

# Nuclear structure with exotic beams

Lecture 2:

Nuclear reactions I

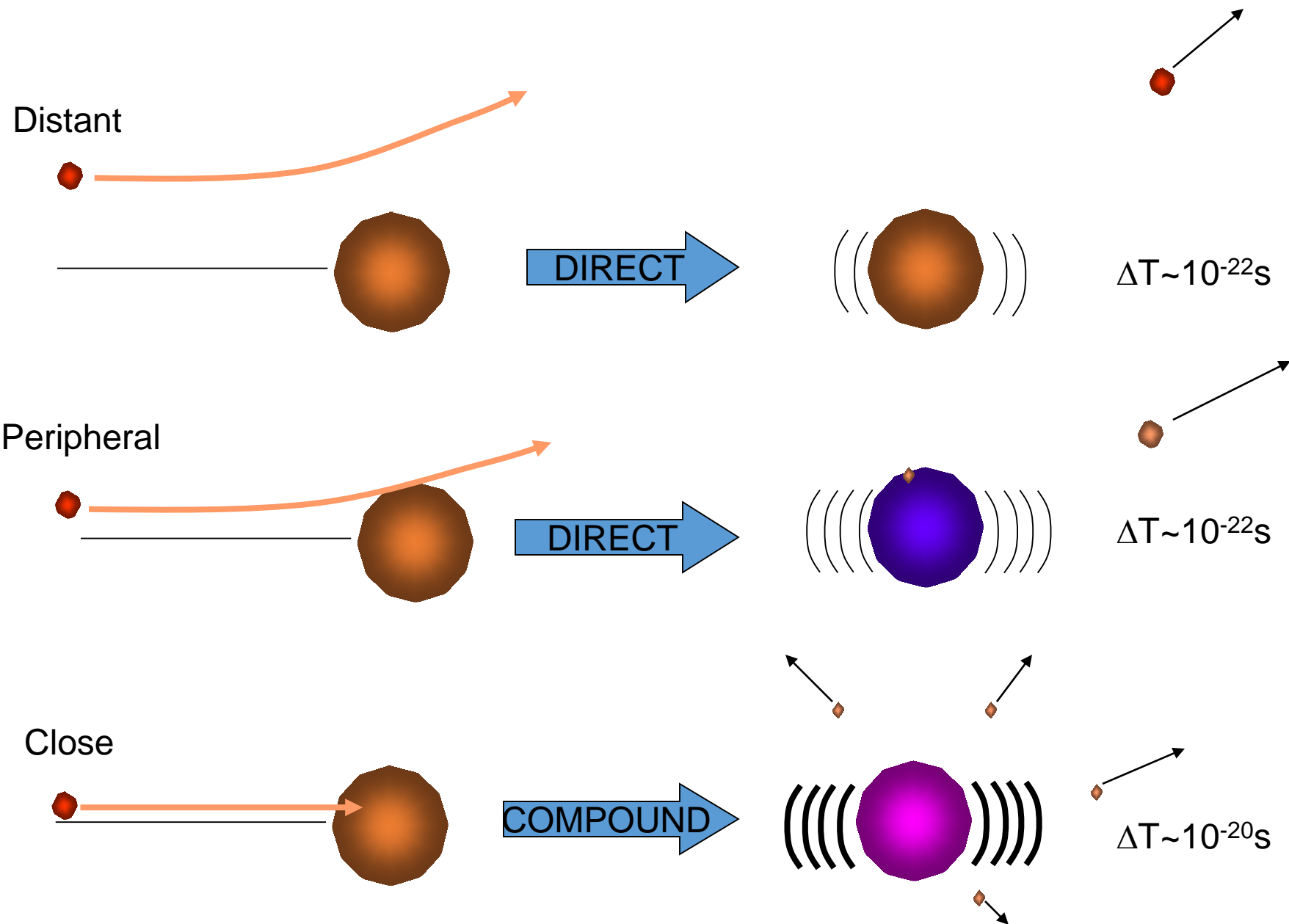
Elastic and Inelastic Scattering



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# Some types of collisions



# Very simple measurements can tell us important things

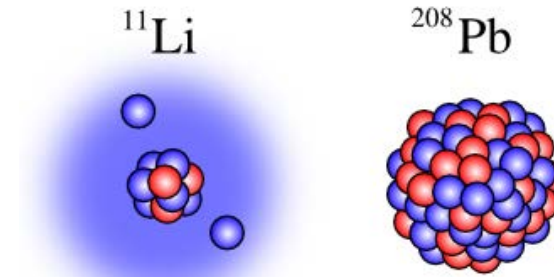


TABLE I. Interaction cross sections ( $\sigma_I$ ) in millibarns.

| Beam             | Be          | Target C                        | Al            |
|------------------|-------------|---------------------------------|---------------|
| $^6\text{Li}$    | $651 \pm 6$ | $688 \pm 10$                    | $1010 \pm 11$ |
| $^7\text{Li}$    | $686 \pm 4$ | $736 \pm 6$                     | $1071 \pm 7$  |
| $^8\text{Li}$    | $727 \pm 6$ | $768 \pm 9$                     | $1147 \pm 14$ |
| $^9\text{Li}$    | $739 \pm 5$ | $796 \pm 6$                     | $1135 \pm 7$  |
| $^{11}\text{Li}$ |             | <b><math>1040 \pm 60</math></b> |               |
| $^7\text{Be}$    | $682 \pm 6$ | $738 \pm 9$                     | $1050 \pm 17$ |
| $^9\text{Be}$    | $755 \pm 6$ | $806 \pm 9$                     | $1174 \pm 11$ |
| $^{10}\text{Be}$ | $755 \pm 7$ | $813 \pm 10$                    | $1153 \pm 16$ |

First experimental hint that  $^{11}\text{Li}$  was special – simply a total interaction cross section

I. Tanihata et al, PRL **55**, 2676 (1985)

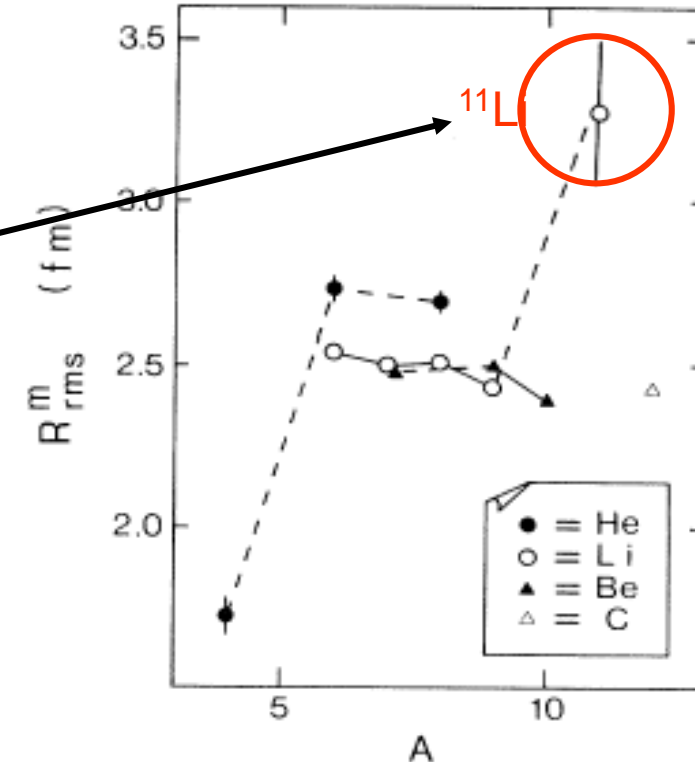
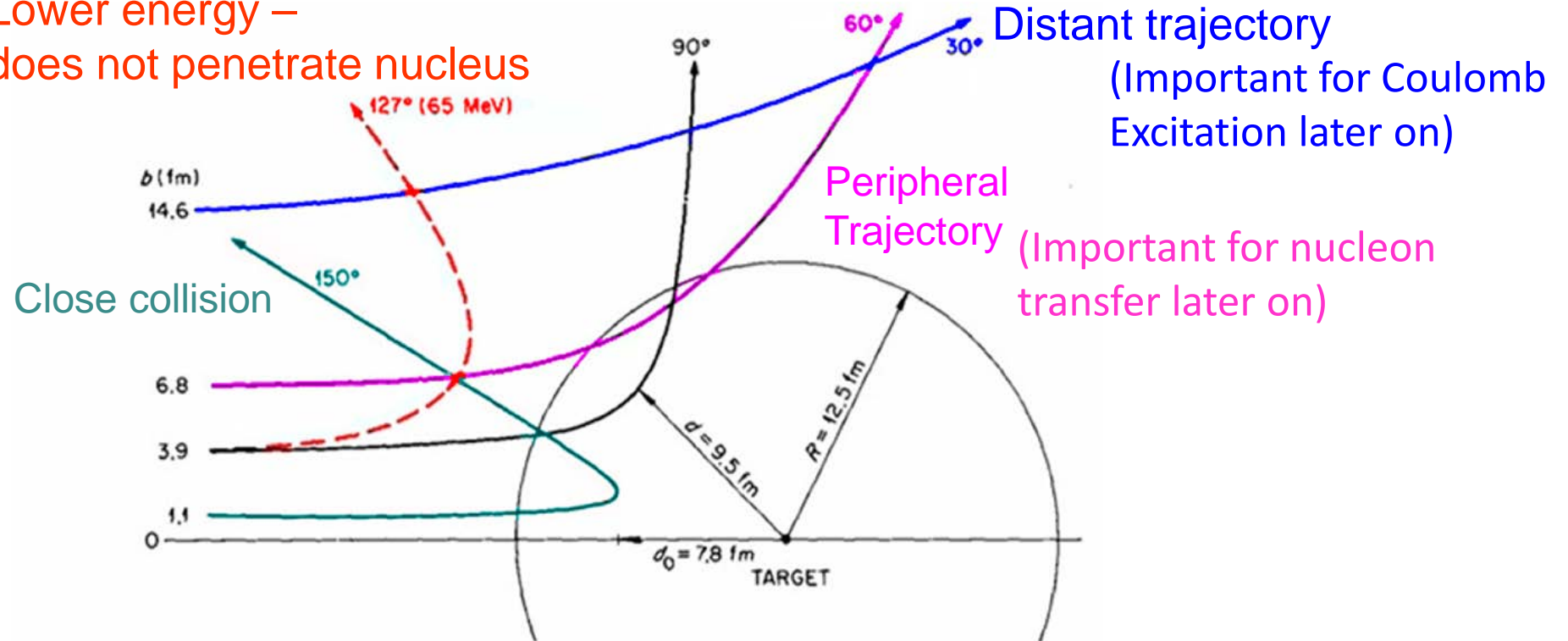


FIG. 3. Matter rms radius  $R_{\text{rms}}^m$ . Lines connecting isotopes are only guides for the eye. Differences in radii are seen for isobars with  $A = 6, 8,$  and  $9$ . The  $^{11}\text{Li}$  isotope has a much larger radius than other nuclei.

