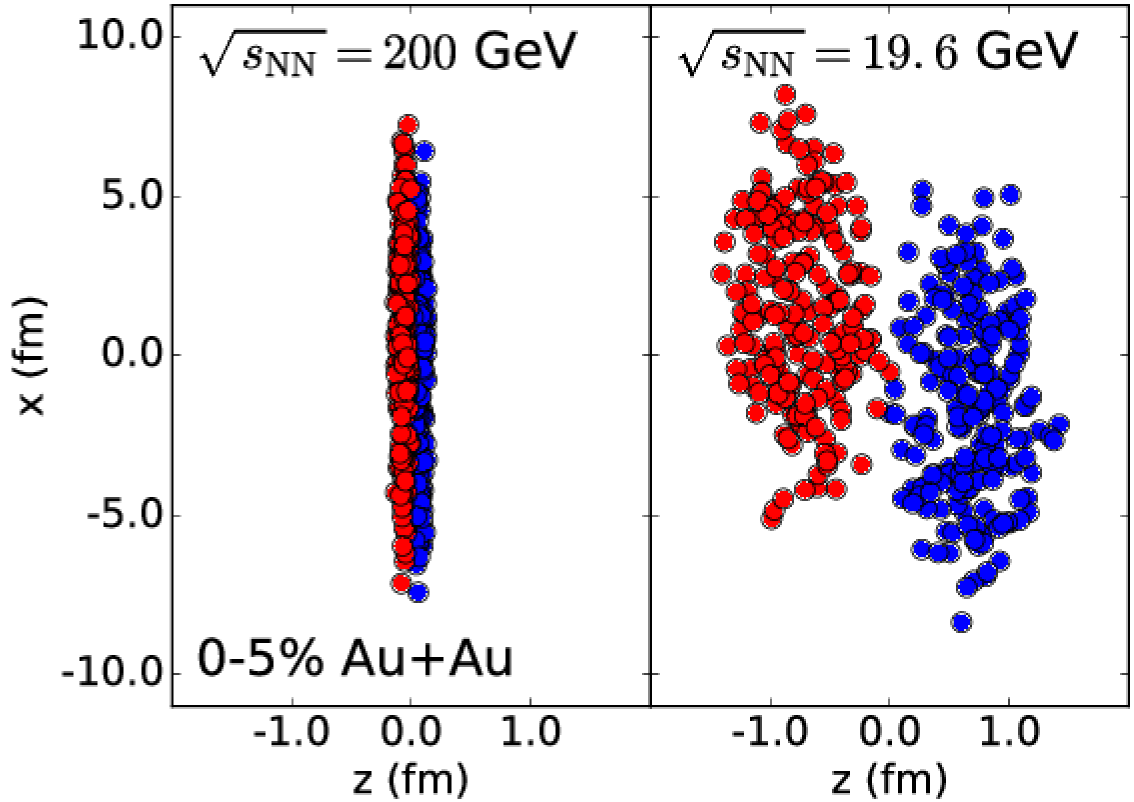
e.g. CGC + hydrodynamics + energy loss of jets + hadron transport model

Initial nucleon positions

Assumption of single thermalization time breaks down at low beam energies,

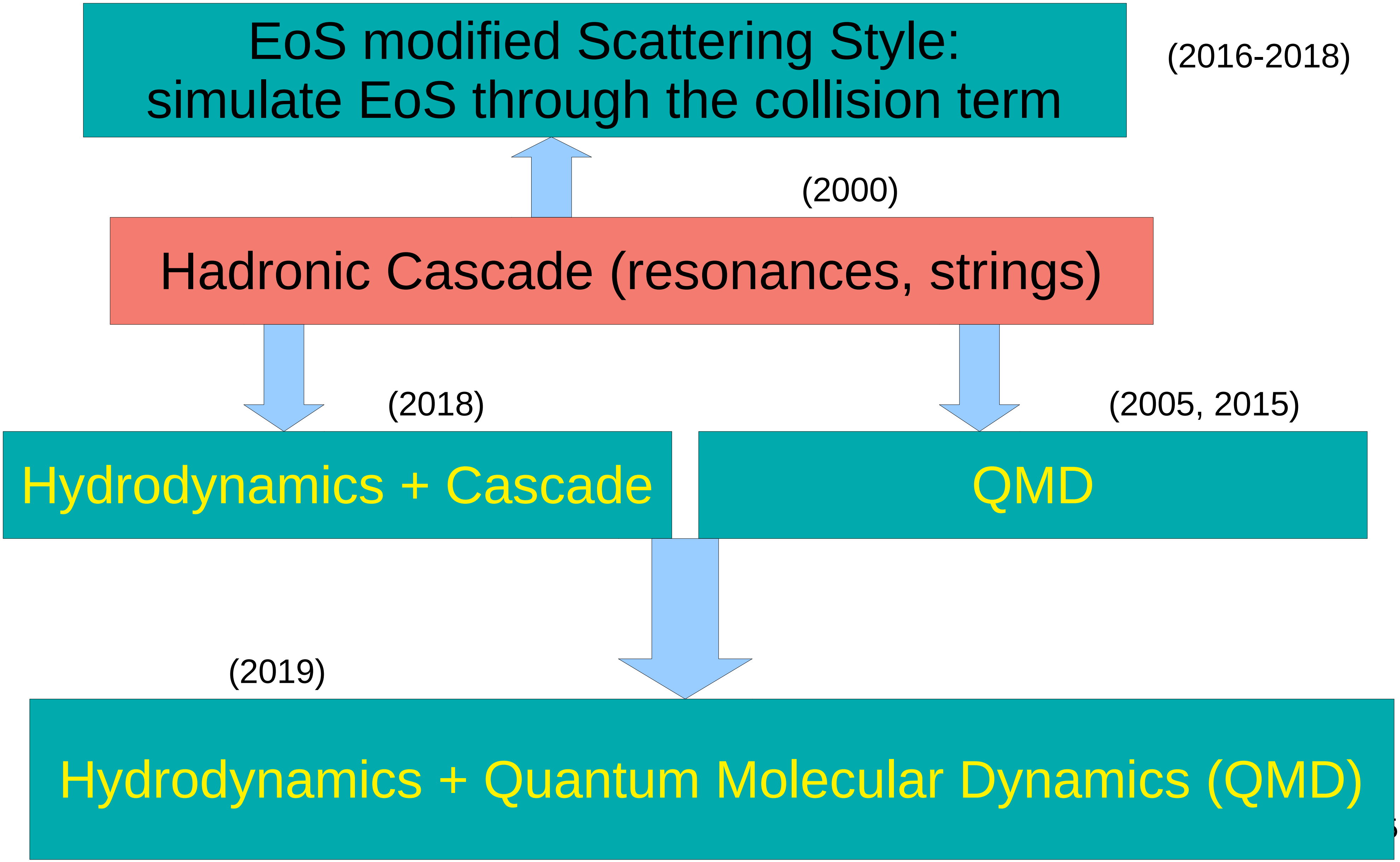
$$\sqrt{s_{NN}} < 30\,\mathrm{GeV}$$

since secondary interactions start before two nuclei pass through each other.



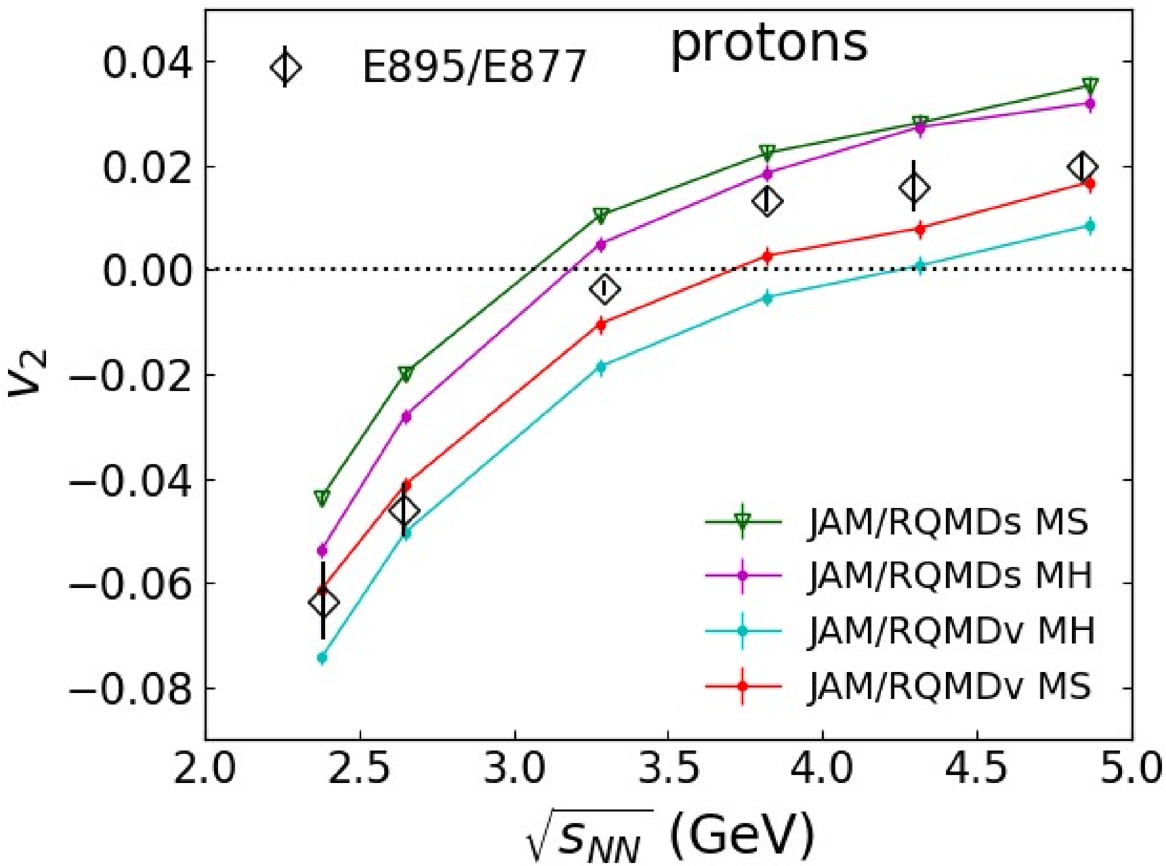
Chun Shen and Bjorn Schenke Phys.Rev. C97 (2018) no.2, 024907

Recent developments in JAM



EoS dependence on v2

Mom. Dep. Soft EoS (K=270MeV), hard (K=370 MeV)



