QCD Phase Transitions and Relativistic Heavy-Ion Collisions

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References: arXiv:1005.4814 (chiral symmetry)
arXiv:1108.2939 (deconfinement)
arXiv:12xx.xxxx (nuclear matter)
Lecture Plan

Part 1  (Monday)  [mainly on theory background – 31pp]
QCD Phase Transitions
  
  *Quark and gluon confinement / deconfinement*
  *Chiral-symmetry breaking / restoration*

Part 2  (Tuesday)  [mainly on phenomenology (+theory) – 38pp]
Relativistic Heavy-Ion Collisions
  
  *QCD critical point or something else?*
  *Initial fields and topological effects*
Part 1: Theory Background
Today's Goal

Understanding of the QCD phase diagram

Fukushima-Hatsuda (2010)
QCD Phase Transitions

Intuitive picture of the QCD phase transitions

Color (or Quark) Deconfinement \((\text{d.o.f})\)

- Hadron
  - \ (~1\,\text{fm})
  - Current (bare) quark \(m \sim \text{a few MeV} (< 5\,\text{MeV})\)

- Percolation transition
  - \(T^{-1}\)
  - or \(\rho_B^{-1/3}\)

Chiral Symmetry Restoration \((\text{mass})\)

- Constituent quark \(m \sim M_N / 3 \sim 350\,\text{MeV}\)
- Current (bare) quark \(m \sim \text{a few MeV} (< 5\,\text{MeV})\)
Historical Phase Diagram

Baym (1983)

Phase Diagram of Nuclear Matter
Confinement and Deconfinement

Physical degrees of freedom up to the scale $\sim \Lambda_{\text{QCD}}$ at zero baryon density

**Confined Phase**

Hadron: $(3 \text{ pions}) + \cdots = 3 \sim \cdots$

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**Deconfined Phase**

Gluon: $(2 \text{ polarization}) \times (8 \text{ colors}) = 16$

Quark: $\frac{7}{8} \times (2 \text{ spin}) \times 2 \left( q \text{ and } \bar{q} \right)$

$\times (2 \sim 3 \text{ flavors}) \times (3 \text{ colors}) = 21 \sim 31.5$
**Thermodynamics**

**Stefan-Boltzmann law**

\[ p(T) = \nu \frac{\pi^2}{90} T^4 \]

\[ s(T) = \frac{\partial p}{\partial T} = \nu \frac{2\pi^2}{45} T^3 \]

\[ \epsilon(T) = -p + sT = \nu \frac{\pi^2}{30} T^4 \]

*The most favorable state has the largest \( p \)*

(i.e. the smallest free energy)

\[ \nu : \text{physical degrees of freedom} \]

\[ \nu \approx 3 \quad \text{in the pion gas} \]

\[ \nu = 37 \sim 47.5 \quad \text{in the quark-gluon gas} \]
**Bag Model – Estimate of $T_c$**

**Phase transition in a bag model**

**Confined Phase**

$$ p_{\text{hadron}}(T) = \frac{3\pi^2}{90} T^4 + B $$

**Deconfined Phase**

$$ p_{\text{pert}}(T) = \frac{37\pi^2}{90} T^4 $$

(Two-flavor assumed, $B^{1/4} \sim 220\text{MeV}$)

**1st-order Phase Transition**

$$ T_c = \frac{90}{(37 - 3)\pi^2} B \sim 160 \text{ MeV} $$
Jump from the pion gas to the quark-gluon gas

No 1st-order phase transition, but only smooth crossover
[Note that $p$ is always continuous even for the 1st-order]
Hagedorn Transition

Hagedorn limiting temperature

\[ Z = N \int dm \rho(m) e^{-m/T} \quad \rho(m) \sim e^{m/T_H} \] string-type model

Integration diverges for \( T > T_H \rightarrow \) Limiting temperature
Thermal Statistical (Hadron Resonance) Model

Non-interacting stable and unstable hadrons at temperature $T$, baryon chemical potential $\mu_B$, strangeness chemical potential $\mu_s$, electric chemical potential $\mu_Q$, where $\mu_s$ and $\mu_Q$ are constrained by the collision condition.