# Coulomb trajectories

Lower energy –  
does not penetrate nucleus



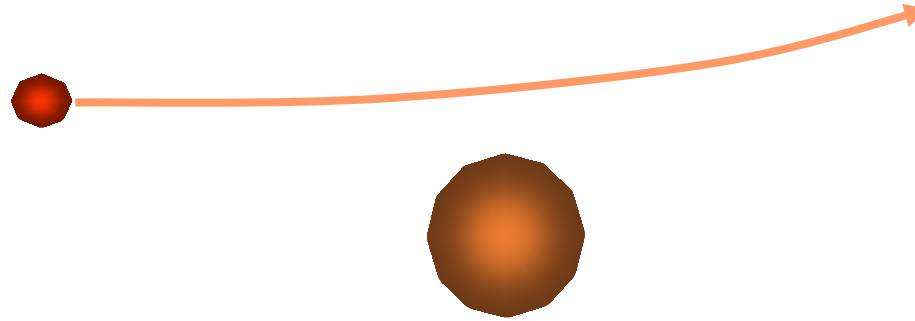
“Grazing angle”:  
angle corresponding to the trajectory  
for which the two nuclei just touch  
each other

$^{16}\text{O} + ^{208}\text{Pb}$   $E(^{16}\text{O}) = 130$  MeV,  $V_C \sim 93$  MeV

(Satchler, *Direct Nuclear Reactions* 1980, pp 36)

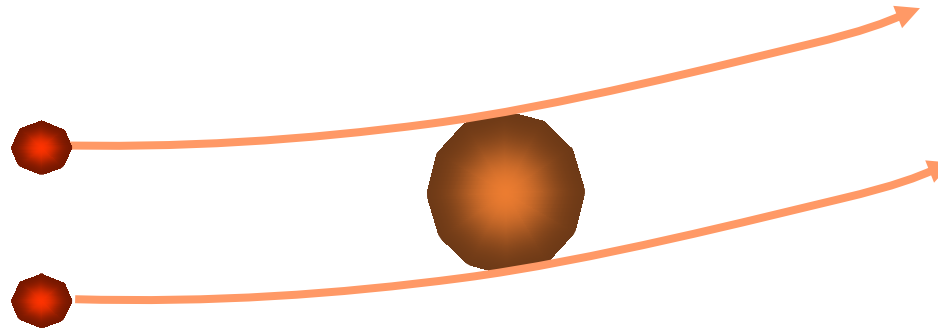
# Schematic evolution of elastic scattering with energy and angle

Low energy/large impact parameter -  
1 dominant trajectory  
 $d\sigma/d\Omega \sim d\sigma_R/d\Omega$



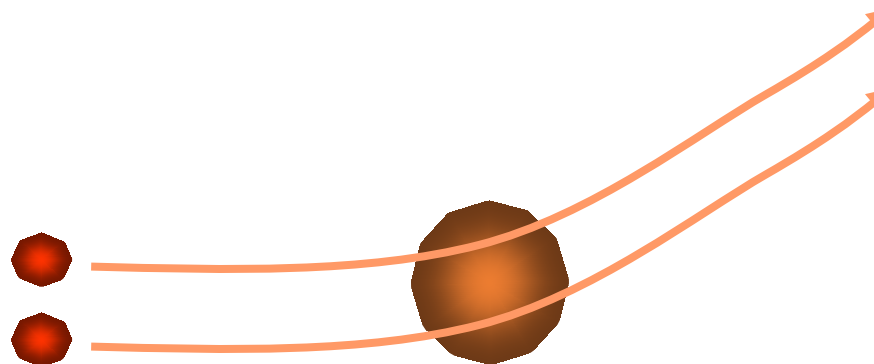
$$\theta < \theta_{GR}$$
$$b > R_{12}$$

Grazing trajectories from either side interfere constructively –  
“Fresnel” or “Coulomb-nuclear” interference  
and  $d\sigma/d\Omega > d\sigma_R/d\Omega$



$$\theta \sim \theta_{GR}$$
$$b \sim R_{12}$$

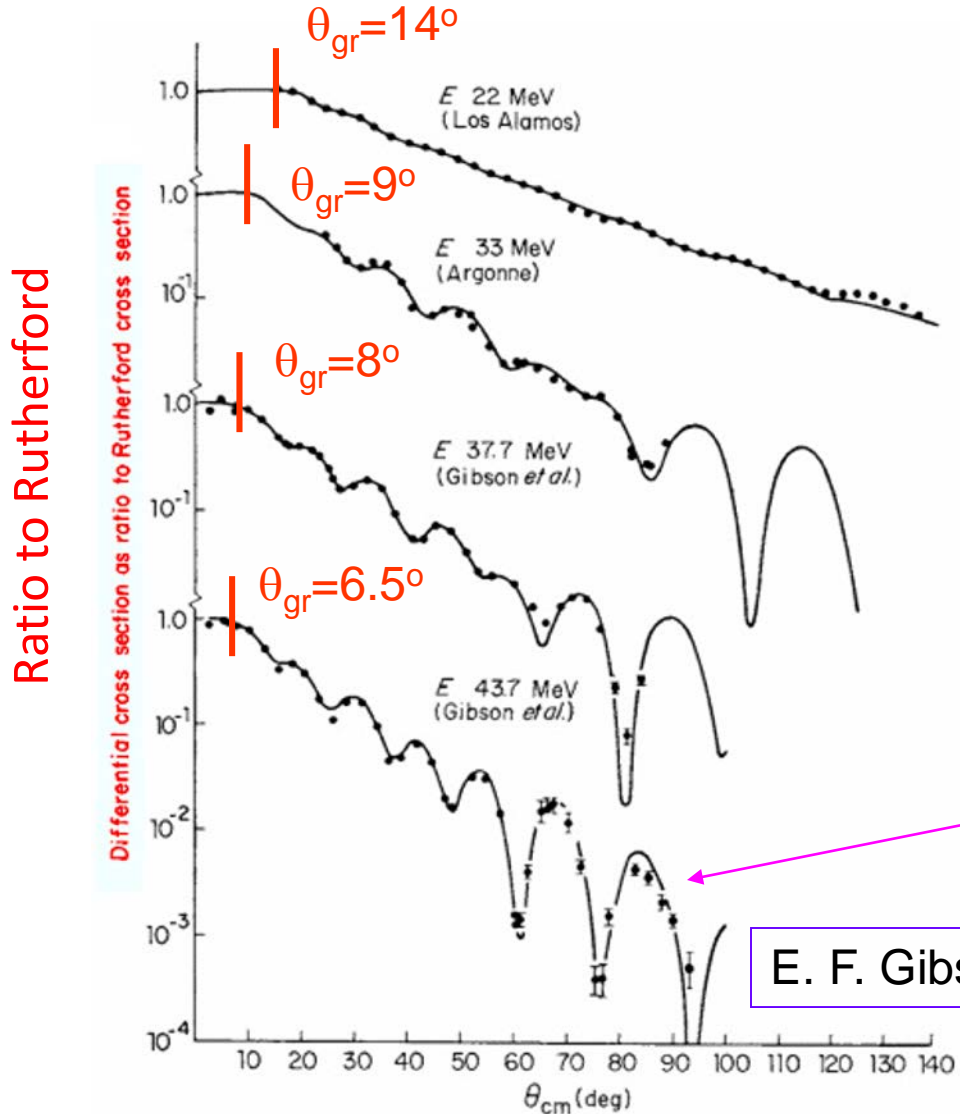
Higher energy/small impact parameter:  
absorption, 2  
interfering trajectories:  
 $d\sigma/d\Omega \ll d\sigma_R/d\Omega$



$$\theta > \theta_{GR}$$
$$b < R_{12}$$

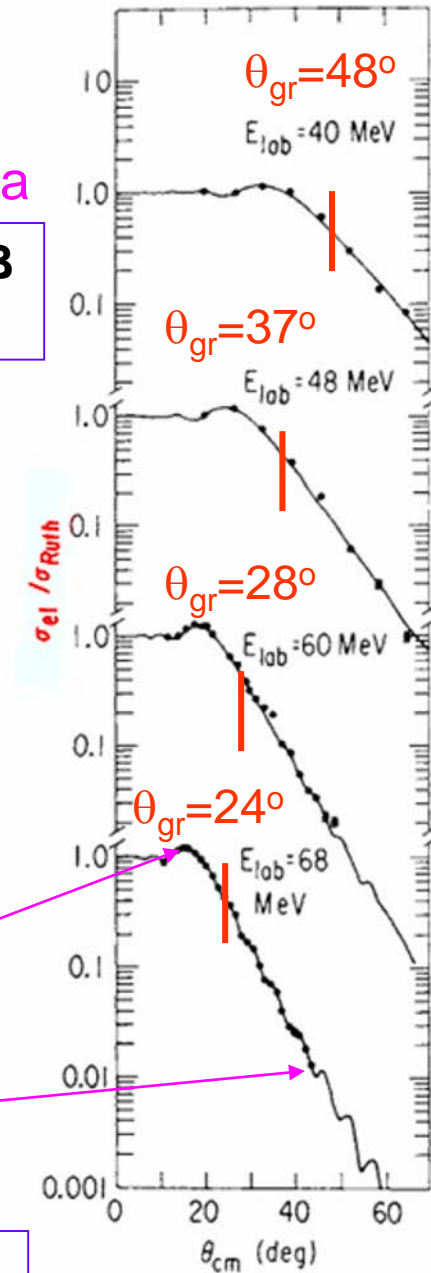
# Evolution of elastic-scattering angular distributions with energy