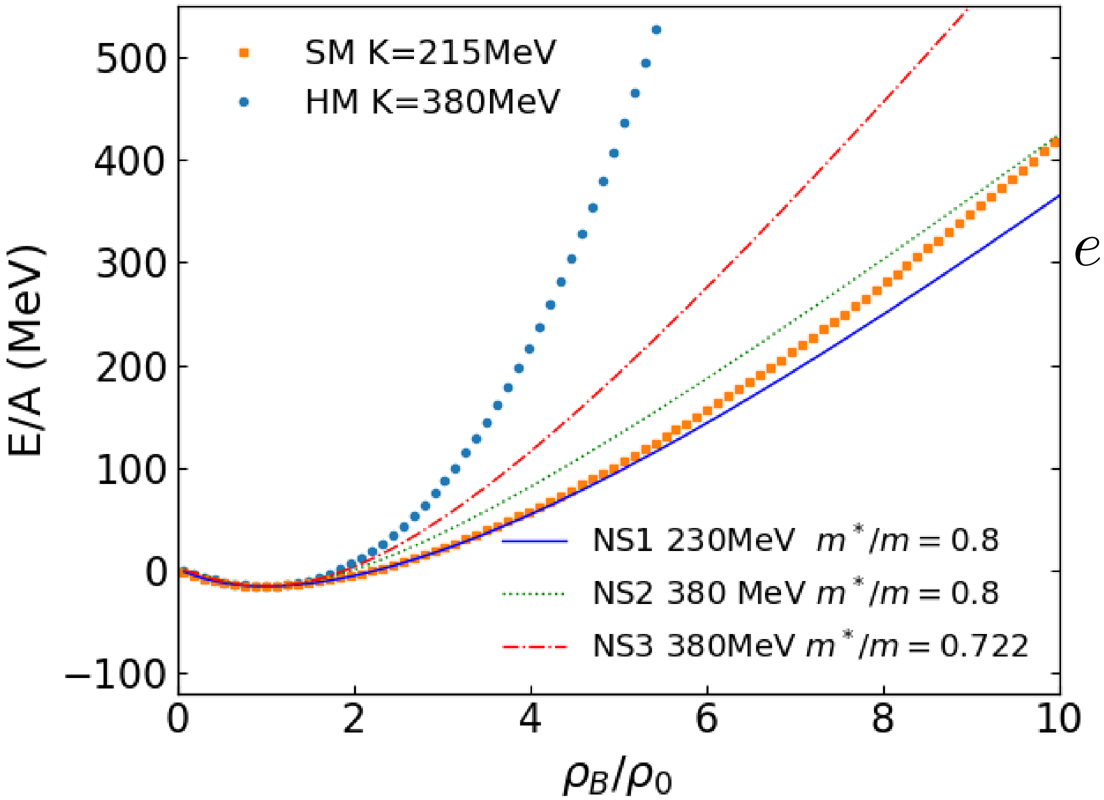
Skyrme type potential cannot explain the excitation function of v2.

$$V_i = \frac{\alpha}{2\rho_0}\rho_i + \frac{\beta}{(1+\gamma)\rho_0^\gamma}\rho_i^\gamma + \sum_{k=1,2}\frac{C_{ex}^{(k)}}{2\rho_0}\sum_{j\neq i}\frac{1}{1+[p_{ij}/\mu_k]^2}\rho_{ij}$$

EOS from the relativistic mean-field theory

The RMF is first employed by RVUU transport models by
C. M. Ko, Q. Li and R. C. Wang, Phys. Rev. Lett. 59, 1084 (1987)
B. Blattel, V. Koch, W. Cassing and U. Mosel, Phys. Rev. C 38, 1767 (1988)

$$[p^*_\mu \partial^\mu_x + (p^*_\nu F^{\mu\nu} + m^* \partial^\nu_x m^*)\partial^{p^*}_\mu] f(x, p^*) = C_{\text{coll}}$$



Non-linear sigma-omega model

$$e = \int d^3p E^* f(p) + \frac{1}{2}\frac{g_v^2}{m_\omega^2}\rho_B^2 + U(\sigma)$$

$$m^* = m - g_s\sigma$$

$$U(\sigma) = \frac{m_\sigma^2}{2}\sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4$$

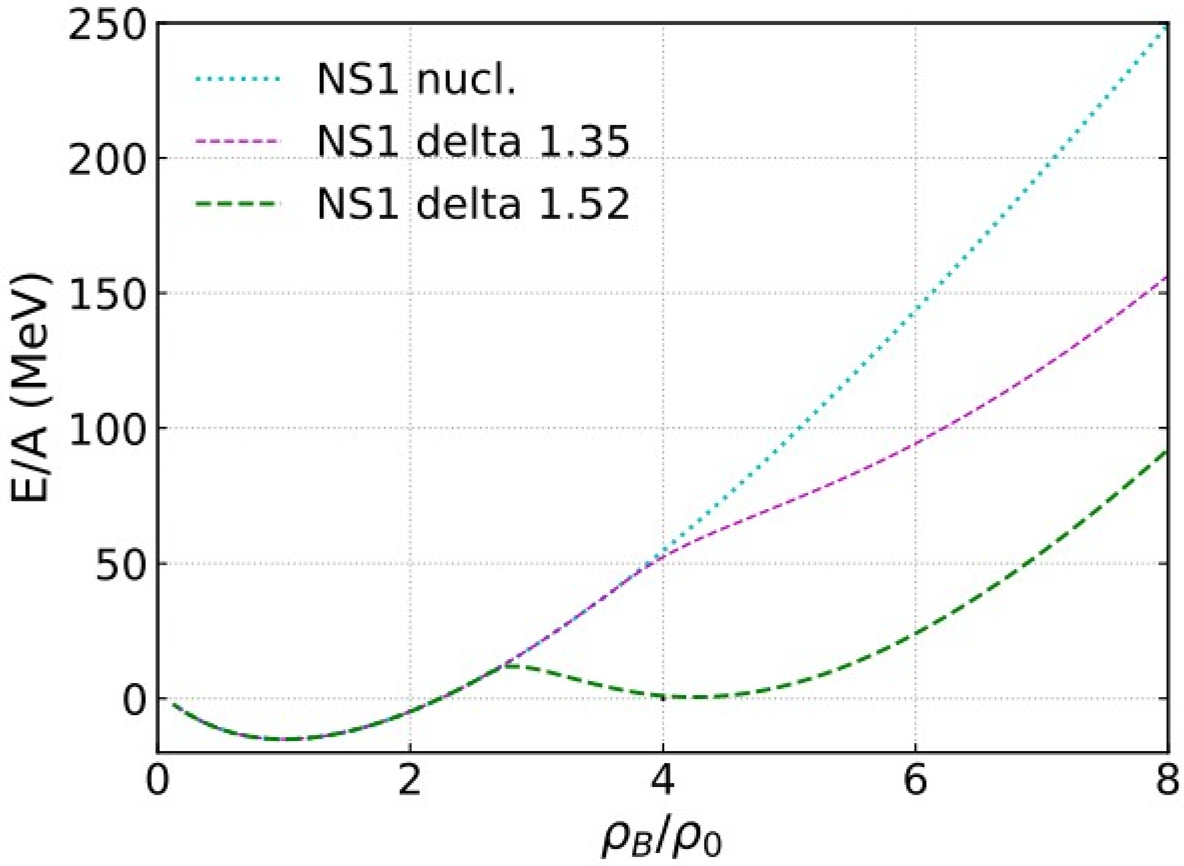
Delta-isomer state in RMF

J. Boguta, Phys. Lett.B109 (1982)251,
B. M.Waldhauser,et. al, PRC36(1987) 1019

$$g_{\Delta\omega} = g_{N\omega}$$

$$g_{\Delta\sigma} = \alpha g_{N\sigma}$$

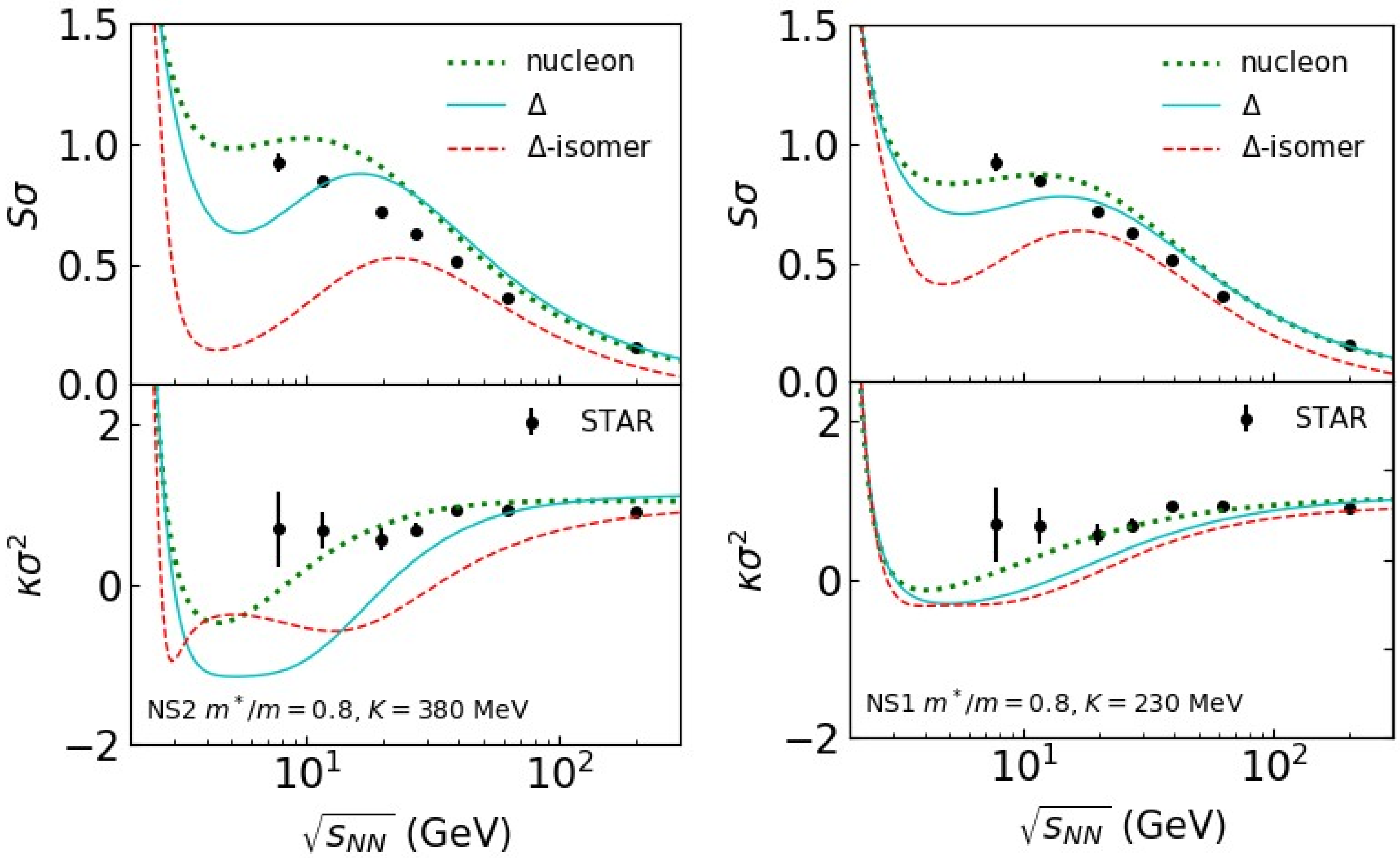
$$\alpha = 1.35, 1.52$$



Effects of Delta-isomer state on kurtosis

Compute baryon number fluctuations according to
K. Fukushima, PRC91 (2015) 044910

large EoS dependence !