Contained mesons (blue) and baryons (red): figure from THERMUS2.0 Manual
Consistent with Experimental Data

Particle yield ratio determined by the thermal weight

Taken by PBM's slides
Consistent with Experimental Data

Particle yield ratio determined by the thermal weight
Thermodynamics

Phase boundary inferred by the SM
Determination of $T_c$ at Zero Density

Some (many) people talk about $T_c$ but...

Below $T_c$: No quark? ← Suppressed by random gluon fields

Above $T_c$: No hadron? ← Many hadrons and resonances (HRG)

Fukushima-Hatsuda (2010)

What really changes below and above $T_c$?
Order Parameter for Deconfinement

Quark potential and confinement / deconfinement

Test Quark (Probe)

$x = 0$

$x = \infty$

$f_q(x) = \infty$ Confined

$f_q(x) < \infty$ Deconfined

$f_q : \text{Free energy gain induced by the test quark}$

$\Phi = e^{-f_q/T}$

\[
\begin{cases}
\Phi = 0 & \text{Confined} \\
\Phi > 0 & \text{Deconfined}
\end{cases}
\]

$\Phi : \text{Polyakov loop} = \text{Deconfinement order parameter}$
Inter-quark Potential

Confinement by the linear potential

\[ f_{qq}(r) \]

\[ e^{-f_{qq}(r)/T} \sim e^{-f_q(0)/T} e^{-f_q(r)/T} \quad (r \to \infty) \]

Confining (linear) potential

\[ f_{q\bar{q}}(r) \sim \sigma r + \cdots \]

\[ \sigma \approx 1 \text{ GeV/fm} \]

\[ \Phi = 0 \]

RB-Tokyo (2005)
String Breaking

Screening effect by the quark-pair creation

$m_h \sim 0.5 \sim 1.0 \text{GeV}$

$f_q(x) < 0.8 \text{GeV}$

Confined? Deconfined? Indistinguishable!

What really changes below and above $T_c$?

Nothing qualitatively

RB-Tokyo (2005)

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Remarks

Without a clear jump (e.g. the 1st-order phase transition) there is no way to define where the deconfinement takes place.

The problem is similar to the magnetization in a finite external magnetic field.
→ Order parameter is never vanishing!

The change of the physical degrees of freedom (hadrons → quarks) should be smooth. There is no sudden change to quarks and gluons.
Pseudo-critical Temperature

Smooth crossover: approximate order parameter

It is pointless to try to locate the deconfinement temperature

Very smooth phenomenon!

Pseudo-critical Temperature
Pragmatic (working) definition

Inflection point in the order-parameters
Polyakov loop, Strange-quark susceptibility, etc...
Caution

Do not believe any phase diagram like:

\[
T_c = 176(3)(4) \text{ MeV} \quad T_c = 175(2)(4) \text{ MeV}
\]

This part (crossover) is strongly depending on the prescription. Error estimate does not include such prescription dependence.

The inflection point is well-defined, but \( T_c \) is not well-defined!
Chiral Symmetry

\[
U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V
\]

Massless Theory  Massive Theory

or, equivalently

\[
SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A \rightarrow SU(N_f)_V \times U(1)_V
\]

Baryon number conservation  Isospin symmetry \((N_f=2)\)

\[U(N_f)_L, U(N_f)_R, U(N_f)_V\] rotations in \(N_f\) flavor space

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Left-handed and Right-handed

Decomposing the Dirac Lagrangian

\[
\bar{\psi} \left( i \gamma^\mu D_\mu - m \right) \psi = \phi_L^\dagger i \bar{\sigma}^\mu D_\mu \phi_L + \phi_R^\dagger i \sigma^\mu D_\mu \phi_R
\]

\[-m \left( \phi_L^\dagger \phi_R + \phi_R^\dagger \phi_L \right)\]

\[
\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \sigma^\mu = (1, \sigma), \quad \bar{\sigma}^\mu = (1, -\sigma)
\]

\[
\psi = \begin{pmatrix} \phi_L \\ \phi_R \end{pmatrix}
\]

Solving the free massless Dirac equation (for L)

\[
i \bar{\sigma}^\mu \partial_\mu \phi_L = 0 \quad \rightarrow \quad \left( p_0 + \sigma \cdot p \right) \phi_L(p) = 0
\]

\[
\lambda \phi_L(p) = -\frac{1}{2} \phi_L(p) \quad \text{helicity} \quad \lambda = \frac{1}{2} \sigma \cdot \frac{p}{|p|}
\]
Chirality and Helicity