$^3\text{H}+^{58}\text{Ni}$



$^{13}\text{C}+^{40}\text{Ca}$

P. D. Bond, PL 47B 231 (1971)



Fresnel peak

Interference Oscillations

E. F. Gibson, PR 155, 1208 (1967)

Figs. from Glendenning 2004, pp 38 and 39

# The Optical Model

- The **optical model** is a schematic model of nuclear scattering that sweeps all of the microscopic nuclear structure under the rug.
- It is called “**optical**” because it treats the incident and outgoing particles as waves scattered by some ~spherical region. Those waves can also be **absorbed** (“cloudy ball”) and we can lose flux (particles), thus reducing the elastic scattering cross section.
- The combined effects of many complex states are averaged into a single nucleus-nucleus potential – called the **Optical Potential**

# Formalism and Optical Model scattering

Need a solution to Schrödinger's equation

$$-\frac{\hbar^2}{2m} \frac{d^2 u_l(r)}{dr^2} + \left[ U(r) + \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u_l(r) = E u_l(r)$$

The asymptotic solutions are waves and an approximate (Born) solution is:

$$\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2; \quad f(\theta, \phi) = -\frac{1}{4\pi} \int \exp(-i\vec{k}' \cdot \vec{r}') U(r') \exp(i\vec{k} \cdot \vec{r}') dr'$$

A better treatment uses asymptotic solutions that are waves distorted by the Coulomb Potential:

“**Distorted-Wave Born Approximation**” or **DWBA**

$U(r)$  is the **Optical Potential**, consisting of:  
Volume, surface, and  $l \cdot S$  (real and imaginary)



# Contributions to $U(r)$

- Coulomb part:  $V_C(r) = Z_1 Z_2 e^2 / r$
- Real Nuclear part:  $V(r)$ 
  - Comes from the nuclear attraction
- Imaginary Nuclear Part (!):  $W(r)$ 
  - **Why?** Other things can happen so we can lose elastic flux! There must be “**absorption**” of waves.
- Spin-Orbit ( $\mathbf{l} \cdot \mathbf{S}$ ) part:  $V_{SO}(r)$ 
  - **Why?** There is a spin-orbit component to the nuclear force so it seems natural to have one between nuclei. Also, it seems to be needed to explain **polarization** data!

$$U(r) = V_C(r) + V(r) + iW(r) + V_{SO}(r)$$

Only the real parts contribute to elastic scattering

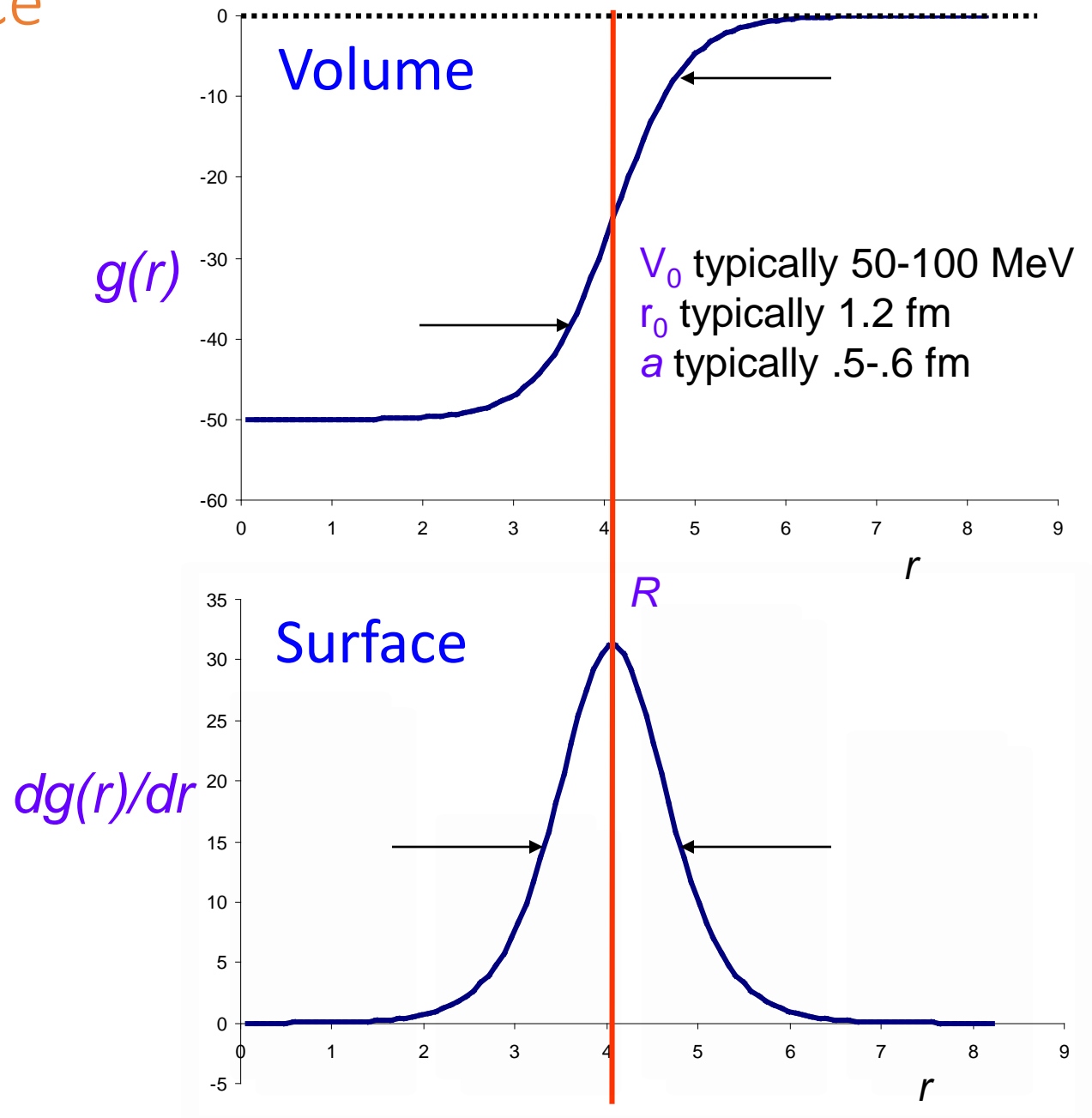
# Volume and surface potentials

$R=r_0(A_1^{1/3}+A_2^{1/3})$  is the radius where the potential is  $\frac{1}{2}$  its maximum.

“ $a$ ” is the “diffuseness” parameter. It describes the “spread” of the potential about  $R$ .

Typically, the spin-orbit potential is described as

$$V_{so} = \frac{C}{r} \frac{dg(r)}{dr} \vec{l} \cdot \vec{s}$$



# Inelastic scattering and channel coupling

Optical potential

Coupled differential equations

$$(E_\alpha - T_{\alpha l} - U_\alpha) u_\alpha^0 = 0$$

$$(E_{\alpha'} - T_{\alpha l} - U_\alpha) u_{\alpha'}^0 = V_{\alpha\alpha'} u_\alpha^0$$

Coupling potential

$$V_{\alpha\alpha'} \sim \langle \phi_{\alpha'} | V_{INEL}(r) | \phi_\alpha \rangle$$

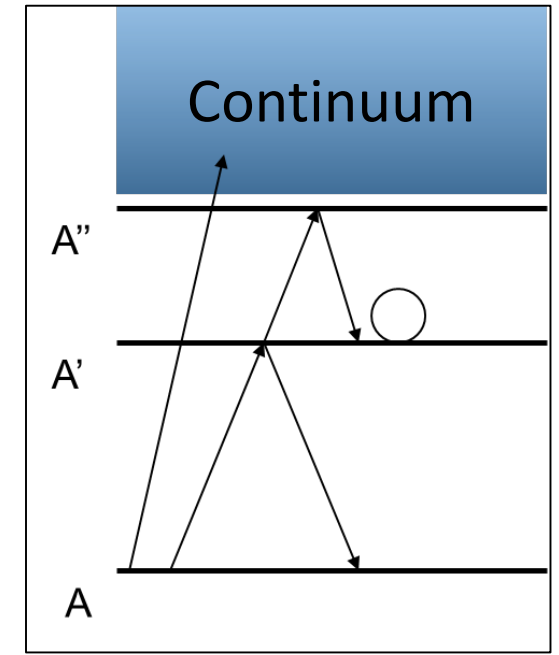
$\phi_\alpha$  are the intrinsic states in a collective model, and  $V_{INEL}$  is a coupling potential

$$V_{VIB}(r) \sim R_0 \frac{dU}{dr} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\vec{r}) \quad \text{Vibrational model}$$

$$V_{ROT}(r) \sim R_0 \frac{dU}{dr} \beta_L Y_{LM}(\vec{r}) \quad \text{Rotational model}$$

These correspond to distortions of the nuclear surface.

The  $\alpha$ 's and  $\beta$ 's tell us about the collectivity of the nuclei  
The  $\beta_L$  in particular give the magnitude of different multipole deformations



# Coupling to rotational states

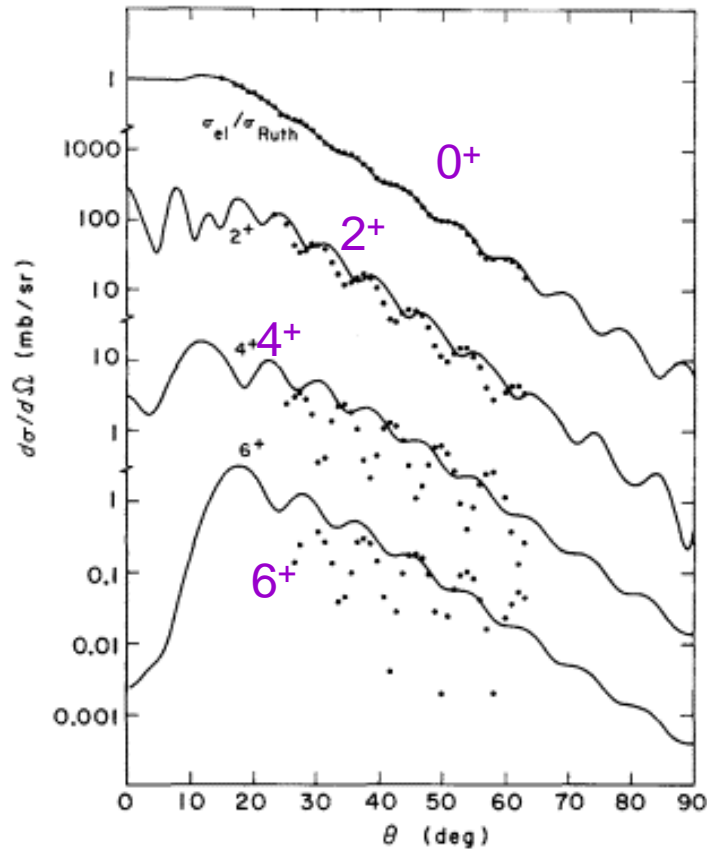


Fig. 13.3. Distorted-wave calculation with optical parameters that fit the elastic section as shown (listed in Chapter 4).  $\beta_2 = 0.3$ ,  $\beta_4 = 0.15$ ,  $\beta_6 = 0.075$ . DWBA,  $^{154}\text{Sm}$  (Glendenning, 1969a).