Delta and its isomer state has large effects on the Net-baryon number fluctuations.
What about the dynamical effect? We can do it by RQMD.

The Quantum Molecular Dynamics

Quantum molecular dynamics (QMD) approach is a N-body non-equilibrium theory to describe heavy ion collisions.
J. Aichelin and H. Stoecker, Phys. Lett.B176 (1986)14,
J. Aichelin, Phys. Rep.202 (1991) 233.

Particles are represented by a Gaussian wave packet. The equation of motion is given by the time-dependent variational principle,

$$S = \int dt \left\langle \Phi \left| i\hbar \frac{d}{dt} - H \right| \Phi \right\rangle$$

$$\langle H \rangle = \sum_i \frac{\boldsymbol{p}_i^2}{2m_i} + \sum_{i \neq j} \langle V_{ij} \rangle \qquad \frac{d\boldsymbol{r}_i}{dt} = \frac{\partial \langle H \rangle}{\partial \boldsymbol{p}_i} \quad \frac{\boldsymbol{p}_i}{dt} = -\frac{\partial \langle H \rangle}{\partial \boldsymbol{r}_i}$$

Mean-fields are simulated by the potential interactions, and collision term is also included to simulate Boltzmann type collisions kernel.

How do you extend non-relativistic QMD approach to relativistic version?

The RQMD model (1989)

Relativistic extension of QMD (RQMD) was developed by H. Sorge based on the constrained Hamiltonian dynamics:
H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192, 266 (1989).

Manifestly covariant way: four-vectors $q_i^\mu, \quad p_i^\mu \ (i = 1, N)$

For the description of N-particle system, we have 8N dimension.
In order to reduced the dimension from 8N to 6N, we need 2N constraints.

On-mass shell condition:

$$\phi_i = p_i^2 - m_i^2 - V_i = 0, \quad (i = 1, \cdots, N)$$

Time fixation:

$$\phi_{i+1} = \sum_{j \neq i} \frac{\exp(q_{ij}^2 / L_c)}{q_{ij}^2 / L_c} q_{ij} p_{ij} = 0, \quad (i = 1, \cdots, N)$$

Hamiltonian is a linear combinations of the constraints, and equations of motion are given by

$$H = \sum_i \lambda_i \phi_i \qquad \frac{dq_i}{d\tau} = \{H, q_i\}, \quad \frac{dp_i}{d\tau} = \{H, p_i\}$$

RQMD approach in JAM

On-mass shell condition:

Tubingen group,C.Fuchs, et.al.NPA603(1996)471

$$H_i = (p_i - V_i)^2 + (m_i - S_i)^2 = p_i^{*2} + m_i^{*2} = 0, \quad (i = 1, \cdots, N)$$

Simplified version of RQMD was proposed by T. Maruyama (1996)
T. Maruyama, et. al. Prog. Theor. Phys. 96, 263 (1996).

Time fixation to equate the all time coordinate of the particles:

$$\chi_i = \hat{a} \cdot (q_i - q_N) = 0 \quad (i = 1, \cdots, N-1)$$
$$\chi_N = \hat{a} \cdot q_N - \tau = 0$$
$$\hat{a} = (1,0,0,0) \text{ in a reference frame}$$

We also assume that time-component of the momentum coordinate is replaced by the kinetic energy in the argument of the potential.

JAM Mean-field mode summary

$$H = \sum_{i}^{N} \sqrt{(\boldsymbol{p}_i - \boldsymbol{V}_i)^2 + (m_i - S_i)^2} + V_i^0$$

$$\dot{\boldsymbol{x}}_i = \frac{\boldsymbol{p}_i^*}{p_i^{*0}} + \sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial \boldsymbol{p}_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial \boldsymbol{p}_i} \right), \quad \dot{\boldsymbol{p}}_i = -\sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial \boldsymbol{r}_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial \boldsymbol{r}_i} \right)$$

V_i^μ: ω-field S_i: σ-field

$$m_i^* = m_i - g_s \sigma_i, \quad V^\mu = g_v \omega_i^\mu$$

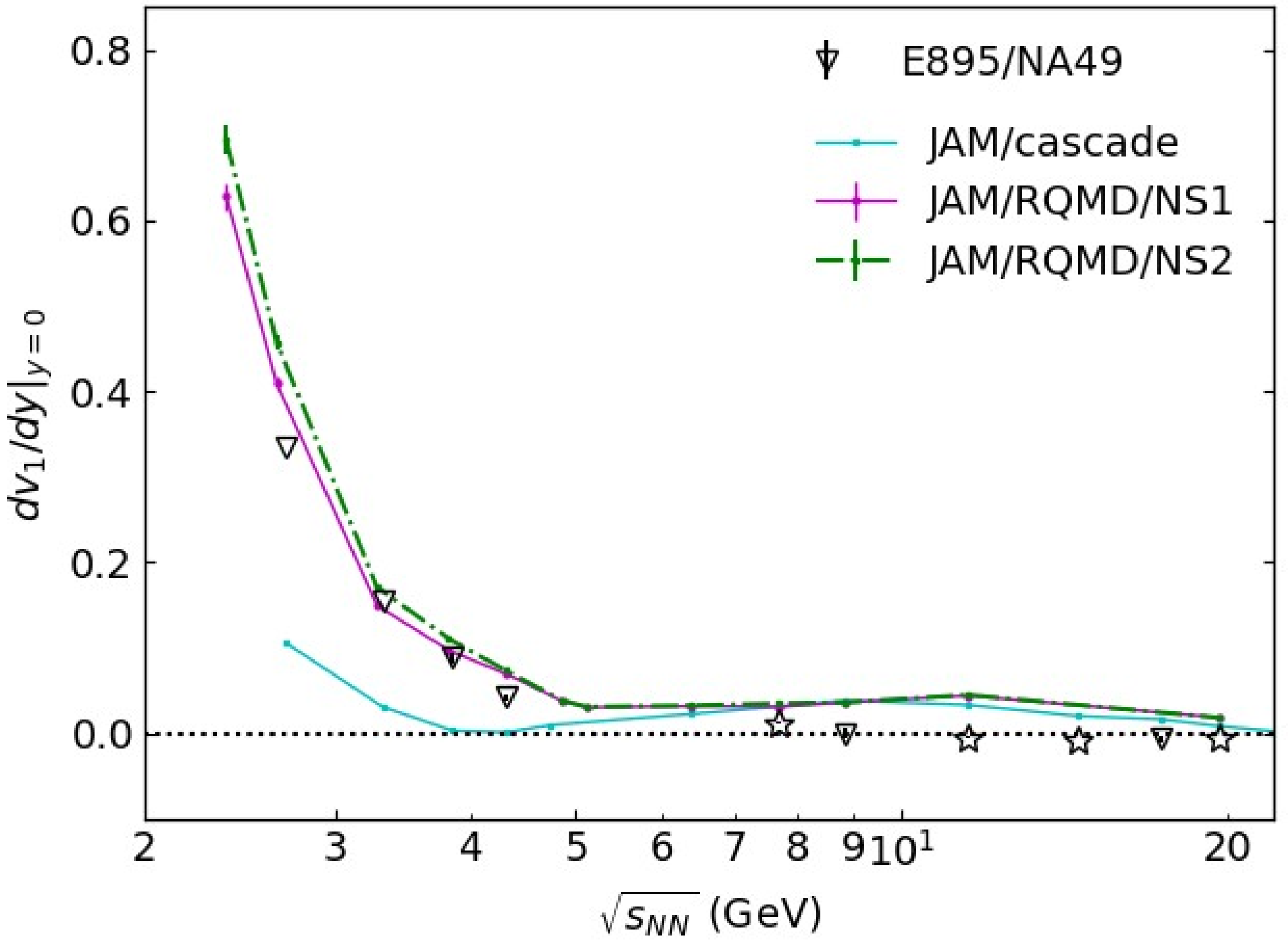
$$m_\sigma^2 \sigma + g_2 \sigma^2 + g_3 \sigma^3 = g_s \rho_s(i) \qquad m_\omega^2 \omega^\mu = g_v J_B^\nu(i)$$

$$\rho_s(i) = \sum_{i \neq j} \frac{m_j^*}{p_j^{0*}} \rho_{ij}, \quad J_B^\mu(i) = \sum_{i \neq j} B_j \frac{p^{*\mu}}{p^{0*}} \rho_{ij}$$