Right-handed Particle

Spin

Momentum

Left-handed Particle

Right-handed Anti-Particle

Left-handed Anti-Particle
Quantum Anomaly

Parallel electric and magnetic fields

Electric field $\rightarrow \pm eEt$ on the Dirac surface

\[
p^{(R)}_F = +eEt \quad p^{(L)}_F = -eEt
\]

\[
\frac{dN_5}{dt \cdot d^3x} = \frac{d(N_R - N_L)}{d^4x} = \frac{e^2}{2\pi^2} E \cdot B
\]

\[
\partial_\mu j^{\mu}_5 = -\frac{e^2}{8\pi^2} F \tilde{F} \quad \text{(QED)}
\]

$U(1)_A$ symmetry is explicitly broken at the quantum level
**Nambu-Goldstone Bosons**

**Chiral symmetry breaking**

\[ \text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V \]

In the \( N_f = 2 \) case:

\[ \text{SU}(2) \times \text{SU}(2) \cong \text{O}(4) \rightarrow \text{SU}(2) \cong \text{O}(3) \]

- Condensate develops in the \( \sigma \) direction
- There are three flat directions \( \rightarrow \pi^+ \pi^- \pi^0 \)

**Flat directions** = Massless excitations

= Nambu-Goldstone (NG) boson
Chiral condensate

\[ \langle \bar{\psi} \psi \rangle = \langle \bar{u}_L u_R \rangle + \langle \bar{u}_R u_L \rangle + \langle \bar{d}_L d_R \rangle + \langle \bar{d}_R d_L \rangle + \ldots \]

- Condensate of the scalar (\( \sigma \)) meson ~ Higgs condensate
- Conjugate to the mass ~ Source of the dynamical mass

**Pion (NG boson) = Fluctuation**

Chiral rotation: \( \psi \rightarrow e^{i \gamma_5 \tau^a \theta^a / 2} \psi \)

\[ \bar{\psi} \psi \rightarrow \bar{\psi} \psi + \theta^a \bar{\psi} i \gamma_5 \tau^a \psi \]
Critical Phenomenon

Second-order phase transition at finite $T$

Critical exponents

$$\langle \bar{\psi} \psi \rangle \sim |1 - T/T_\chi|^\beta$$
$$\chi = G(0) \sim |1 - T/T_\chi|^{-\gamma}$$
$$G(k) \sim k^{-2+\eta}$$
$$\langle \bar{\psi} \psi \rangle \sim m^{1/\delta}$$
$$C = |1 - T/T_\chi|^{-\alpha}$$
$$\xi \sim |1 - T/T_\chi|^{-\nu}$$
Universality

Phase transition and the fixed-point structures

Critical exponents are determined by the fixed-point structure which is determined by the global symmetry of the system.
Universality Class of QCD

Two-flavor with $U(1)_A$ anomaly

$SU(2) \times SU(2) \simeq O(4)$

2nd-order phase transition with
$O(4)$ universality class

Two-flavor without $U(1)_A$ anomaly

$SU(2) \times SU(2) \times U(1)_A \simeq O(4) \times O(2)$

Fluctuation-induced
1st-order transition

Three-flavor with/without $U(1)_A$ anomaly

$\det \bar{\psi} \left(1 \pm \gamma_5\right) \psi \sim \langle \bar{q} q \rangle^3$

1st-order phase transition (tree-level)
Columbia Plot

\[ O(4) \]

\[ 2nd \]

\[ \infty \]

\[ m_s \]

\[ m_{ud} \]

Physical Point?

Tricritical Point

Chiral Critical Line

1st Critical Line

Crossover

Deconfinement Critical Line

1st
Chiral Phase Transition on the Lattice

Smooth crossover due to (small) quark masses

Is this smooth like deconfinement or close to a 2nd-order transition?

Consistence check with the universality – OK

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QCD Critical Point

Critical surface with increasing density

When the physical point hits the critical surface → Critical point
Lattice Results Reliable???