Elastic scattering alone can be fit with an optical model...

$\alpha + ^{154}\text{Sm}$

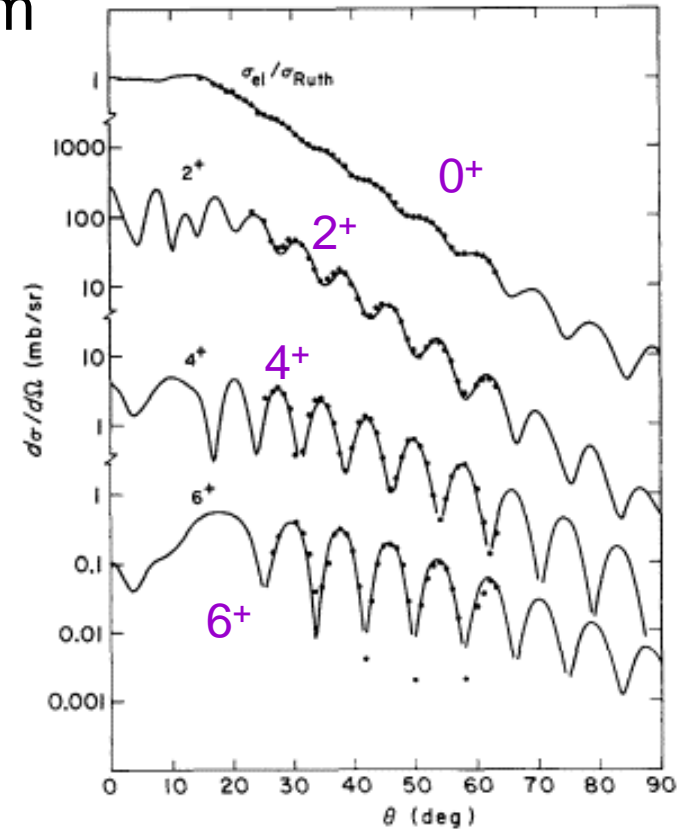


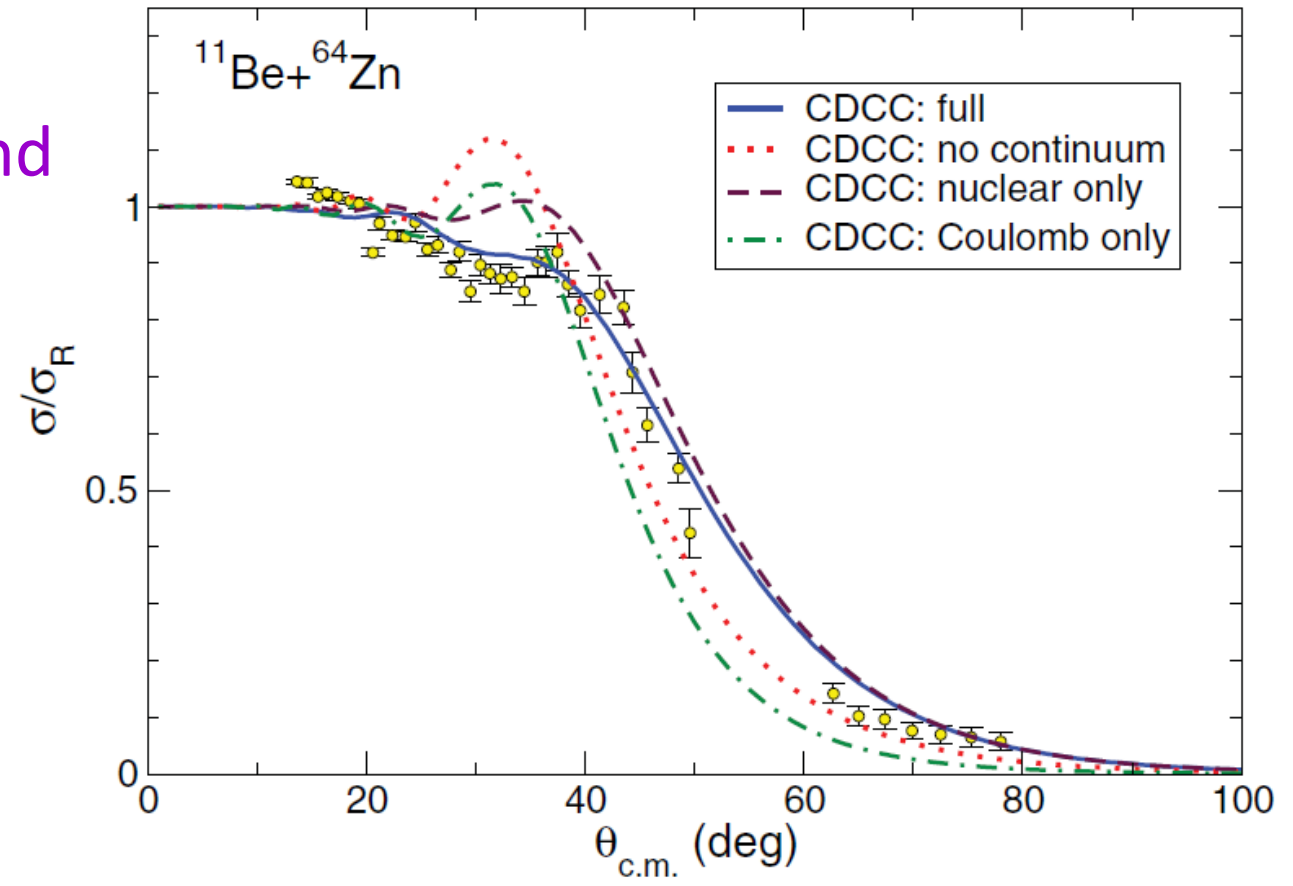
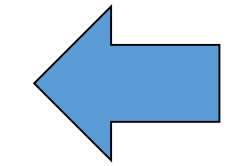
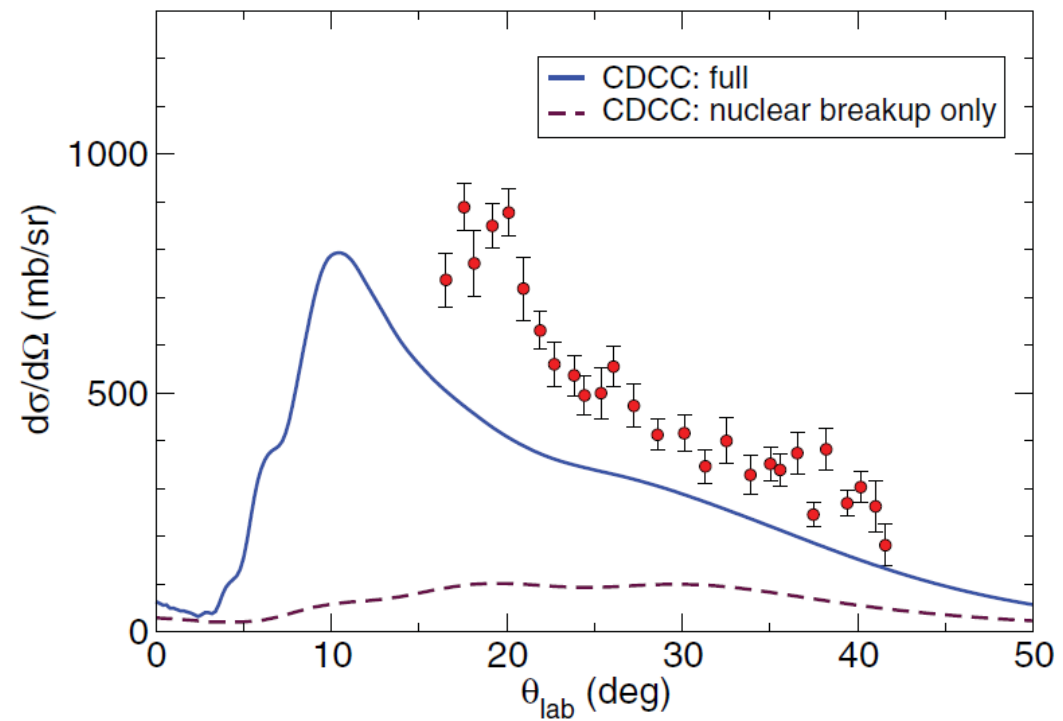
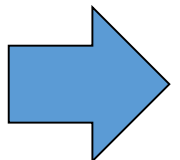
Fig. 13.4. Cross sections for 50-MeV alpha-excitation ground-state rotational band  $^{154}\text{Sm}$ . Curves are coupled-channel calculation as described in text. The data were taken at the Berkeley 88-in. Cyclotron.  $\beta_2 = 0.225$ ,  $\beta_4 = 0.05$ ,  $\beta_6 = -0.015$  (from Harvey *et al.*, 1968; Hendrie *et al.* 1968; calculation by Glendenning 1969a).

Channel-coupling is needed to fit the inelastic channels. Everything is treated simultaneously.

