Quarkonium as an open quantum system in the QGP

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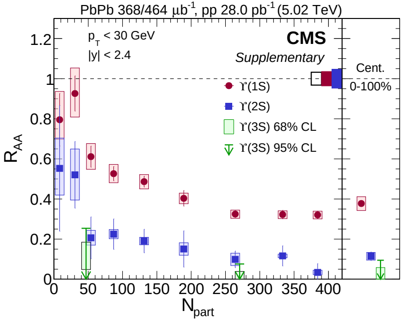
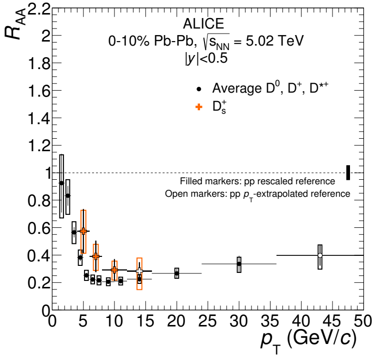
Phenomenology of heavy quark and quarkonium

Data show dissociation/recombination and heavy quark energy loss





Heavy quark phenomenology

- Classical transport theory for heavy quark and quarkonium
- Schrödinger equation for heavy quark-antiquark pair with a complex potential

Theory of **open quantum systems** can provide a more fundamental description

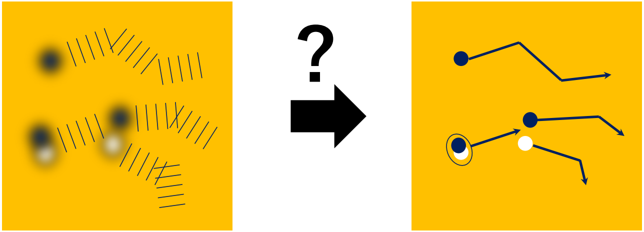
Open quantum system approach to heavy quarks

System (HQ) + environment (QGP)

1. Integrating all the medium effects into effective dynamics of heavy quarks
2. Density matrix for heavy quarks

$$\rho(x,y) \equiv \langle \psi(x)\psi^*(y) \rangle, \quad \dot{\rho} = \frac{1}{i\hbar}[H,\rho] + \mathcal{L}[\rho]$$

Classical transport theory: well-defined phase space trajectories



Today's talk:

- Decoherence as a dynamical mechanism for "classicalization"
- Recent development of open quantum system, based on Lindblad theory

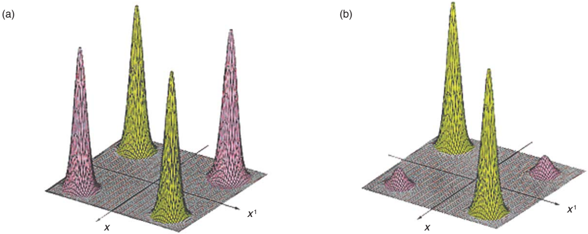
Decoherence

Decoherence of macroscopic superposition state in quantum mechanics

- A cat who is dead and alive in the Schrödinger's thought experiment
- Superposition state made incoherent by fluctuations of the environment
  - e.g. At 300K, a small dust at distance 1mm loses coherence in 10^{-10}s

Off-diagonal part of the density matrix

- Decoherence by Caldeira-Leggett master equation



[Zurek, quant-ph/0306072]

- Decoherence rate depends on the distance

$$\dot{\rho}(x,y) \sim \underbrace{-F(x-y)\rho(x,y)}_{\text{decoherence}}, \quad \underbrace{F(x-y) \geq 0}_{\text{damps off-diagonal part}}, \quad \underbrace{F(0) = 0}_{\text{decoherence ineffective}}$$

Environment fluctuations select localized wave packet $\sim$ "classical particle"

Decoherence rate in QGP

1. Non-relativistic limit

$$\mathcal{L}_I = \rho_Q A_0$$

2. Correlation of scalar potential A_0

$$G^{>}(x) = \langle A_0(x) A_0(0) \rangle$$

3. Heavy quark dynamics is slow compared to QGP time scales

$$D(r) = \underbrace{\int_{-\infty}^{\infty} dt}_{\text{HQ is slow}}\, G^{>}(x) = \underbrace{C_F \alpha T \int_0^{\infty} \frac{2dzz}{(z^2+1)^2} \frac{\sin(zrm_D)}{zrm_D}}_{\text{HTL approx.}} \sim \underbrace{\gamma e^{-r^2/\ell_{\text{corr}}^2}}_{\ell_{\text{corr}} \sim 1/m_D}$$