Endrodi-Fodor-Katz-Szado (2007)

Even at zero density the critical curve is not yet located!!!

How can we believe any lattice results at finite density???
Part 2: Phenomenology (+Theory)
Freeze-out points are located by the particle yields
Two regimes of **meson-dominance** and **baryon-dominance**

**Mesonic Hagedorn Transition**

\[ Z \sim \int dm \rho(m) e^{-\frac{m}{T}} \]

\[ \rho(m) \sim e^{m/T_H} \]

\[ T_c = T_H \]

**Baryonic Hagedorn Transition**

\[ Z \sim \int dm \rho_B(m) e^{-\left(\frac{m_B - \mu_B}{T}\right)/T} \]

\[ \rho(m) \sim e^{m_B/T_B} \]

\[ T_c = \left(1 - \frac{\mu_B}{m_B}\right)T_B \]

Andronic-Blaschke-Braun-Munzinger-Cleymans-KF
-McLerran-Oeschler-Pisarski-Redlich-Sasaki (2010)
Translation in terms of $T$ and $\mu_B$

$$Q(T, \mu_B) \rightarrow Q(\sqrt{s_{NN}})$$

Signature for the QCD CP

Particle distribution

Fluctuations with respect to the conserved quantity (baryon number) should be enhanced near the CP

Some of early-day model results
Fukushima (2008)
Remarks

- Chiral phase transition is consistent with the O(4) universality... Why interested only in the CP?
  No anomaly in experimental data as compared to the hadron resonance gas model, so far.
  
  (Braun-Munzinger, Karsch, Redlich, etc)

- Proton number is not a conserved quantity, but the net baryon number (proton+neutron) is conserved.
  
  (Asakawa, Kitazawa, etc)

- Relation between the chemical freeze-out and the QCD phase transition? SM knows nothing about the chiral-symmetry related properties.
  
  (Braun-Munzinger, Stachel, Wetterich, etc)
Mechanism of the 1st-order Transition

Free energy vs the dynamical quark mass ($T=0$)

$$\Omega \left[ M \right] / V = - \int_0^\mu d\mu' \rho(\mu')$$

Matter part favors $M=0$  
($\rho$ is then the largest)

Vacuum part favors $M = M_0$

Double-well shape  
1st-order phase transition

Simple and robust mechanism
Vector Interaction

Vector interaction in the mean-field approx.

\[ L_v = -g_v \left( \bar{\psi} \gamma_\mu \psi \right) \left( \bar{\psi} \gamma^\mu \psi \right) \rightarrow \Delta \Omega = g_v \rho^2 \]

Pushed up at \( M = 0 \)

With some \( g_v > 0 \)

the double-well shape gone

No 1st-order transition and no critical point
Self-bound fermionic system $\rightarrow$ 1st-order

Schematic picture of the (symmetric) nuclear saturation curve

Weizsäcker-Bethe mass formula

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A} + \delta_A$$

$$\rho_0 = 0.17 \text{ fm}^{-3}$$

$$a_v = 16.3 \text{ MeV}$$
Simple Mean-field Model

σ–ω model (Walecka model)

\[ L = \bar{\psi} \left[ i \gamma_\mu \partial^\mu + \left( \mu_B - g_\omega \omega^0 \right) \gamma_0 - \left( M_N - g_\sigma \sigma \right) \right] \psi - \frac{1}{2} m_\sigma \sigma^2 + \frac{1}{2} m_\omega (\omega^0)^2 \]

\[ \Omega/V = -4 \int \frac{d^3 p}{(2\pi)^3} \left[ T \ln \left[ 1 + e^{-(\omega - \mu^*_B)/T} \right] + T \ln \left[ 1 + e^{-(\omega + \mu^*_B)/T} \right] \right] \]

\[ \frac{\partial \Omega}{\partial M_N^*} = \frac{\partial \Omega}{\partial \mu_B^*} = 0 \]

\[ \frac{\partial \Omega}{\partial M_N^*} = \frac{m_\sigma^2 (M_N^* - M_N)^2}{2 g_\sigma^2} - \frac{m_\omega^2 (\mu_B^* - \mu_B)^2}{2 g_\omega^2} \]

\[ \frac{\varepsilon}{\rho} \bigg|_{\rho=\rho_0} - M_N = -16.3 \text{ MeV} \]

\[ \frac{d(\varepsilon/\rho)}{d\rho} \bigg|_{\rho=\rho_0} = 0 \]

\[ M_N = 939 \text{ MeV}, \ m_\sigma = 550 \text{ MeV}, \ m_\omega = 783 \text{ MeV}, \ g_s = 10.3, \ g_\omega = 12.7 \]

This naturally incorporates the vector interaction. Compressibility needs the potential terms, etc...