$$\rho_{ij} = \frac{\gamma_{ij}}{(4\pi L)^{3/2}} \exp(q_{Tij}^2 / 4L)$$

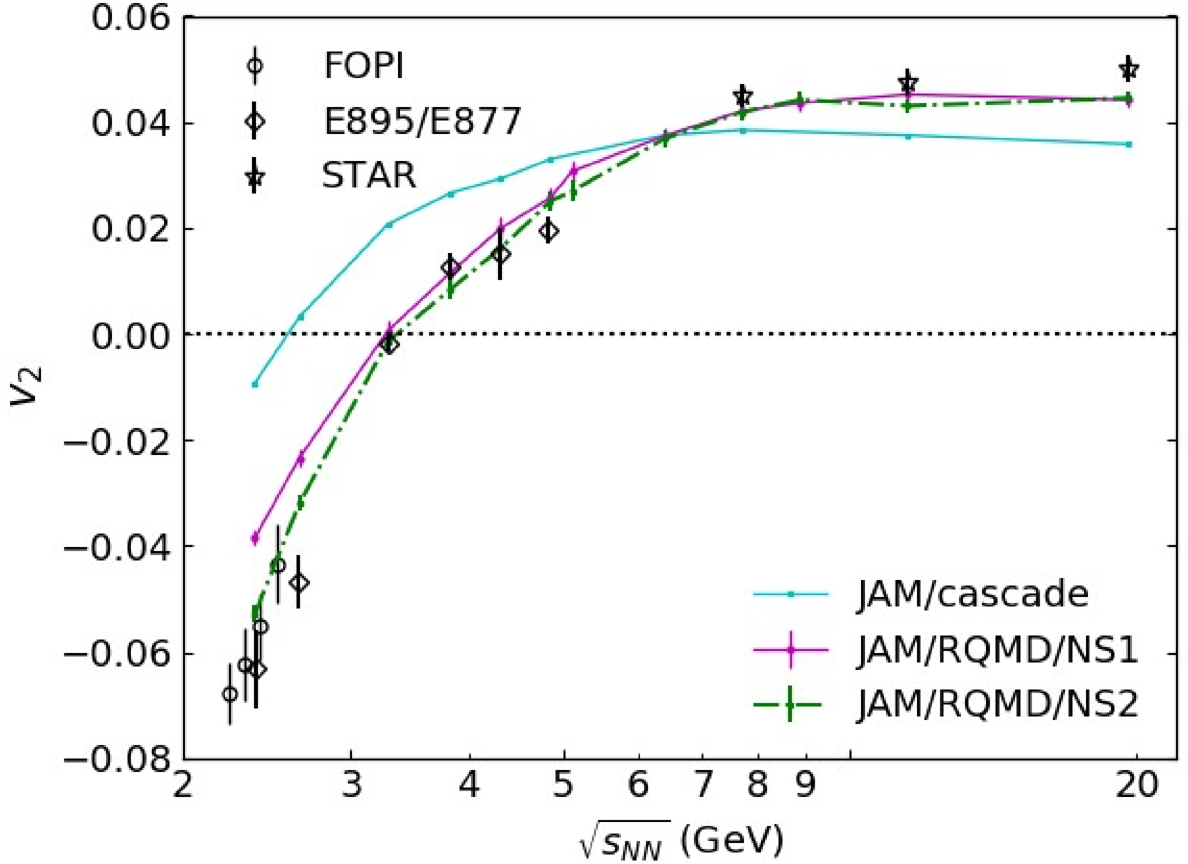
V1 from JAM/RQMDsv mode

RQMD with the sigma-omega model



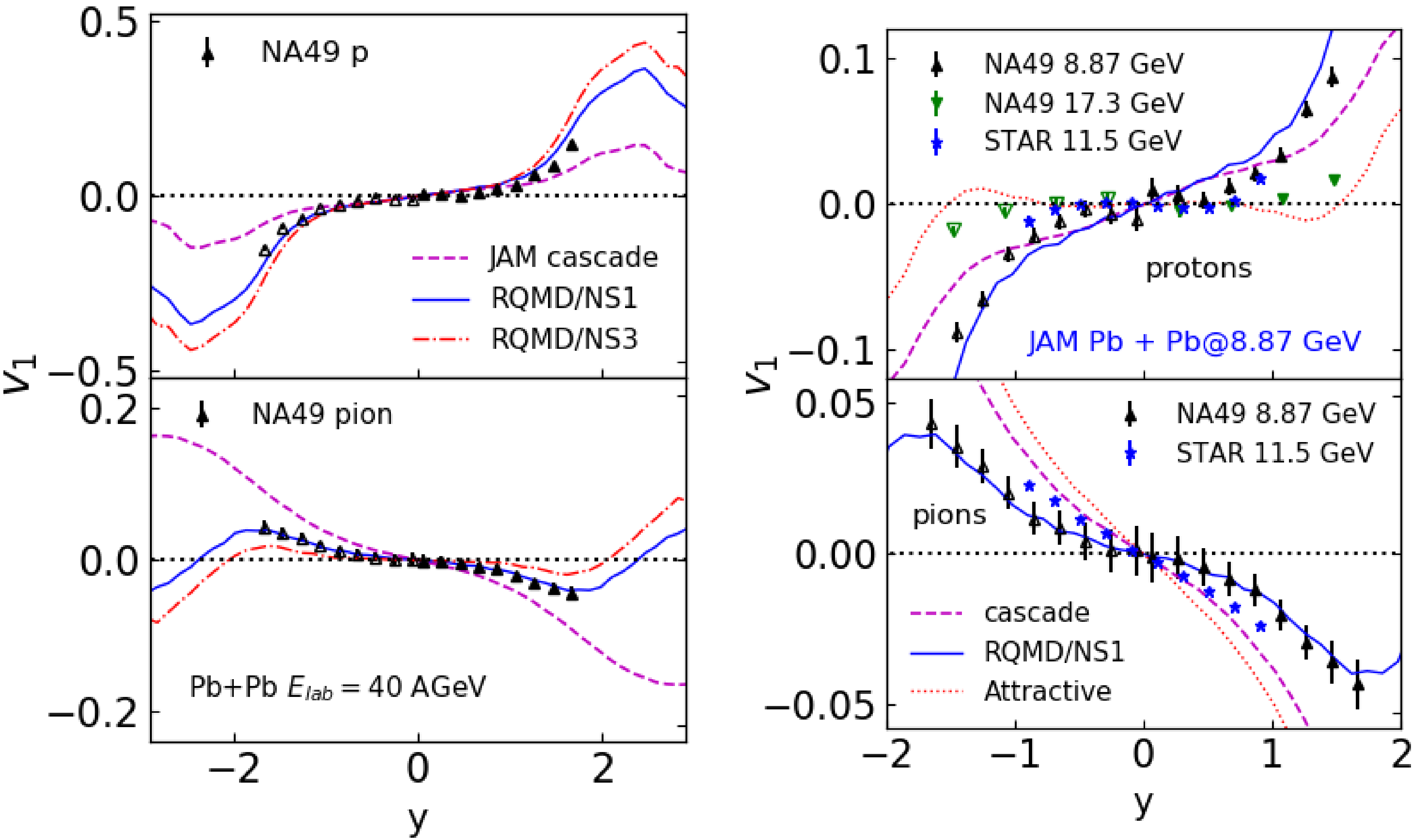
V2 from JAM/RQMDsv

RQMD with the sigma-omega model

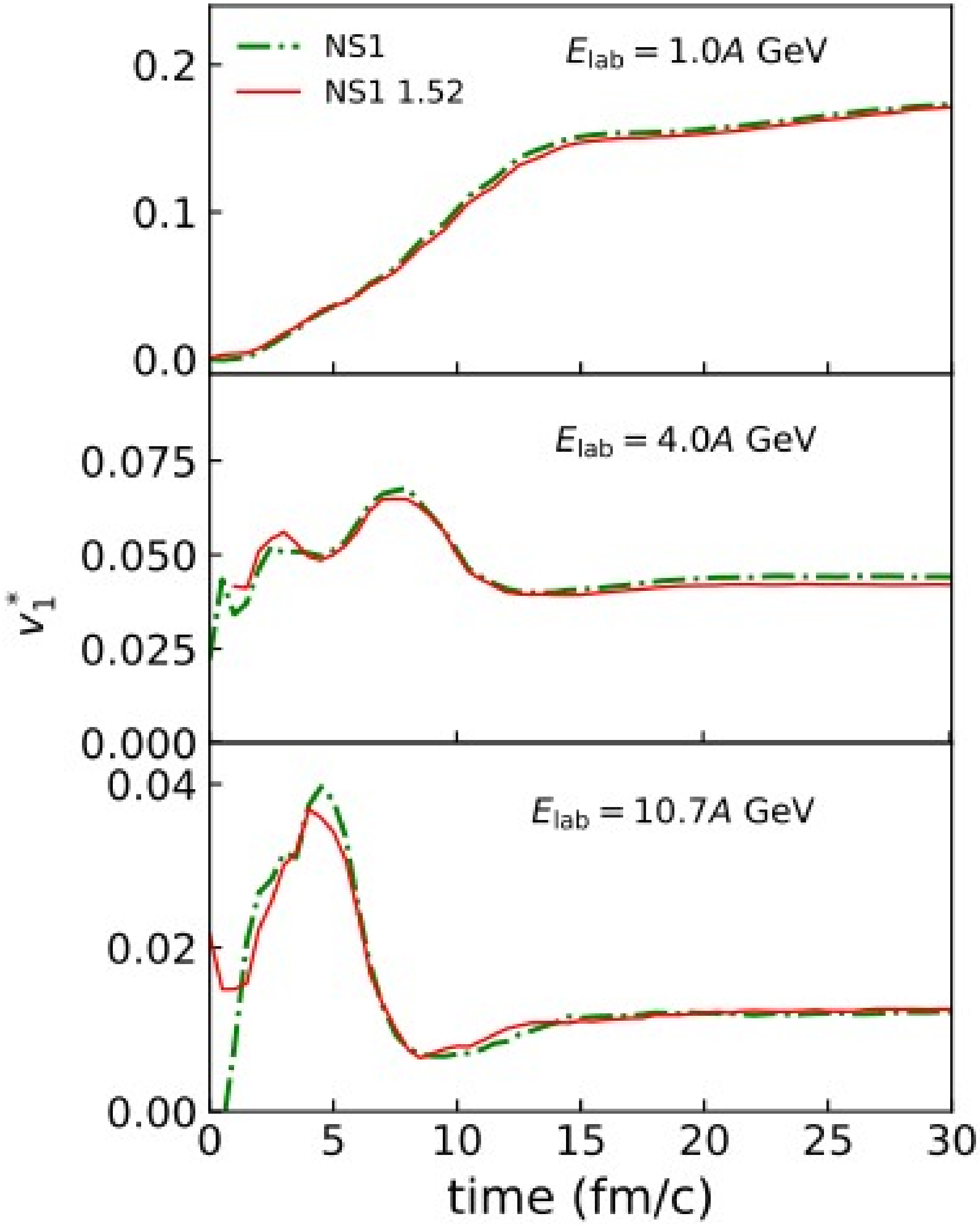


Rapidity dependence of V1 at SPS

JAM/RQMD with the sigma-omega model



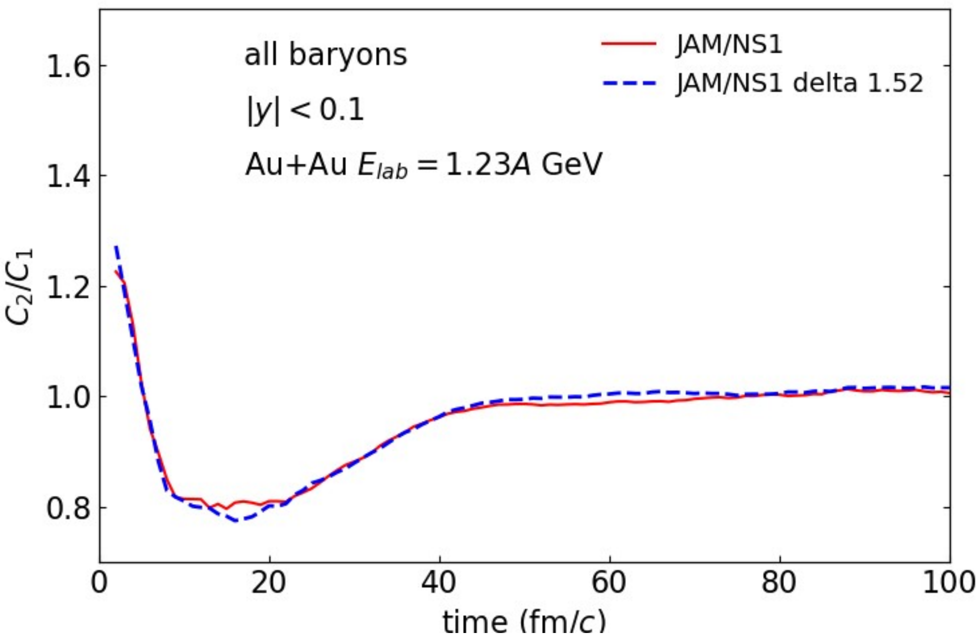
Time evolution of directed flow

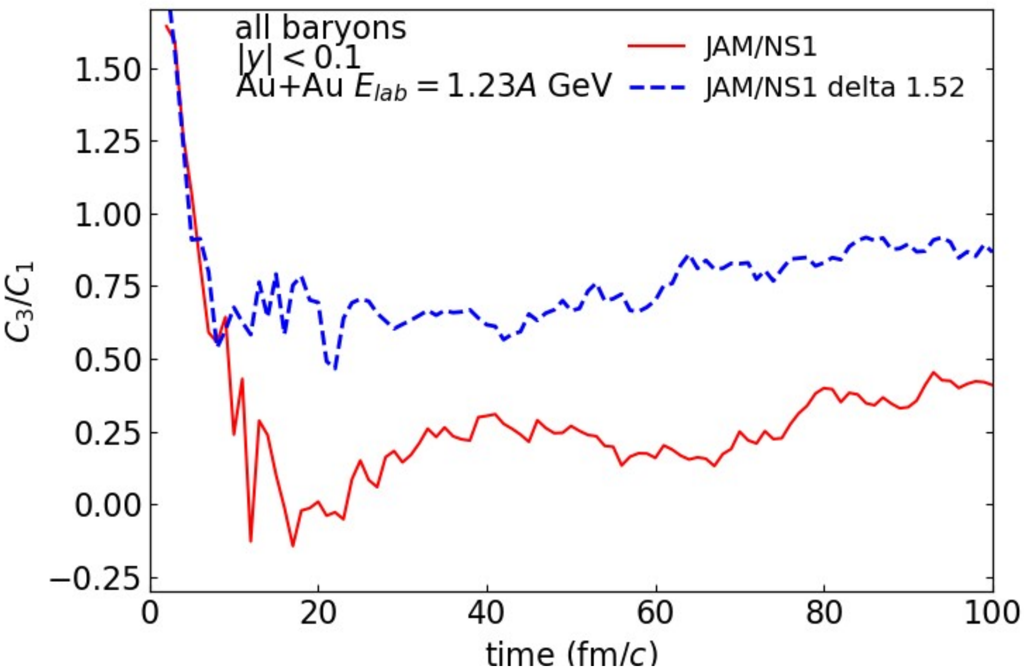


No effects of delta-isomer on the directed flow v1.

In principle, we should also modify collision term for the consistency with the mean-field part.

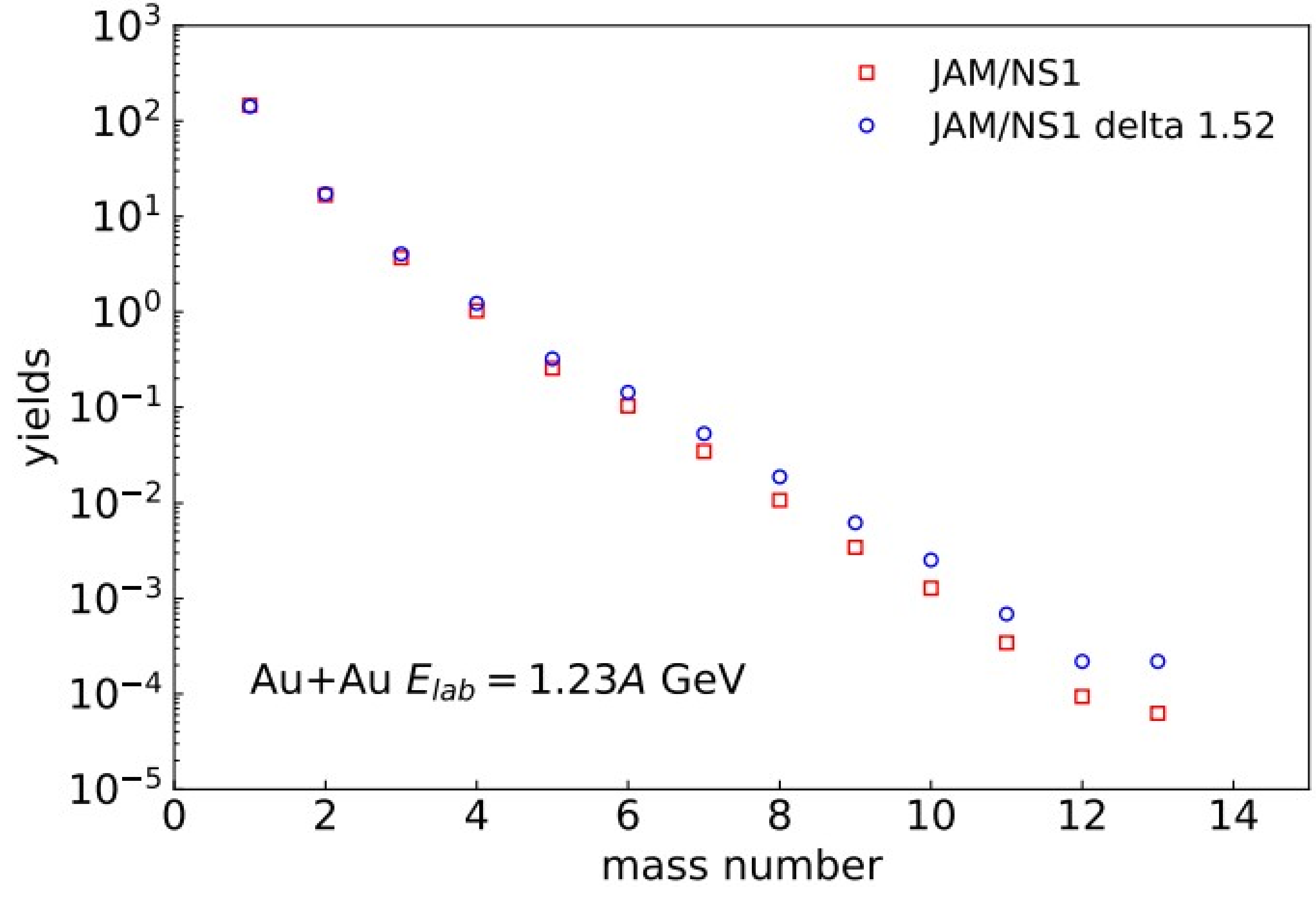
Time evolution of c2/c1 and c3/c1