# Continuum coupling: $^{11}\text{Be}+^{64}\text{Zn}$

## Elastic scattering and Coupled Channels

DiPietro et al., PRC **85**  
054607 (2012).



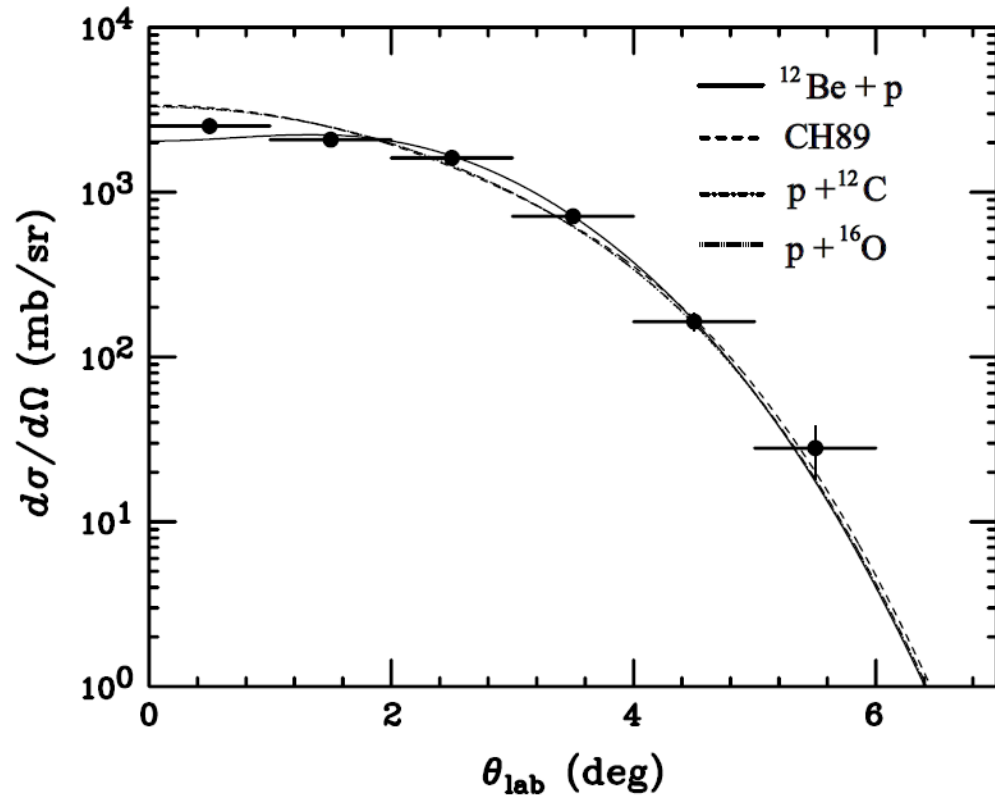
Breakup events and  
Coupled Channels

Continuum Discretized Coupled Channels

# $^{16}\text{C}$ +proton scattering

## Neutron-dominant quadrupole collective motion in $^{16}\text{C}$

H. J. Ong,<sup>1,\*</sup> N. Imai,<sup>2</sup> N. Aoi,<sup>2</sup> H. Sakurai,<sup>1</sup> Zs. Dombrádi,<sup>3</sup> A. Saito,<sup>4</sup> Z. Elekes,<sup>2,3</sup> H. Baba,<sup>4</sup> K. Demichi,<sup>5</sup> Z. S. Fülöp,<sup>3</sup> J. Gibelin,<sup>5,6</sup> T. Gomi,<sup>2</sup> H. Hasegawa,<sup>5</sup> M. Ishihara,<sup>2</sup> H. Iwasaki,<sup>1</sup> S. Kanno,<sup>5</sup> S. Kawai,<sup>5</sup> T. Kubo,<sup>2</sup> K. Kurita,<sup>5</sup> Y. U. Matsuyama,<sup>5</sup> S. Michimasa,<sup>2</sup> T. Minemura,<sup>2</sup> T. Motobayashi,<sup>2</sup> M. Notani,<sup>4,†</sup> S. Ota,<sup>7</sup> H. K. Sakai,<sup>5</sup> S. Shimoura,<sup>4</sup> E. Takeshita,<sup>5</sup> S. Takeuchi,<sup>2</sup> M. Tamaki,<sup>4</sup> Y. Togano,<sup>5</sup> K. Yamada,<sup>2</sup> Y. Yanagisawa,<sup>2</sup> and K. Yoneda<sup>2</sup>



Large ratio of  $M_n/M_p$  indicates particular sensitivity to neutron motion

TABLE I. Nuclear deformation parameter  $\beta_{pp'}$  and deformation length  $\delta_{pp'}$  deduced from DWBA calculations.

| Optical potential       | $\beta_{pp'}$ | $r_0$ (fm) | $\delta_{pp'}$ (fm) |
|-------------------------|---------------|------------|---------------------|
| CH89 [14]               | 0.476(37)     | 1.16       | 1.39(11)            |
| $p+^{16}\text{O}$ [15]  | 0.440(33)     | 1.14       | 1.26(9)             |
| $p+^{12}\text{C}$ [15]  | 0.531(42)     | 1.10       | 1.47(12)            |
| $^{12}\text{Be}+p$ [16] | 0.435(32)     | 1.48       | 1.62(12)            |

$$\frac{M_n}{M_p} = \frac{b_p}{b_n} \left[ \frac{\delta_{pp'}}{\delta_{em}} \left( 1 + \frac{b_n N}{b_p Z} \right) - 1 \right]$$

$M_p, M_n$ : proton and neutron matrix elements

$b_p, b_n$ : proton and neutron interaction strengths

# $^{22}\text{O}$ -proton scattering

## $N = 14$ Shell Closure in $^{22}\text{O}$ Viewed through a Neutron Sensitive Probe

E. Becheva,<sup>1</sup> Y. Blumenfeld,<sup>1</sup> E. Khan,<sup>1</sup> D. Beumel,<sup>1</sup> J. M. Daugas,<sup>2</sup> F. Delaunay,<sup>1</sup> Ch-E. Demonchy,<sup>3</sup> A. Drouart,<sup>4</sup> M. Fallot,<sup>1</sup> A. Gillibert,<sup>4</sup> L. Giot,<sup>3</sup> M. Grasso,<sup>1,5</sup> N. Keeley,<sup>4</sup> K. W. Kemper,<sup>6</sup> D. T. Khoa,<sup>7</sup> V. Lapoux,<sup>4</sup> V. Lima,<sup>1</sup> A. Musumarra,<sup>5</sup> L. Nalpas,<sup>4</sup> E. C. Pollacco,<sup>4</sup> O. Roig,<sup>2</sup> P. Roussel-Chomaz,<sup>3</sup> J. E. Sauvestre,<sup>2</sup> J. A. Scarpaci,<sup>1</sup> F. Skaza,<sup>4</sup> and H. S. Than<sup>7</sup>

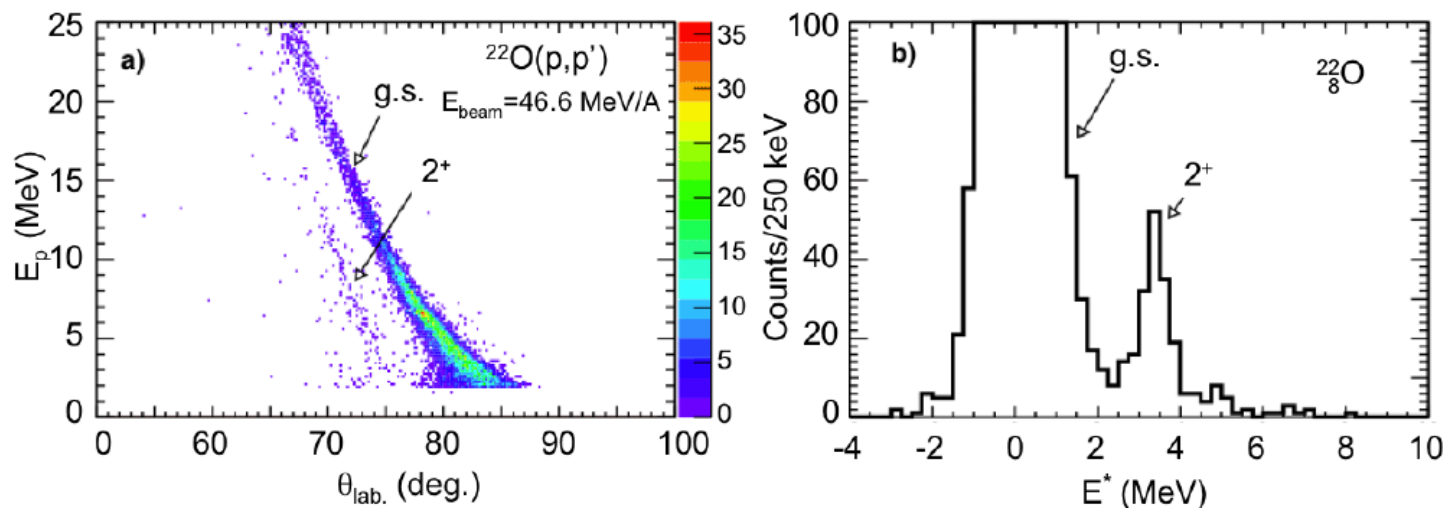


FIG. 1 (color online). (a) Scatter plot of recoiling proton energy versus scattering angle in the laboratory frame for the  $^{22}\text{O}$  beam. (b)  $^{22}\text{O}$  excitation energy spectrum deduced from the proton kinematics.