- Decoherence rate for a heavy quark [Akamatsu-Rothkopf (12)]

$$F(x-y) = \underbrace{D(0) - D(x-y)}_{\text{fluctuation is different at } x \text{ and } y} \geq 0$$

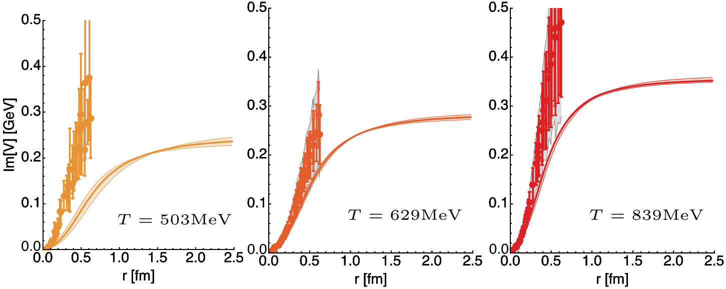
- Imaginary part of HQ potential [Laine+ (07), Beraudo+ (08), Brambilla+ (08), Rothkopf+ (17,...)]

$$\underbrace{V_{\text{Im}}(r) = D(r) - D(0)}_{\text{width from fluctuations}} \leq 0$$

The decoherence rate and the imaginary part of the complex potential are related!

Complex potential

- Spectral decomposition of thermal Wilson loop on the lattice [Rothkopf+ (17, ...)]



- Complex potential in several other setups
  - in a hot wind, anisotropic plasma, magnetic field, etc
- Complex potential as a stochastic potential model [Akamatsu+ (12), Kajimoto+ (17)]

$$\underbrace{\langle \theta(x)\theta(x')\rangle}_{\text{white noise field } \theta(x)} = D(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t'), \quad \underbrace{H = K + V_{\mathrm{Re}} + \theta}_{\text{unitary time evolution}}$$

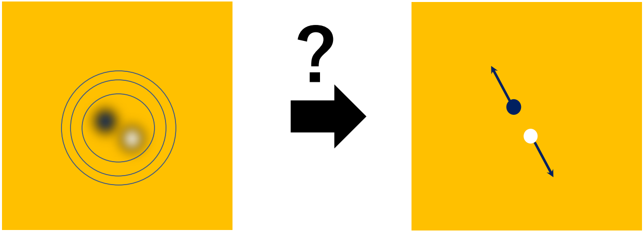
Application: decoherence after singlet-octet transition [Akamatsu, in progress]

A simplified model in large N_c limit ($\lambda = g^2 N_c, C_F\alpha = \lambda/8\pi$)

$$V_{\text{singlet}} = -\frac{C_F\alpha}{r}\exp[-m_D r], \quad V_{\text{octet}} = 0, \quad m_D^2 \sim N_c g^2 T^2 \sim \lambda T^2$$

Singlet-octet transition by an (in)elastic scattering

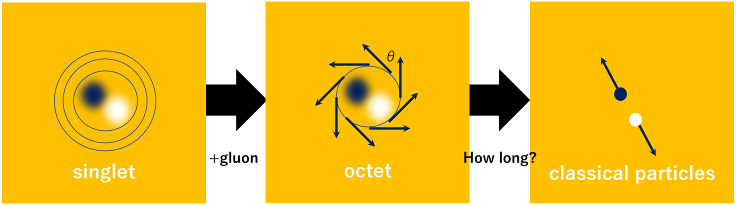
$$\phi_s + g^{(*)} \to \underbrace{\phi_o \to p_o}_{V_o = 0}$$



- A singlet bound state + classical transport for octet [Yao-Mueller (18), Blaizot-Escobedo (18), etc]

  How long does it take for an octet to be regarded as classical particles?

Application: decoherence after singlet-octet transition [Akamatsu, in progress]



1. Decoherence rate

$$\frac{1}{\tau_{\text{dec}}(\Delta x)} \sim D(0) - D(\Delta x) \sim C_F \alpha T \left(\frac{\Delta x}{l_{\text{corr}}}\right)^2 \sim (C_F\alpha)^2 T^3 (\Delta x)^2$$

2. Wave function size $r(t)$ after singlet-octet transition

$$r(t) \sim r_0 + vt \sim \underbrace{\frac{1}{MC_F\alpha} + C_F\alpha t}_{\text{Coulomb bound states}} \sim C_F\alpha t$$

3. Decoherence and evolution comparable at classicalization time t_c

$$\tau_{\text{dec}}(r(t_c)\theta_c) \sim t_c \to t_c \sim \frac{M^{1/2}}{C_F\alpha T^{3/2}} \sim \frac{10}{T} \quad (C_F\alpha \sim 0.3,\ T \sim M(C_F\alpha)^2)$$

Is $t_c \sim 10/T$ long or short in heavy-ion collisions?