$$C_1 = \langle N \rangle, \ C_n = \langle (N - \langle N \rangle)^n \rangle$$

Nuclear cluster formation



Summary

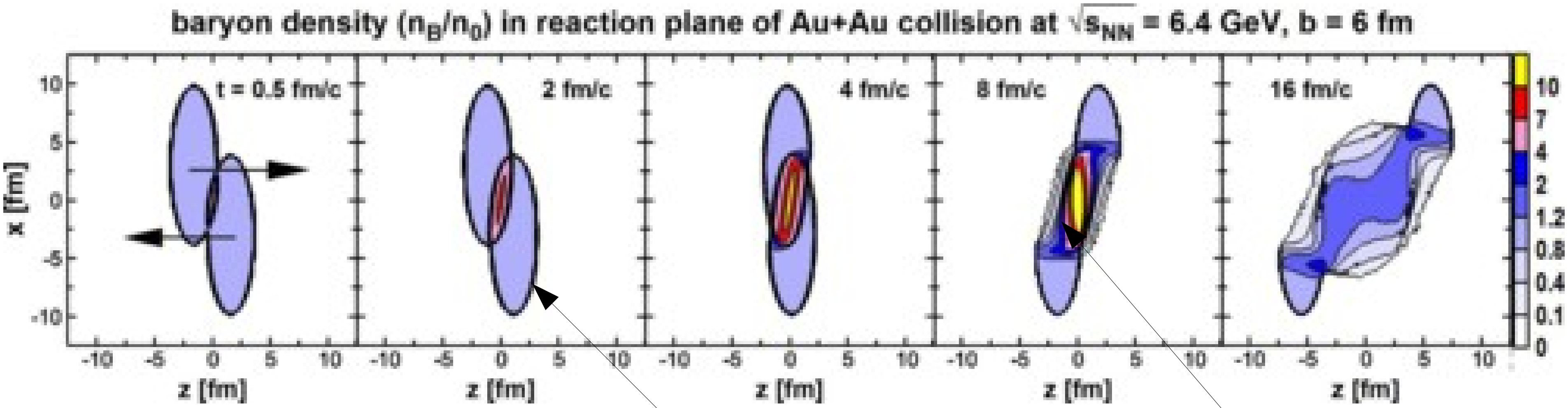
- We extend the JAM+hydro approach by including the EoS effects in the non-equilibrium phase within the QMD approach: HyQMD.
- Relativistic quantum molecular dynamics in JAM is extended by implementing the sigma-omega interactions.
- Description of collective flows are significantly improved over non-relativistic Skyrme type potential.
- Effects of Delta-isomer state on the flow, cluster formation, and baryon number fluctuations are studied within RQMD.
- RQMD can be applied for the description of final hadron gas stage at RHIC/LHC energeis.