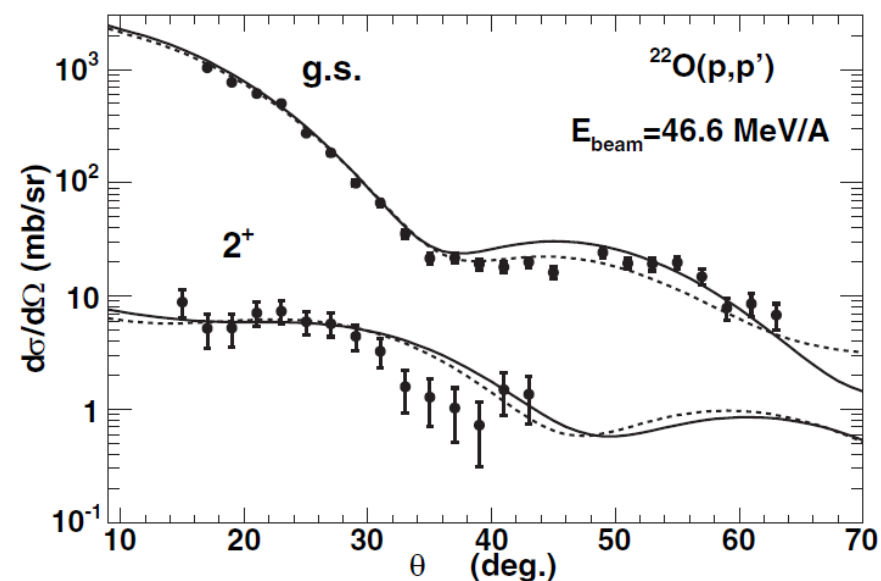
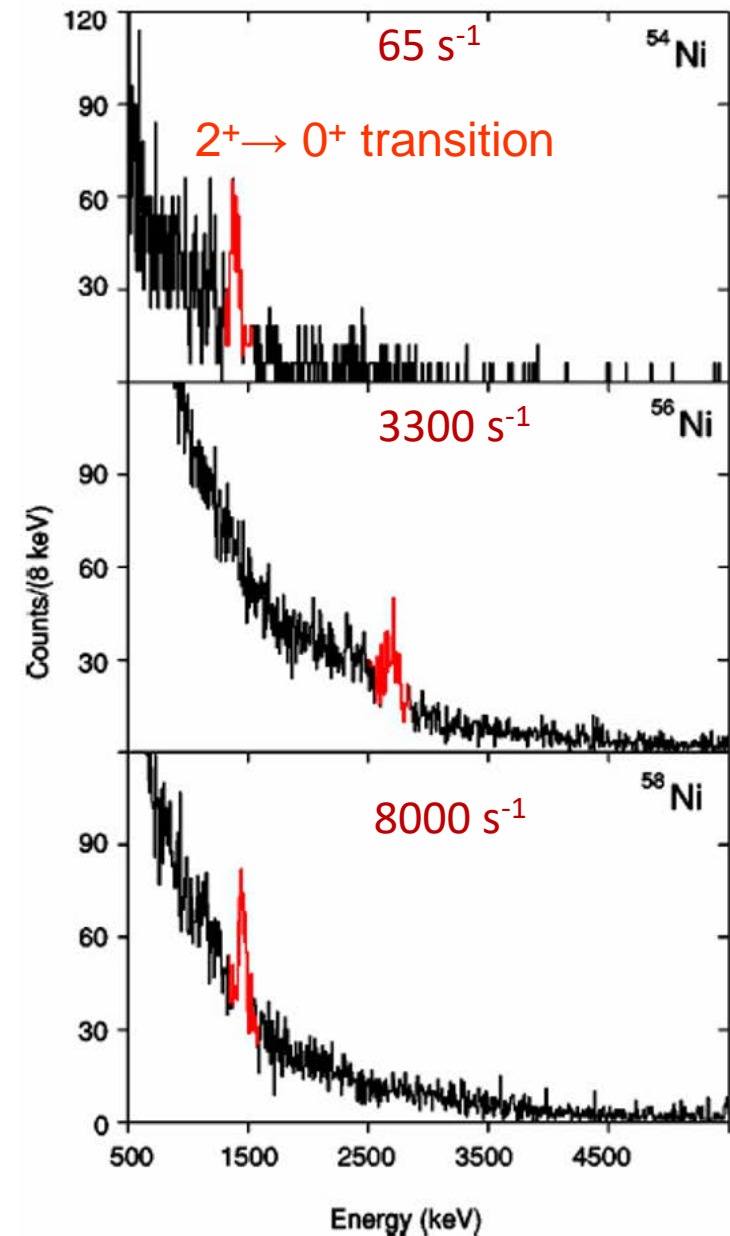
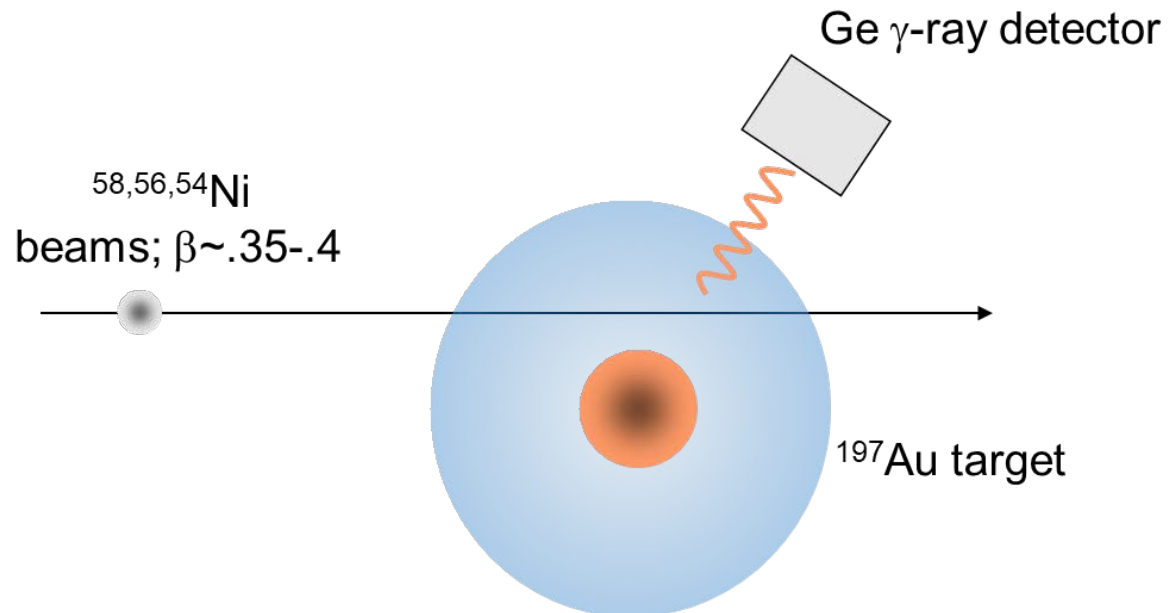


FIG. 2. Elastic and  $2_1^+$  inelastic angular distributions of  $^{22}\text{O}$  at 46.6 MeV (dots). DWBA using the phenomenological KD global optical potential (solid lines) and folding model (dashed lines) calculations are shown (see text).

# Coulomb Excitation as a spectroscopic tool

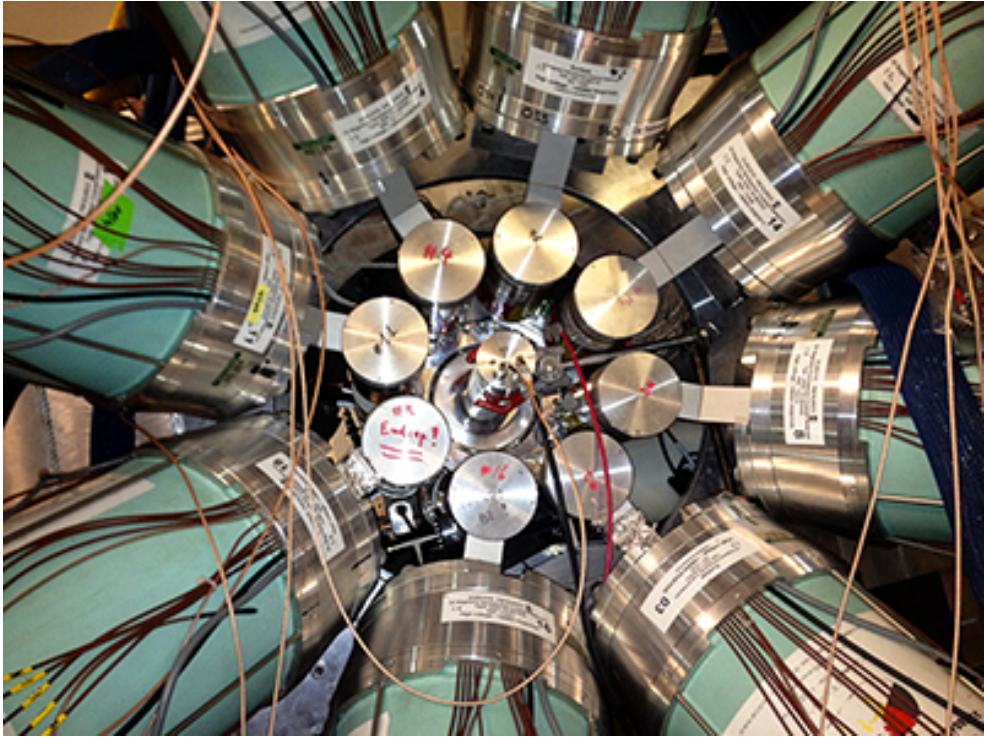
|  | $^{58}\text{Ni}^a$ | $^{56}\text{Ni}$ | $^{54}\text{Ni}$ |
|--|--------------------|------------------|------------------|
| Experimental results                   |                    |                  |                  |
| $E_\gamma$ (keV)                       | 1453(8)            | 2695(15)         | 1396(9)          |
| $\sigma$ (mb)                          | 175(36)            | 107(26)          | 134(36)          |
| $v/c$ (midtarget)                      | 0.373              | 0.391            | 0.346            |
| $\theta_{lab}^{max}$ (degrees)         | 3.2                | 2.9              | 3.5              |
| $b_{min}$ (fm)                         | 13.9               | 14.3             | 16.2             |
| $B(E2 \uparrow)$ ( $e^2 \text{fm}^4$ ) | 707(145)           | 494(119)         | 626(169)         |
| Adopted values                         |                    |                  |                  |
| $E_\gamma$ (keV)                       | 1454.28(10)        | 2700.6(7)        |                  |
| $B(E2 \uparrow)$ ( $e^2 \text{fm}^4$ ) | 695(20)            | 600(120)         |                  |



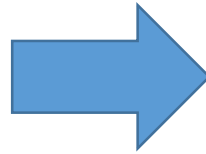
K. L Yurkewicz et al, PRC **70**, 054319 (2004)