Application: decoherence after singlet-octet transition [Akamatsu, in progress]

In terms of the density matrix

1. Octet wave function just after a singlet-octet transition

$$\phi_s(\boldsymbol{x}, 0) + g^{(*)} \to \phi_o(\boldsymbol{x}, 0)$$

2. Octet density matrix evolves

$$\underbrace{\rho_o(\boldsymbol{p}, \boldsymbol{p}'; 0)}_{\text{superposition of } p\text{s}} = \tilde{\phi}_o(\boldsymbol{p}, 0)\tilde{\phi}_o^*(\boldsymbol{p}', 0) \xrightarrow[\text{classicalization}]{\text{decoherence}} \underbrace{\rho_o(\boldsymbol{p}, \boldsymbol{p}'; t_c)}_{\text{nearly diagonal}}$$

3. Distribution of a classical octet particle at t_c

$$n_o(\boldsymbol{p}, \boldsymbol{x}; t_c) = \text{Wigner transform of } \rho_o(\boldsymbol{p}, \boldsymbol{p}'; t_c)$$

Classical Boltzmann equation should use $n_o(\boldsymbol{p}, \boldsymbol{x}; t_c)$ for initial distribution
If $t_c \sim$ QGP lifetime, classical description must fail

Recent developments in open quantum system approach to quarkonium

1. Lindblad equation for quarkonium
   - Any Markovian equation preserving positivity & probability must be: [Lindblad (76)]

$$\frac{d}{dt}\rho(t) = -i[H,\rho] + \sum_n \left(2L_n\rho L_n^\dagger - L_n^\dagger L_n \rho - \rho L_n^\dagger L_n\right)$$

2. Numerical simulation by stochastic unravelling
   - Solve Lindblad equation by generating stochastic ensemble of $\{\phi_i(t)\}$

$$\rho(t) = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \frac{|\phi_i(t)\rangle\langle\phi_i(t)|}{\underbrace{||\phi_i(t)||^2}_{\phi(t)\ \text{unnormalized}}} = \text{Average of}\left[\frac{|\phi(t)\rangle\langle\phi(t)|}{||\phi(t)||^2}\right],$$

   - Quantum state diffusion method for solving Lindblad equation [Gisin-Percival (92)]

$$|d\phi\rangle = -iH|\phi(t)\rangle dt + \sum_n \Big(\underbrace{2\langle L_n^\dagger\rangle_\phi L_n}_{\text{nonlinear in }\phi} - L_n^\dagger L_n\Big)|\phi(t)\rangle dt + \sum_n L_n|\phi(t)\rangle d\xi_n,$$

$$\underbrace{\langle d\xi_n d\xi_m^*\rangle}_{\text{complex noise}} = 2\delta_{nm}dt$$

We will derive the Lindblad equation and compute by stochastic unravelling

Lindblad operators in various regimes

1. Perturbation theory in the influence functional formalism [Akamatsu (15), De Boni (17)]
   - Regime of quantum Brownian motion

$$\underbrace{\tau_E \ll \tau_{S,R}}_{\text{HQ motion and relaxation is slow}} \quad \to \quad g \ll 1, \quad g^3 \ln(1/g) \ll \frac{M}{T} \ll \frac{g}{\alpha^2} \sim \frac{100}{g^3}$$

   - Lindblad operators describe scatterings with $\Delta p = k$ and recoil

$$L_k = \underbrace{\sqrt{D(k)}}_{\to \text{ rate } D(k)} \underbrace{e^{ikx/2}\left[1 + \frac{ik\cdot\nabla_x}{4MT}\right]e^{ikx/2}}_{\Delta p = k,\ \Delta x \sim k/MT} \underbrace{(t^a \otimes 1)}_{\text{color rotation}} \quad +\text{heavy antiquark}$$

2. Dyson-Schwinger equation for density matrix in pNRQCD [Brambilla+ (17, 18)]
   - Weak and strong coupling regimes

$$\underbrace{1/a_0 \gg T}_{\mathcal{L}_{\text{pNRQCD}} \text{ in } T=0} \gg E \gg m_D, \quad 1/a_0 \gg T \sim m_D \gg E$$