JAM microscopic transport model

- space-time propagation of particles based on cascade method
- Resonance (up to 2GeV) and string excitation and decays
- Re-scattering among all hadrons
- DPM type string excitation law as in HIJING.
- Use Pythia6 for string fragmentation
- Nuclear cluster formation and its statistical decay
- Propagation by the hadronic mean-fields within relativistic quantum molecular dynamics (RQMD/S) (2005, 2016)
- EoS controlled collision term (2017)
- Dynamical coupling of Fluid dynamics through source terms (2018) (Hydro + hadronic cascade)
- RQMD with scalar and vector potentials based on RMF (2019)
- Hydrodynamic Quantum Molecular Dynamics (HyQMD) approach (2019)

New dynamically integrated transport model

Picture from 3FD model: P. Batyuk et.al. PRC94(2016)044817



Low density: hadronic transport model JAM

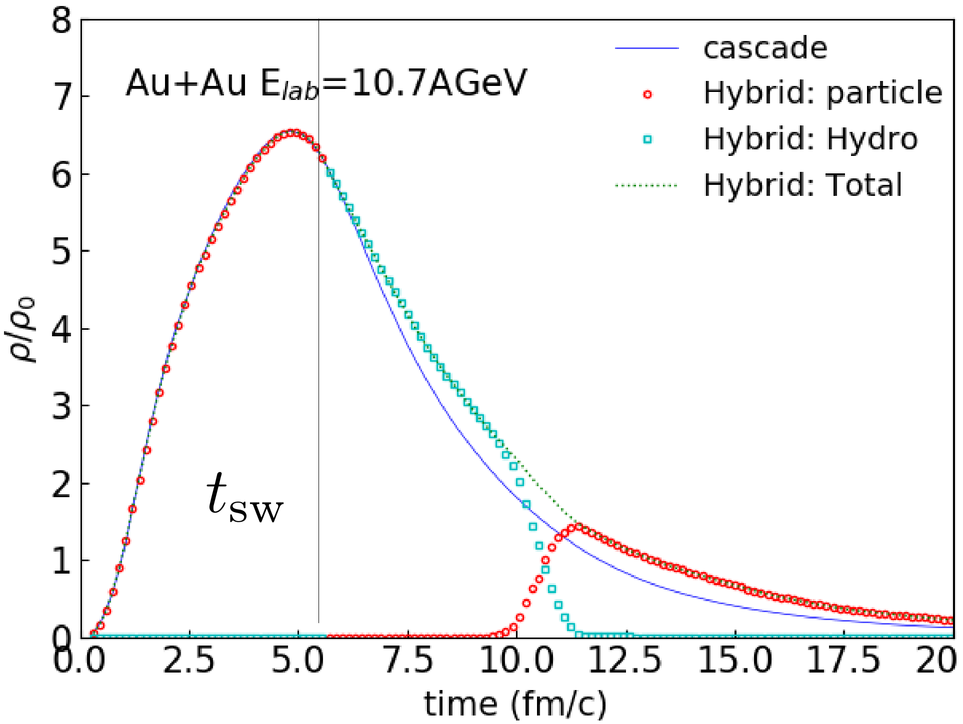
High density: hydrodynamics

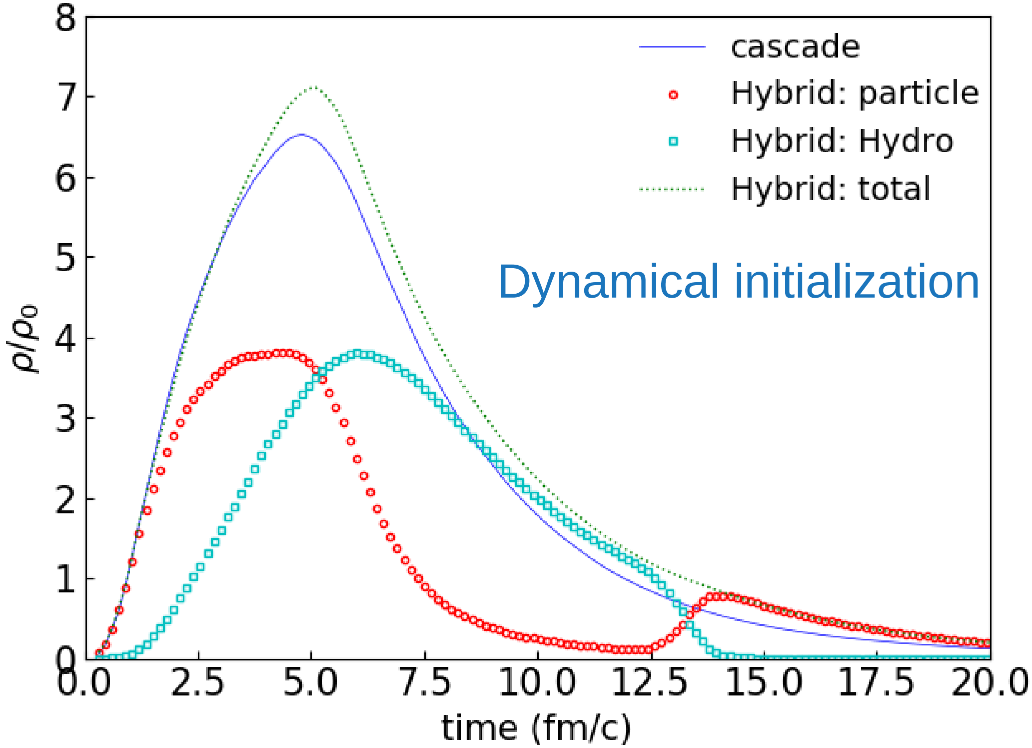
Solve the space-time evolution of both particles and fluids through the source term:

$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_B^\mu = \rho_B \qquad p_\mu \partial^\mu f(x,p) = I_{coll} + S$$

Y. Akamatsu, M. Asakawa, T. Hirano, M. Kitazawa,K. Morita, K. Murase, Y. Nara, C. Nonaka, A. Ohnishi, Phys.Rev. C98 (2018) no.2, 024909

Hybrid model for AGS and SPS energies





Switch to hydro evolution
after two nuclei pass each other.

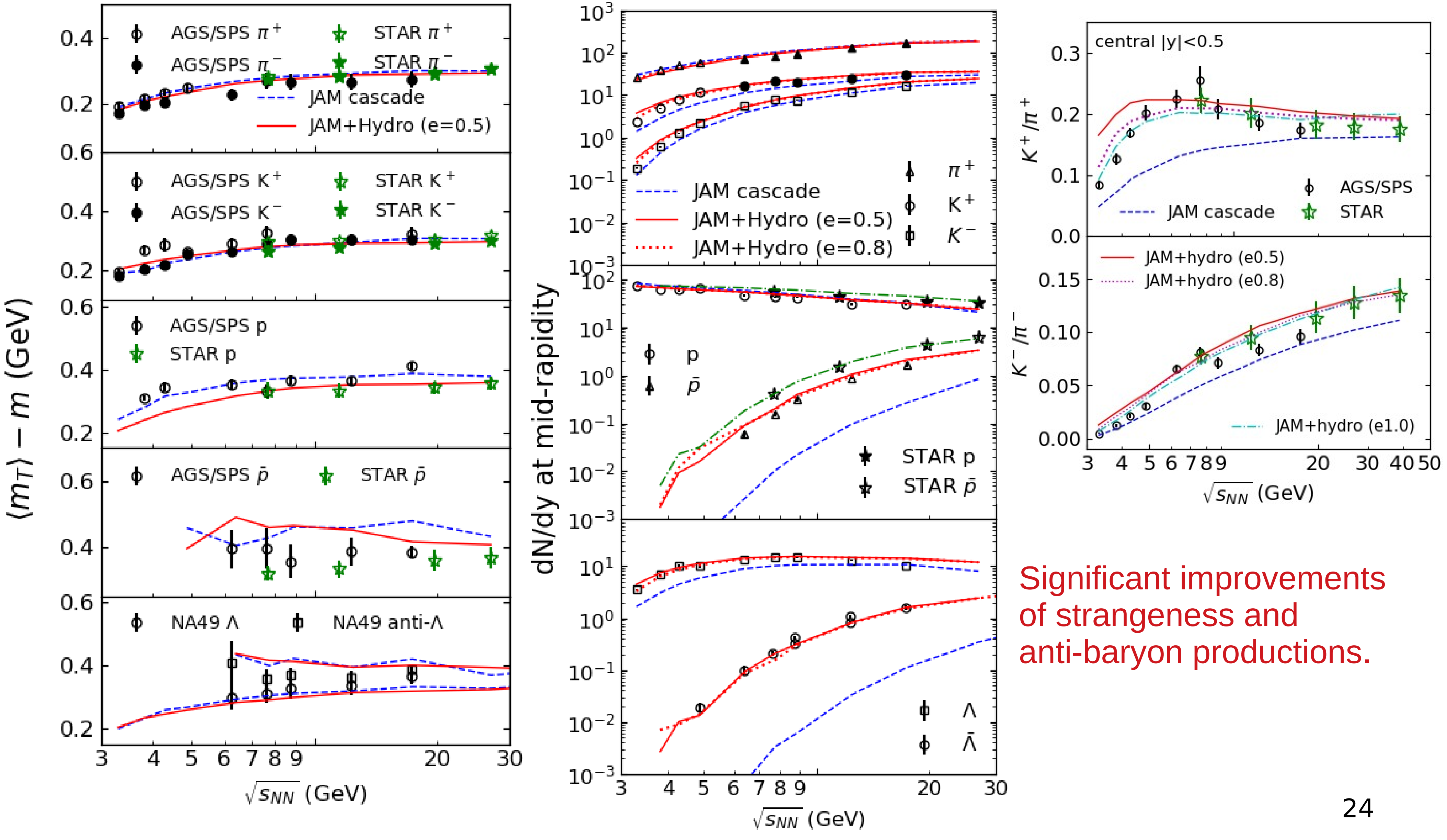
Hadronic EoS is used

Switch to hadron transport below a critical energy density.

It is important to take into account potential effect in the Cooper-Fry formula to ensure smooth transition from fluid to particles.