# SEGA(MSU, 2004) and GRETINA (Now)



SEGA MSU Circa 2004



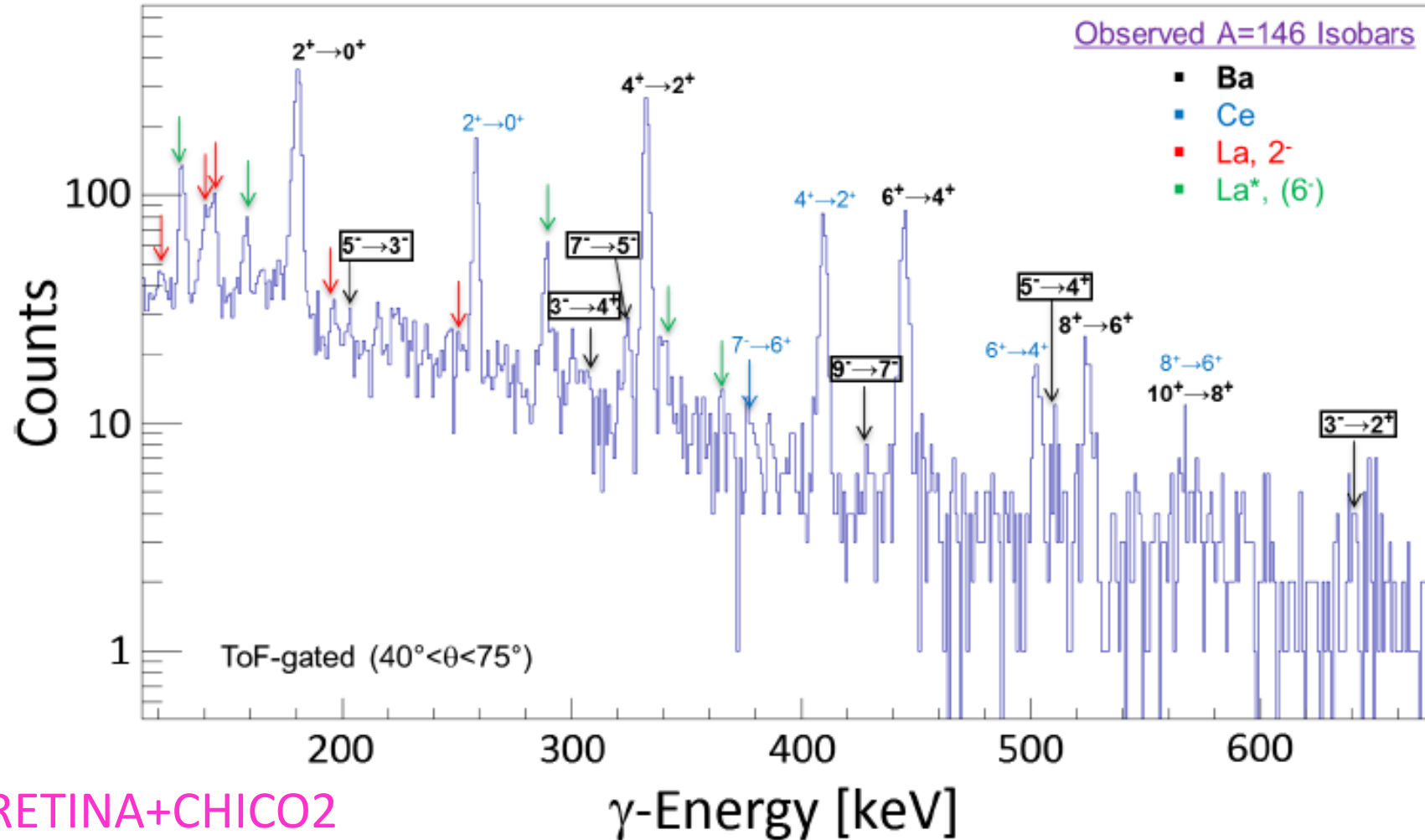
GRETINA circa 2017

# Coulomb excitation and octupole deformation

## Direct Evidence for Octupole Deformation in $^{146}\text{Ba}$ and the Origin of Large $E1$ Moment Variations in Reflection-Asymmetric Nuclei

B. Bucher,<sup>1,2,\*</sup> S. Zhu,<sup>3,†</sup> C. Y. Wu,<sup>1</sup> R. V. F. Janssens,<sup>3</sup> R. N. Bernard,<sup>4</sup> L. M. Robledo,<sup>4</sup> T. R. Rodríguez,<sup>4</sup> D. Cline,<sup>5</sup> A. B. Hayes,<sup>5</sup> A. D. Ayangeakaa,<sup>3</sup> M. Q. Buckner,<sup>1</sup> C. M. Campbell,<sup>6</sup> M. P. Carpenter,<sup>3</sup> J. A. Clark,<sup>3</sup> H. L. Crawford,<sup>6</sup> H. M. David,<sup>3,‡</sup> C. Dickerson,<sup>3</sup> J. Harker,<sup>3,7</sup> C. R. Hoffman,<sup>3</sup> B. P. Kay,<sup>3</sup> F. G. Kondev,<sup>3</sup> T. Lauritsen,<sup>3</sup> A. O. Macchiavelli,<sup>6</sup> R. C. Pardo,<sup>3</sup> G. Savard,<sup>3</sup> D. Seweryniak,<sup>3</sup> and R. Vondrasek<sup>3</sup>

$\sim 3000$   $^{146}\text{Ba}$   $\text{s}^{-1}$



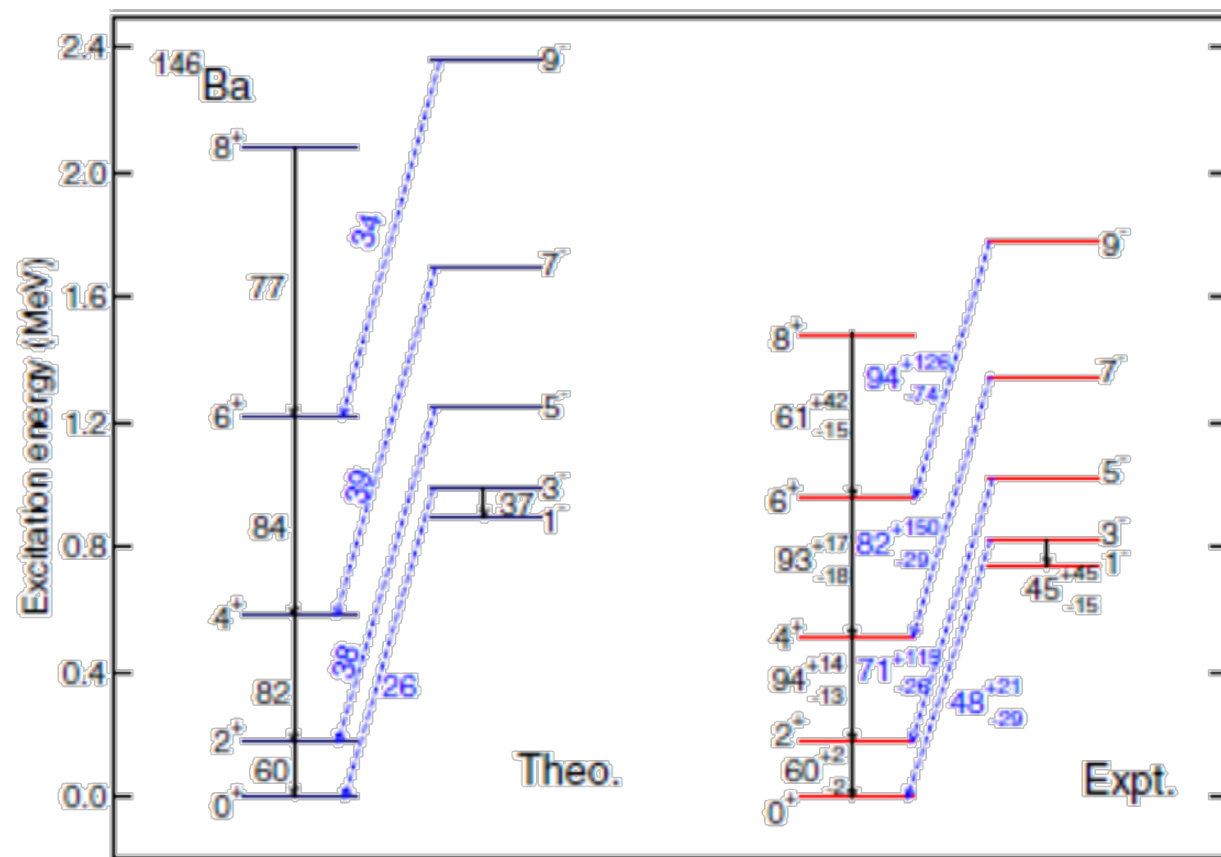
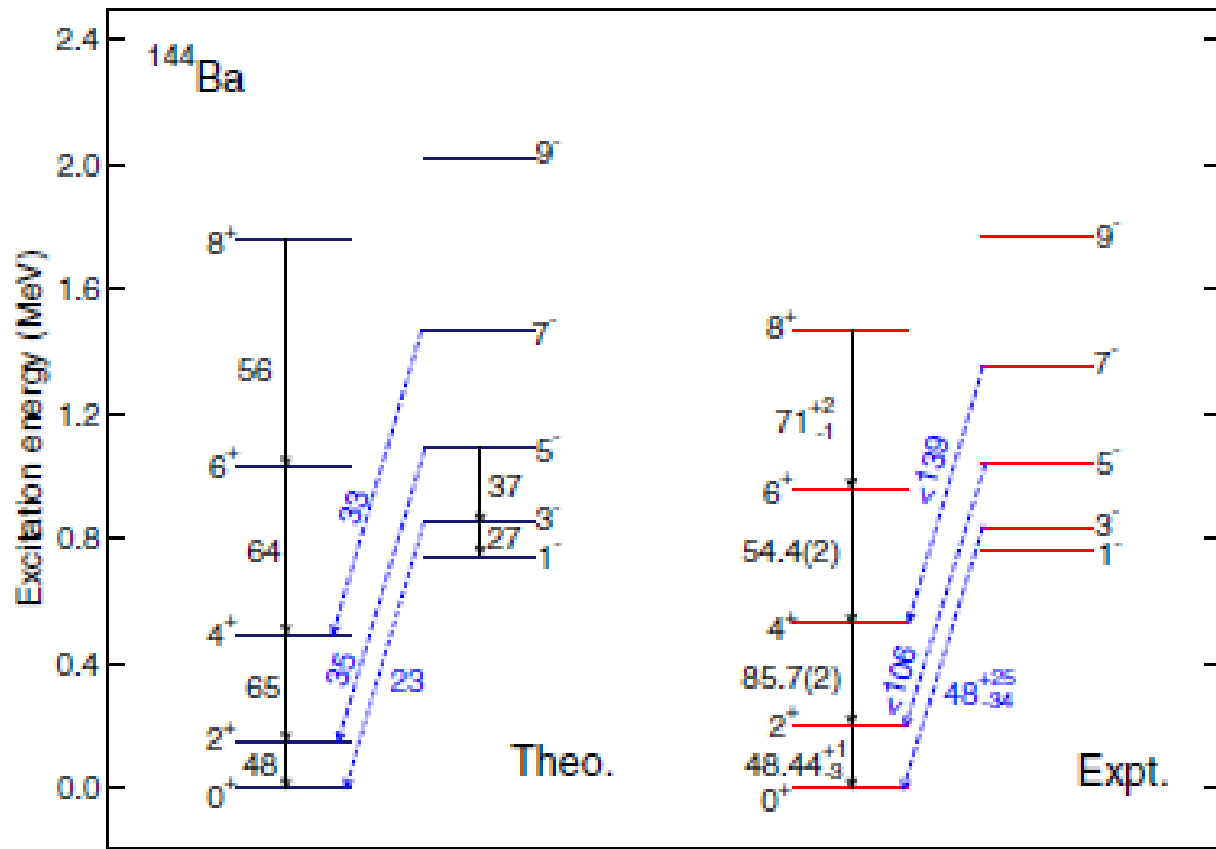
CARIBU+GRETINA+CHICO2