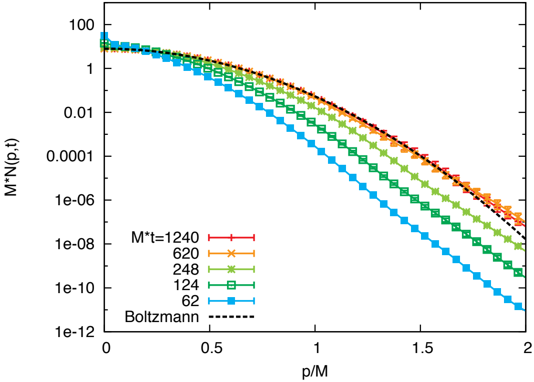
   - Lindblad op. from gluon-dissociation (weak) and inelastic scattering (strong)

Specific form of the Lindblad operators depends on the regime

Equilibration of a single heavy quark

- QSD equation turns out to be a nonlinear stochastic Schrödinger equation
  - Constructing mixed-state density matrix with solitonic basis
- Time evolution of momentum distribution
  - Relaxation time of corresponding classical system is $M\tau_{\text{relax}} \sim 300$

[Akamatsu+ (18)]



Equilibration is achieved with classical relaxation time

Quarkonium survival probability in 1d Bjorken expansion

- Time evolution of occupation number of bottomonium

Bound state in Cornell potential

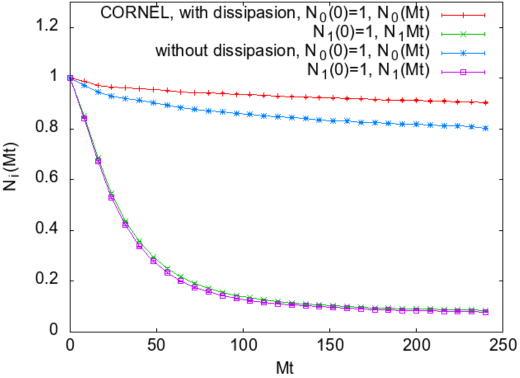
⬇

QSD in Bjorken expanding QGP with HQ potential = Debye screened

⬇

Projection to Cornell bound states

[Miura+, in preparation]



Effect of dissipation is important for quantitative analysis of the ground state

Summary and outlook

Is classicalization time short enough to apply kinetic models?

$$t_c \sim \frac{M^{1/2}}{C_F\alpha T^{3/2}} \sim \frac{1}{(C_F\alpha)^2 T} \sim \frac{10}{T} \quad (C_F\alpha \sim 0.3,\ T \sim M(C_F\alpha)^2)$$

1. For naive theorists, it is rather long because $C_F\alpha \sim 0.3$
2. For HQ phenomenologists, it is subtle because $C_F\alpha \sim 1$
3. For hydro practitioners, it is short enough because $C_F\alpha \gg 1$ or $t_c \sim t_{\mathsf{mft}}$

Open system provides a more fundamental descriptions for quarkonium

1. Different Lindblad operators in different regimes
   - Is quarkonium really confined to one particular regime during evolution?
2. Nonlinear stochastic Schrödinger equation connected with microscopic theory
   - Quarkonium evolution in heavy-ion collisions [Miura+, in preparation]
3. Stochastic potential with color SU(3)
   - Simulate decoherence and classicalization in the octet sector [Kajimoto+, in progress]

Back Up

How is θ_c determined?

My definition of classicalization (in free space + thermal environment)

1. Localized compared to noise correlation $\Delta x \ll l_{\rm corr}$
2. Uncertainty relation $\Delta x \cdot \Delta p \sim 1$ is saturated, and keep saturated

$$\Delta x(t) \sim \Delta x + \frac{\Delta p}{M} t \sim \Delta x + \frac{t}{M \Delta x}, \quad \Delta p(t) \sim \Delta p,$$
$$\to t \sim M(\Delta x)^2 \sim \tau_{\mathsf{dec}}(\Delta x) \sim \frac{l_{\mathsf{corr}}^2}{D(0)(\Delta x)^2}$$
$$\to \left(\frac{l_{\mathsf{corr}}^2}{MD(0)}\right)^{1/4} < \Delta x < l_{\mathsf{corr}}$$

Then, θ_c is obtained

$$\theta_c \sim \frac{\Delta p_{\mathsf{max}}}{p} \sim \frac{1}{p \Delta x_{\mathsf{min}}}$$