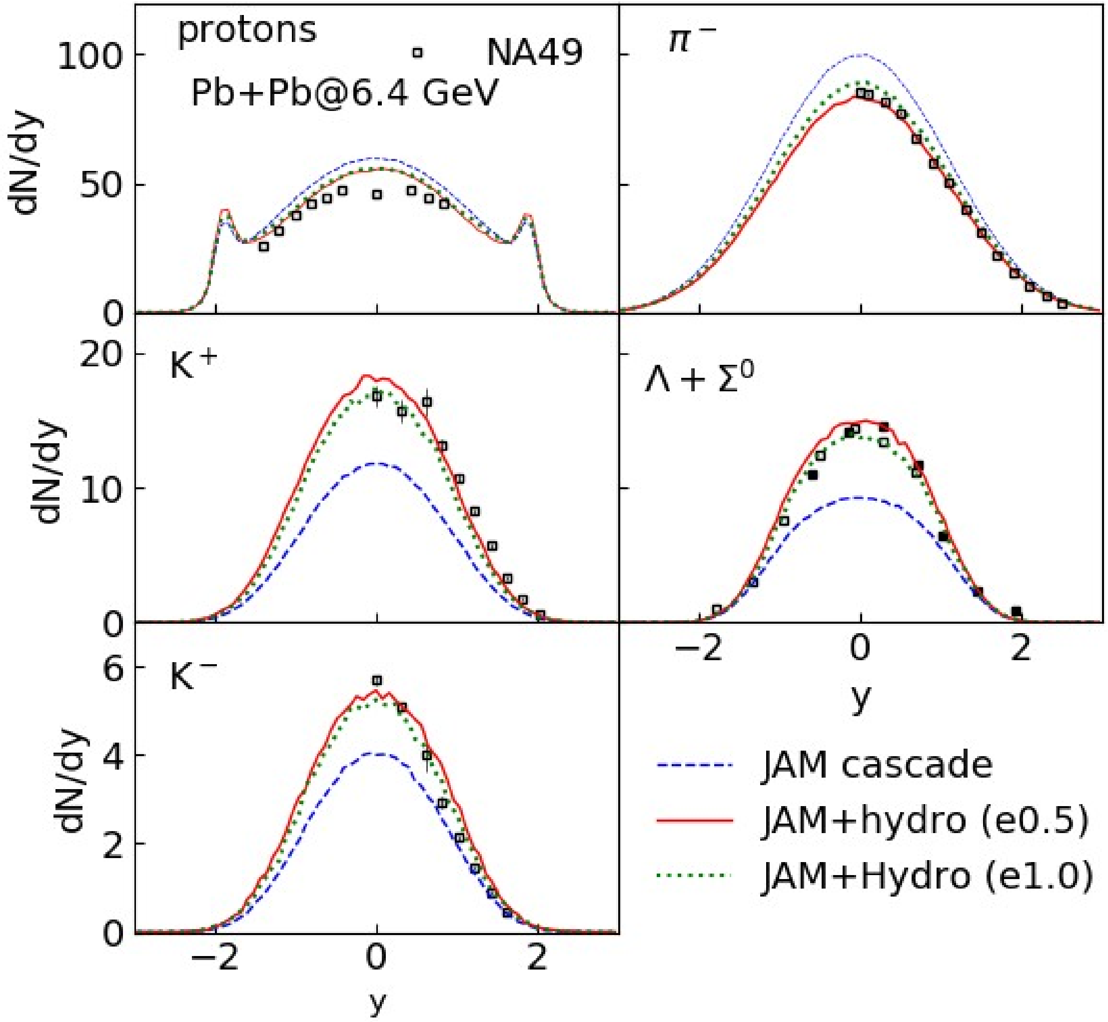
$$\mu = B\mu_B + S\mu_S \rightarrow B(\mu_B - V(\rho_B)) + S\mu_S$$

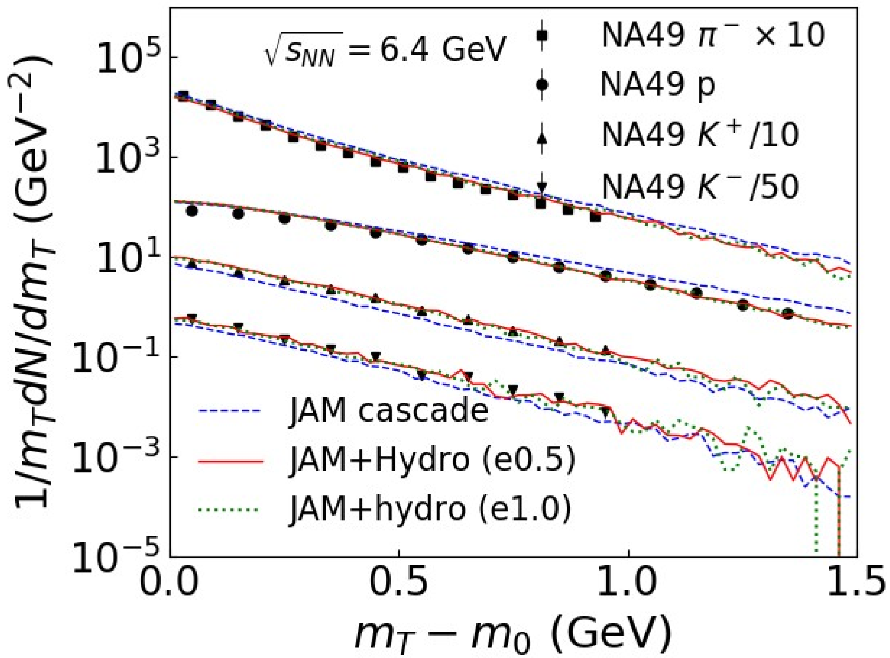
Beam energy dependence of transverse mass and multiplicities from a new integrated model



Significant improvements of strangeness and anti-baryon productions.

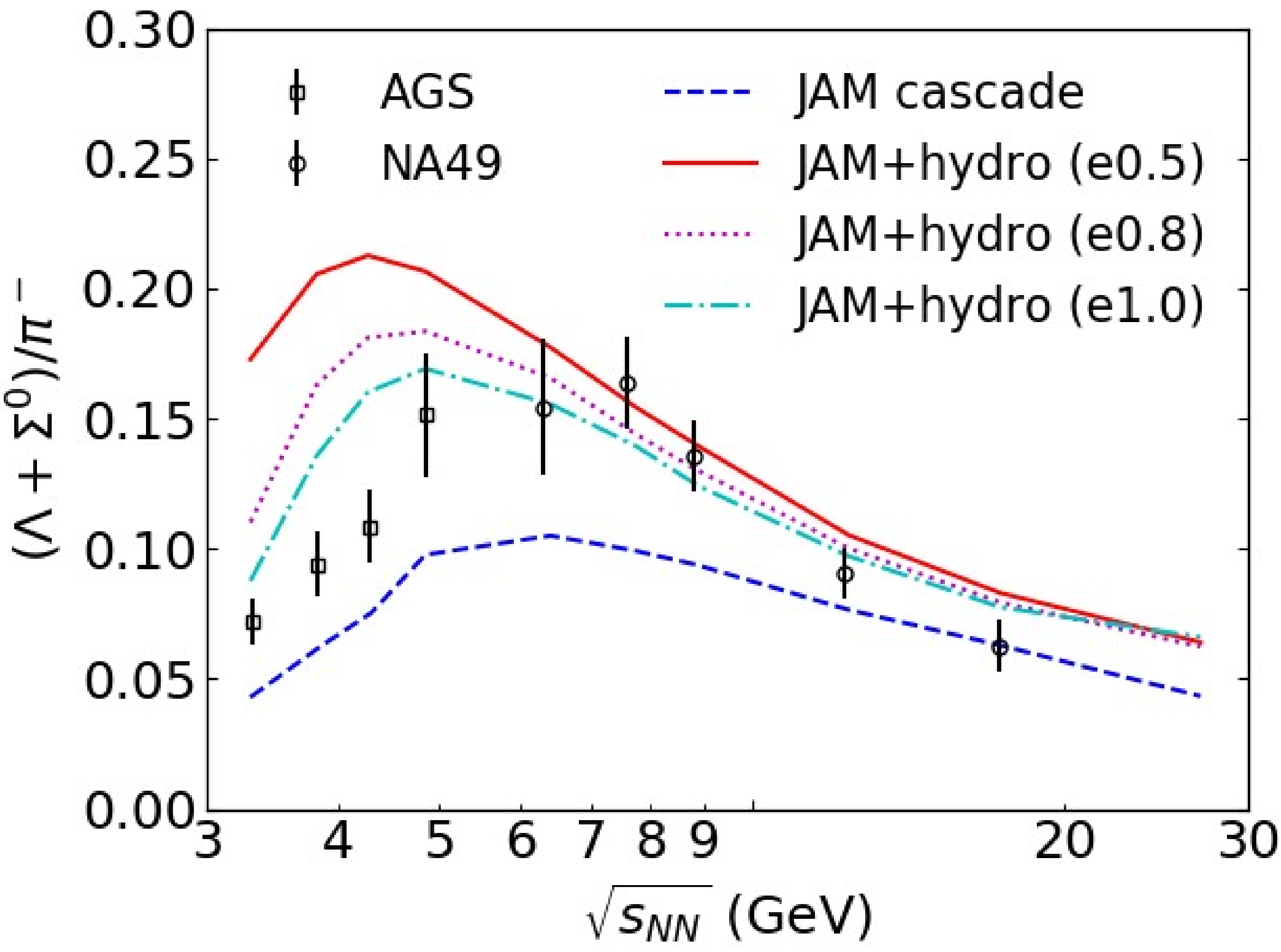
Particle spectra from a new hybrid model in Pb+Pb at Elab=20AGeV