Buballa (1996)
Mean-field Solution

Mean-field variables

At the saturation point
\[ \frac{d \left( \frac{\varepsilon}{\rho} \right)}{d \rho} = \frac{\mu_B}{\rho} - \frac{\varepsilon}{\rho^2} = \frac{p}{\rho^2} = 0 \]

1st-order phase transition
Liquid-gas transition
Chiral phase transition
Critical Point of Nuclear Matter

Intuitive picture

How to realize the density $\rho < \rho_0$ when $\rho = \rho_0$ is the most stable?

This should be realized as a mixed phase (or a non-trivial configuration depending on the surface energy).

Pion (thermal) loops are not important yet at such low temperature.
Is it a self-bound system? → Quark droplet?

Even when the quark droplet is only meta-stable

\[ \frac{d(\varepsilon/\rho)}{d\rho} = \frac{\mu_B}{\rho} - \frac{\varepsilon}{\rho^2} = \frac{p}{\rho^2} = 0 \]

1st-order phase transition → QCD critical point

Any more stable state would exhibit the 1st-order one too.
Vector Interaction Again

\[ L_v = -g_v (\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma^\mu \psi) \rightarrow \Delta \Omega = g_v \rho^2 \]

It is obvious at a glance that the vector interaction would wash the 1st-order transition out.

Is there any chance to find another branch of solution?
Alternative Scenario

CP is unlikely, so something else strongly needed as an alternative scenario
What is there?

Transitional change from the meson-dominant regime to the baryon-dominant regime

Oeschler (2006)
New Regime at Large $\mu_q$ and $N_c$

Phase diagram of large-$N_c$ QCD

Deconfined Phase

Gluons $P \sim O(N_c^2)$

Deconfinement Phase Transition of 1st Order

$\sim \Lambda_{QCD}$

Hadronic Phase

Quarkyonic Phase

$P \sim O(1)$

Baryonic Interaction $P \sim O(N_c)$

$N_c$-quark exchange

McLerran-Pisarski (2007)

$\sim M_B$

$\mu_B$

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Large $N_c$ Limit

Instead of taking account of the screening effects, one can take a limit in which the screening is suppressed.

Forward scattering with soft-gluon exchange should be treated non-perturbatively.

Confinement remains at high density

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Quarkyonic Matter

At large $N_c$ quarks interact strongly through soft-gluon exchange → Quarks are confined (Quarkyonic Matter)

Chiral condensate = $\sigma$ meson condensate

- For confined $\sigma$ meson at rest, gluons must carry a momentum of order of $\mu_q$
- Gluons can be soft if quarks and holes move and $\sigma$ meson has a momentum of order of $2\mu_q$

→ Inhomogeneous condensate favored

Hidaka, Kojo, McLerran, Pisarski
Interpretation of Quarkyonic Matter

Structure of the Fermi sphere

Ground state of large-$N_c$ quark matter at $\mu_q >> \Lambda_{QCD}$

Quarks

$P \sim O(N_c)$

Baryons

~ $\sim \Lambda_{QCD}$

Interacting Baryon Crystal

~ Quasi-quark Gas

McLerran, Pisarski, Hidaka, Kojo
**Inhomogeneity: Simplest Case**

**Chiral spiral in one direction**

\[ \psi(x) = e^{i\gamma_5 \tau_3 q z} \psi'(x) \text{ with } \chi = \langle \bar{\psi}' \psi' \rangle \]

\[ \langle \bar{\psi} \psi \rangle = \chi \cos(2qz) \]

\[ \langle \bar{\psi} \gamma_5 \tau_3 \psi \rangle = \chi \sin(2qz) \]