# Theory for octupole-deformed Ba nuclei

PHYSICAL REVIEW C 97, 024317 (2018)

Signatures of octupole correlations in neutron-rich odd-mass barium isotopes

K. Nomura, T. Nikšić, and D. Vretenar



Note positive and negative parity rotational sequences, with cross-transitions

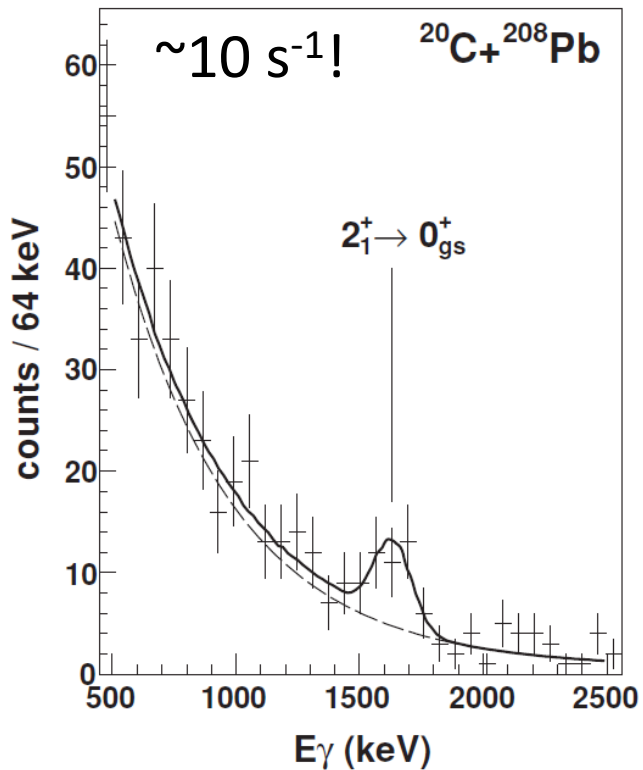
# Combining Coulomb and nuclear scattering

PHYSICAL REVIEW C **79**, 011302(R) (2009)

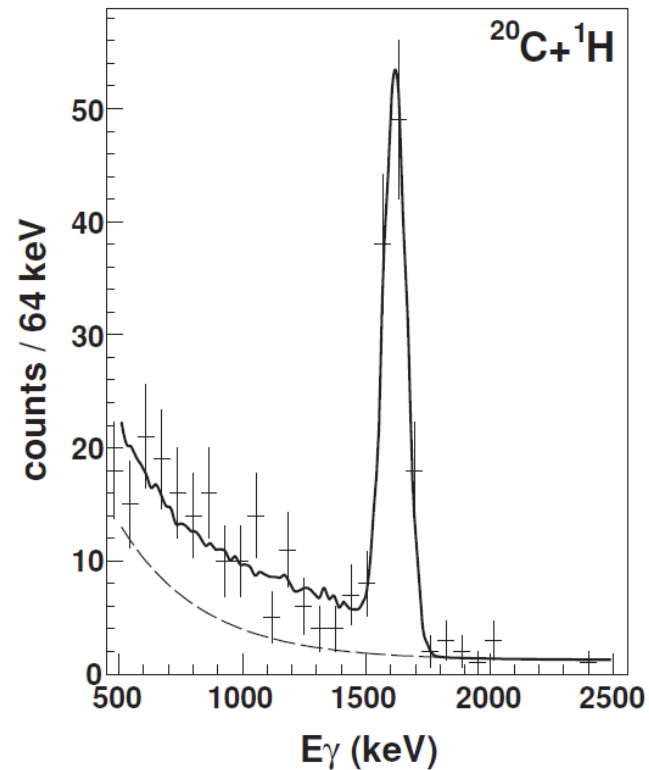
## Persistent decoupling of valence neutrons toward the dripline: Study of $^{20}\text{C}$ by $\gamma$ spectroscopy

Z. Elekes,<sup>1</sup> Zs. Dombrádi,<sup>1</sup> T. Aiba,<sup>2</sup> N. Aoi,<sup>3</sup> H. Baba,<sup>3</sup> D. Bemmerer,<sup>4</sup> B. A. Brown,<sup>5</sup> T. Furumoto,<sup>6</sup> Zs. Fülöp,<sup>1</sup> N. Iwasa,<sup>7</sup> Á. Kiss,<sup>8</sup> T. Kobayashi,<sup>7</sup> Y. Kondo,<sup>9</sup> T. Motobayashi,<sup>3</sup> T. Nakabayashi,<sup>9</sup> T. Nannichi,<sup>9</sup> Y. Sakuragi,<sup>3,6</sup> H. Sakurai,<sup>3</sup> D. Sohler,<sup>1</sup> M. Takashina,<sup>10</sup> S. Takeuchi,<sup>3</sup> K. Tanaka,<sup>3</sup> Y. Togano,<sup>11</sup> K. Yamada,<sup>3</sup> M. Yamaguchi,<sup>3</sup> and K. Yoneda<sup>3</sup>

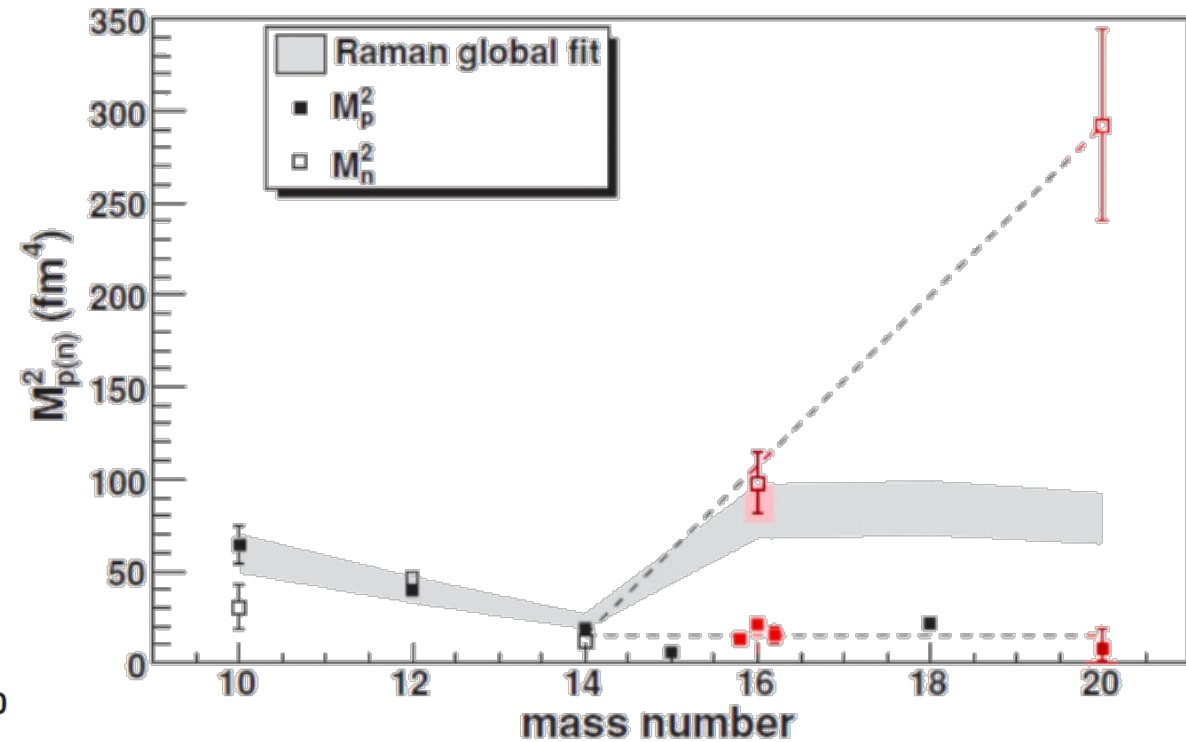
### Coulomb Excitation



### Nuclear Excitation



### $M_p$ and $M_n$



# Conclusions

- Simple scattering reactions can already inform us about nuclear structure
- Since cross sections tend to be large, these can be extremely useful if the intensity of the beam is small
- We should understand scattering reactions before we turn to the more complex problem of extracting nuclear-structure information from re-arrangement (transfer) reactions.