Parameter choice of numerical simulations

$$H_{\text{Debye}} = \frac{p^2}{M} - \frac{\alpha}{\sqrt{x^2 + x_{\text{c}}^2}} \mathrm{e}^{-m_{\text{D}}|x|},$$

$$H_{\text{Cornell}} = \frac{p^2}{M} - \frac{\alpha}{\sqrt{x^2 + x_{\text{c}}^2}} + \sigma x,$$

$$D(x) = \gamma \exp(-x^2/\ell_{\mathsf{corr}}^2)$$

Δx	Δt	N_x	γ	l_{corr}	α	m_{D}	x_{c}	σ
$1/M$	$0.1M(\Delta x)^2$	254	T/π	$1/T$	0.3	T	$1/M$	$0.01M^2$

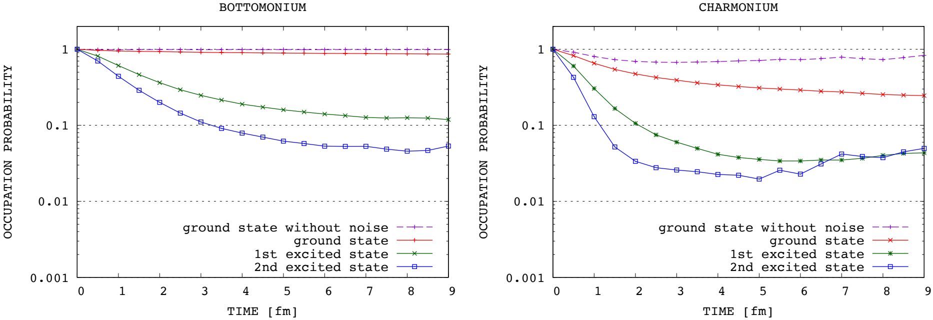
Application: stochastic potential in QGP [Kajimoto+ (17)]

1. In-medium potential V and correlation function D in a Bjorken expansion

$$V(r) = -\frac{0.3}{r}e^{-Tr}, \quad D(r) = 0.3Te^{-T^2r^2}, \quad T(t) = 0.4\text{GeV}\left(\frac{1\text{fm}}{1\text{fm}+t}\right)^{1/3}$$

2. Start from vacuum eigenstates and calculate their survival probability

$$V_{\text{vac}}(r) = -\frac{0.3}{r} + (1\text{GeV/fm})\cdot r, \quad N_{\Upsilon}(t) = \langle \|\Psi_{\Upsilon}^* \cdot \Psi_{b\bar{b}}(t)\|^2 \rangle$$



Decoherence gives an additional dynamical mechanism for dissociation

Nonlinear stochastic Schrödinger equation for a heavy quark

- Nonlinear stochastic Schrödnger equation

$$
\begin{aligned}
d\phi(x,t) &= \phi(x,t+dt) - \phi(x,t) \\
&\simeq \left(i\frac{\nabla^2}{2M} - \frac{1}{2}D(0) \right) \phi(x)dt + d\xi(x)\phi(x) \\
&\quad + \frac{dt}{||\phi(t)||^2} \int d^3y D(x-y)\phi^*(y)\phi(y)\phi(x) + \mathcal{O}(T/M)
\end{aligned}
$$

- Correlation of complex noise field

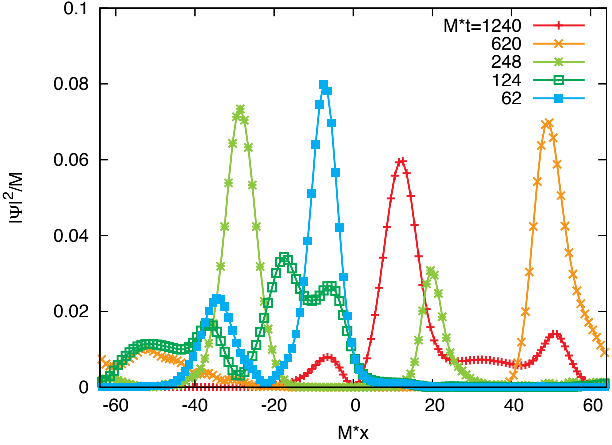
$$
\langle d\xi(x) d\xi^*(y) \rangle = D(x-y)dt, \quad \langle d\xi(x) d\xi(y) \rangle = \langle d\xi^*(x) d\xi^*(y) \rangle = 0
$$

- Density matrix for a heavy quark

$$
\rho_Q(x,y,t) = \mathsf{M}\left[\frac{\phi(x,t)\phi^*(y,t)}{||\phi(t)||^2} \right]
$$

What is the equilibrium solution of the Lindblad equation?
How does a heavy quark approach equilibrium?

Solitonic wave function in one sampling



Wave function is localized because of the nonlinear evolution equation