**Quasi-particle dispersion relation**

\[ \omega = \sqrt{p_{\perp}^2 + \left( \sqrt{p_z^2 + M^2} \pm q \right)^2} \]

The system can develop a density however large \( M \) is if \( q \sim M \) is chosen!
c.f. (1+1)-dimensional System

Dirac Lagrangian in (1+1) dimensions

\[ L = \bar{\psi} \left[ i (\partial_4 - \mu) \gamma_4 + i \partial_z \gamma_z \right] \psi \]
\[ = \bar{\psi}' \left[ i \partial_4 \gamma_4 + i \partial_z \gamma_z \right] \psi' \]

Thermodynamic potential

\[ \Omega/V = -\int_{-\Lambda+\mu}^{\Lambda-\mu} \frac{dp}{2\pi} \frac{|\varepsilon(p)|}{2} - \int_{-\Lambda-\mu}^{\Lambda+\mu} \frac{dp}{2\pi} \frac{|\varepsilon(p)|}{2} + \ldots \]
\[ = \Omega(\mu=0)/V - \frac{\mu^2}{2\pi} \]

Surface integral: Anomaly origin

\[ \text{No suppression by } M \]

Push down the energy as compared to the homogeneous case:

\[ \Omega(\mu=0)/V + \left[ -\frac{p_F\mu}{2\pi} + \frac{M^2}{2\pi} \ln \left| \frac{p_F+\mu}{M} \right| \right] \theta(\mu-M) \]

\[ n = \frac{p_F}{\pi} \theta(\mu-M) \]
Competing Terms

- From the matter (density effect)
  \[ \omega = \sqrt{p_\perp^2 + (\sqrt{p_z^2 + M^2} \pm q)^2} \]
  \[ - \int_0^\mu d\mu' \rho(\mu') - 4 N_c N_f T \int \frac{d^3p}{(2\pi)^3} \ln \left( 1 + e^{-\omega/T} \right) \Rightarrow q \sim M \to \infty \]

- From the vacuum (chiral symmetry breaking)
  \[ a(M_0^2 - M^2) - bM \quad (b \sim \text{bare quark mass}) \Rightarrow M \sim M_0 \]

- From the vacuum (kinetic term)
  \[ (\alpha M^2 + \beta b)q^2 \Rightarrow q \sim 0 \]

- From the interaction (vector-type)
  \[ g_v \rho^2 \Rightarrow M \neq 0 \]
It is natural (but not necessary) that the 1st-order transition with a smaller energy occurs at smaller density. Less affected by the vector interaction then.

\[ a = 0.05 \ (\sim \text{LSM}) \]
\[ M_0 = 340 \text{ MeV} \]
\[ \alpha = 0.25 \]
\[ \beta = 0.25 / M_0 \]

~ Conventional QCD phase diagram

Fukushima (2012)
Phase Diagram with Inhomogeneity

Inhomogeneity survives even with $g_\nu$ that washes the CP out.

Note that the order of the transition and $P$ are not a robust conclusion...

$q \sim M$
**Patch Problem and Successive Phase Transitions**

Kojo-Hidaka-Fukushima-McLerran-Pisarski (2011)

1-D modulation = 1 patch

How to cover the Fermi surface by patches?

Quasi-crystal?

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**cf. p-wave Pion Condensation**

\[
\Pi(\omega, k) \rightarrow D^{-1}(\omega = 0, k = k_c) = 0 \text{ at } \rho = \rho_c
\]

Landau-Migdal (short-range) interaction

\[
f + g \sigma_1 \cdot \sigma_2 + f' \tau_1 \cdot \tau_2 + g' \sigma_1 \cdot \sigma_2 (\tau_1 \cdot \tau_2)
\]

OPEP

\[
V = \frac{m_\pi^2}{3} g^2 \sigma_1 \cdot \sigma_2 \frac{e^{-m_\pi r}}{r} + S_{12} \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) \frac{e^{-m_\pi r}}{r} - \frac{g^2}{3} (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) \delta(r)
\]

Large \(g'\) kills the pion cond.

\[\rightarrow\] Gamow-Teller resonance

Majority thinks negative, but some people still believe.

Spirals in quark matter are similar to the pion condensation
Back to the Old Phase Diagram

Baym (1983)

So, after all, Baym's diagram is correct?
Remarks

- No reliable theory at high density – some lattice-QCD results are reported but not yet under good theoretical control.