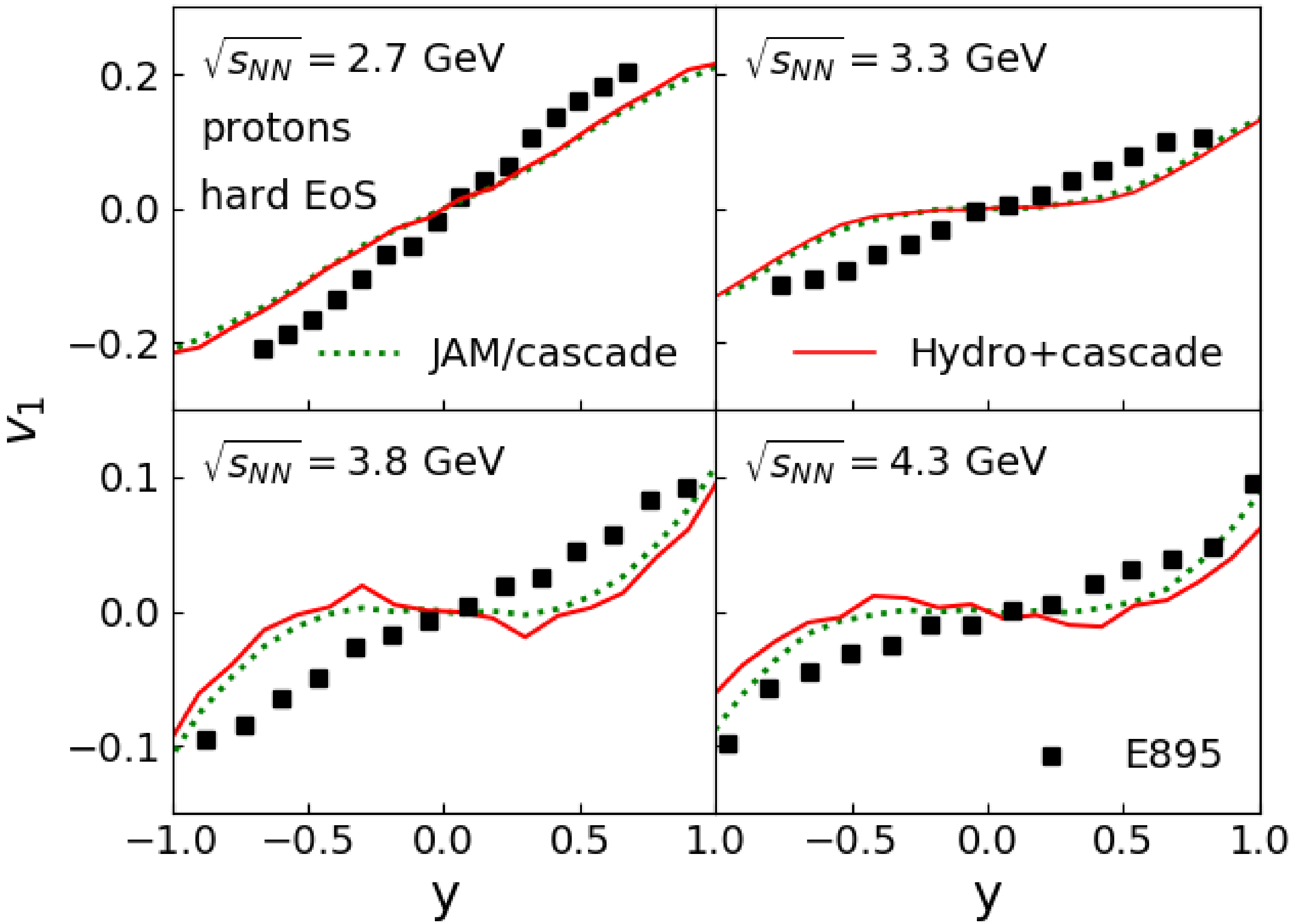
Fluidization energy density 0.5 or 1.0

Beam energy dependence of Lambda/pi ratios from a new hybrid model.



V1 from the Hydro + JAM/cascade model

V1 from the Hydro+JAM/cascade mode is the same as that of cascade calculations.



Single particle potential:

$$V_i = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Hard EoS: K=380MeV
+ first-order PT (Bag model)

Improvement of the hybrid model

EoS in the hadronic cascade is inconsistent with the EoS in the hydrodynamcis
We need to include consistent EoS in non-equilibrium dynamics.

Hydrodynamics + Cascade model

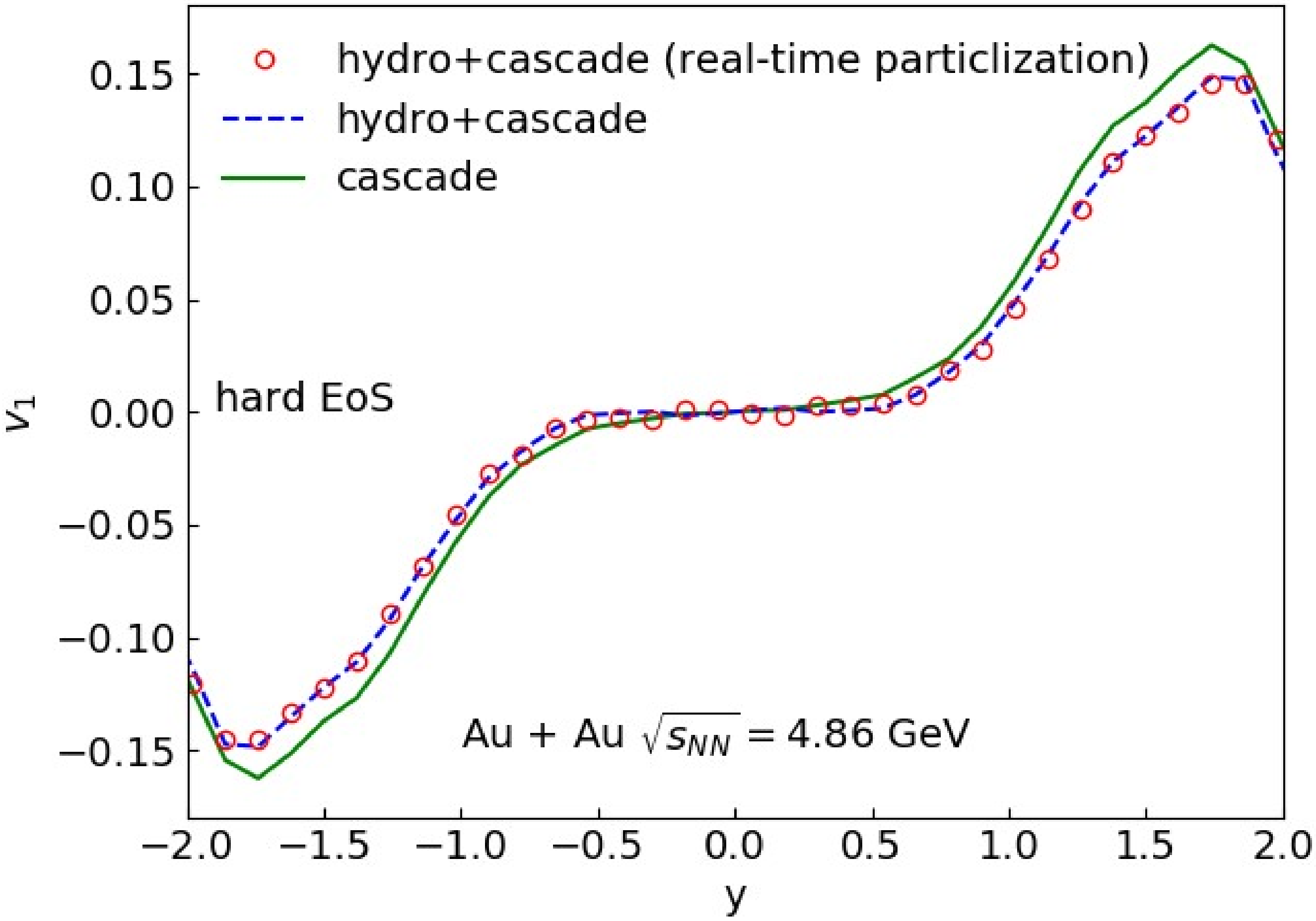
EOS: Hadron Gas

Hydrodynamics + Quantum Molecular Dynamics (QMD)

QMD: N-body non-equilibrium microscopic transport approach

Introduce Real-time particlization

Real-time particlization: fluid elements are conveyed into particles at each time step by using Cooper-Frye formula.

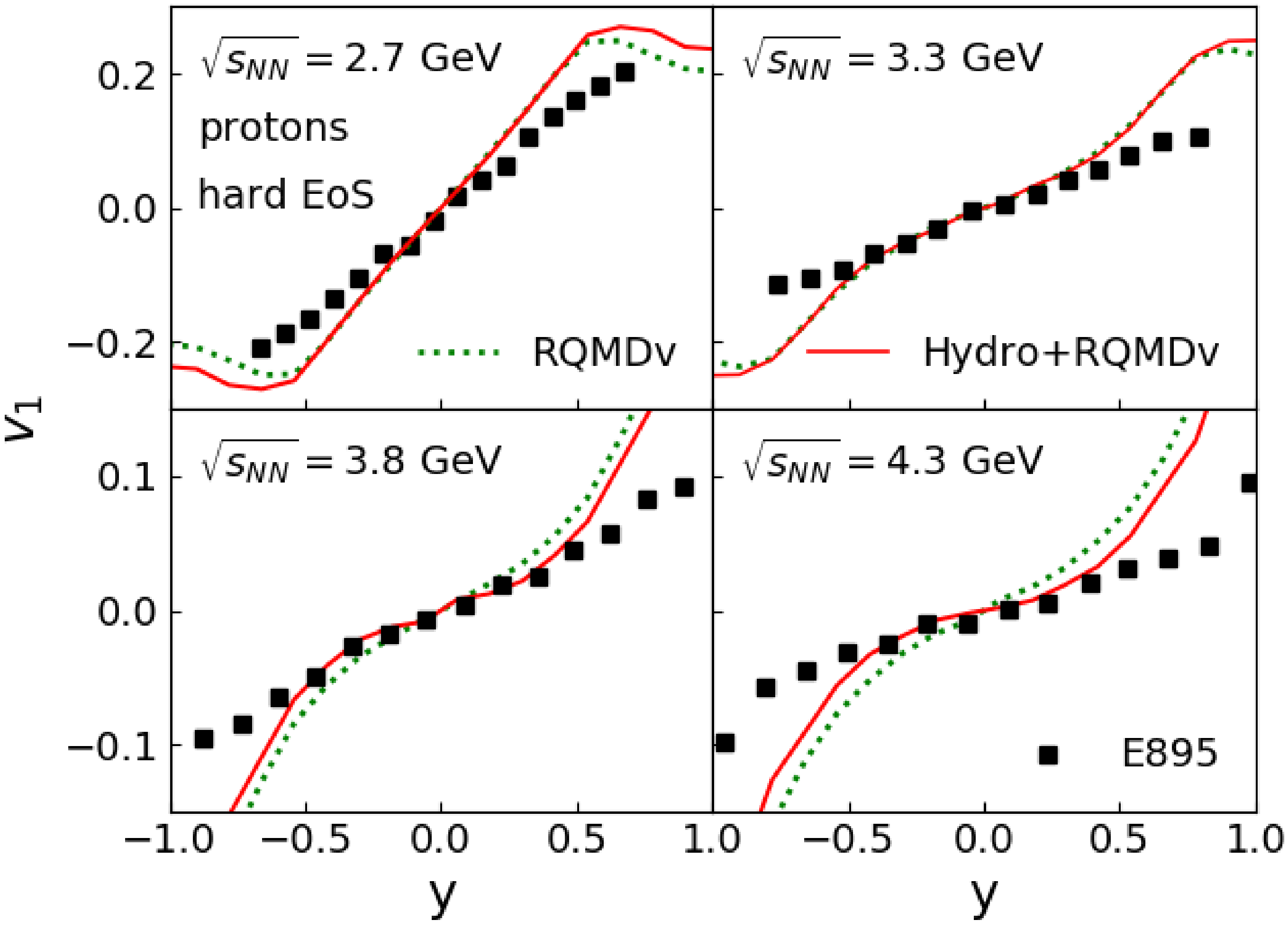


Real-time particlization yields the same results as the particlization after hydro.

It is also important to use the same EOS between hydro and QMD.

HyQMD (RQMDv mode) result

Results from Hydro + RQMDv in which potentials are implemented as a vector.



EOS: 1st-order PT

Mean-field in the particle phase is very important for the flow.
Note that proton flow is positive even if first-order phase transition is included.