- No evidence for CP nor inhomogeneity
  - *Most of realistic models predict no CP*
  - *Any models favor inhomogeneity generally*

- Experimental signature for further complicated structures of the baryon-rich states?
  - *Critical point(s), mixed phase, inhomogeneity*
  - *Fluctuations, and anything else?*
Another Axes

Finite isospin density
No sign problem in the lattice-QCD simulation.
No nuclear matter but only pion condensation
How to achieve large isospin density?  Unrealistic

Strong Magnetic Field
No sign problem and a close interplay with $S\chi B$
Theoretically speaking, similar to $\mu_B$
Generated shortly after the collision. Realistic?
Moving almost at the speed of light

Quark-Gluon Plasma

Impact parameter $\sim b$
(event-by-event measurable quantity)
Magnetic Field

Strong B generated due to Electrodynamics

on top of the Quark-Gluon Plasma
Order Estimate of $B$

Lienard-Wiechert potential

$$eB(t) = eB_0 \left[ 1 + \left(\frac{t}{t_0}\right)^2 \right]^{3/2}$$

$$eB_0 = \left(47.6 \text{ MeV}\right)^2 \left(\frac{1 \text{ fm}}{b}\right)^2 Z \sinh(Y), \quad t_0 = \frac{b}{2 \sinh(Y)}$$

Strongest $B$ in the Universe (QCD scale!)
Chiral Magnetic Effect

Classical Picture

Right-handed Quarks
= momentum parallel to spin

Left-handed Quarks
= momentum anti-parallel to spin

\[ J \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0 \]

Kharzeev-McLerran-Warringa (2007)
Fukushima-Kharzeev-Warringa (2008)
Current from Quantum Theory

Anomaly Relation

\[ j = N_c \sum_{i=\text{flavor}} q_i \frac{\mu_5}{2\pi} B \]

Chiral Magnetic Effect = QCD anomaly \times QED anomaly

Zhitnitsky, Fukushima-Kharzeev-Warringa, Son-Stephanov, Gao-Liang-Pu-Wang-Wang, etc...
Results from STAR@RHIC

Charge-asymmetry fluctuation

\[ \langle \cos(\Delta \phi_\alpha + \Delta \phi_\beta) \rangle \equiv \frac{1}{N_\alpha N_\beta} \sum_{i=1}^{N_\alpha} \sum_{j=1}^{N_\beta} \cos(\Delta \phi_{\alpha,i} + \Delta \phi_{\beta,j}) \]

\[ = \langle \cos \Delta \phi_\alpha \cos \Delta \phi_\beta \rangle - \langle \sin \Delta \phi_\alpha \sin \Delta \phi_\beta \rangle \]

\[ = \left( \langle v_{1,\alpha} v_{1,\beta} \rangle + B_{\alpha\beta}^{\text{in}} \right) - \left( \langle a_\alpha a_\beta \rangle + B_{\alpha\beta}^{\text{out}} \right) . \]

No longer considered as any evidence for the chiral magnetic effect.

STAR results

STAR (2009)
New Possibility – Chiral Magnetic Wave

\[ j^z = \frac{q^2 j^5}{2\pi^2} B \quad j^z = \frac{q^2 j^0}{2\pi^2} B \]

\[ \partial_0 j^0 - \partial_z j^z = 0 \quad \partial_0 j^5 - \partial_z j^z = 0 \]

\[ \text{Wave eq.} \quad \partial_0^2 j^0 - \frac{q^2 B}{2\pi^2} \partial_z^2 j^0 = 0 \]

Kharzeev-Yee (2010)
Observable Effect

Quadrupole moment $q_e$

$B$

$\mu_B$

$j^z_5$

Sep. 3, 4, 2012 @ CNS
Observable Effect

Quadrupole moment $q_e$

$B$

Sep. 3, 4, 2012 @ CNS
Elliptic Flow Difference

Charge asymmetry

\[ A_\pm = \frac{\bar{N}_+ - \bar{N}_-}{\bar{N}_+ + \bar{N}_-} \]

\[ v_2^\pm = v_2 + \frac{q_e}{\rho_e} A_\pm \]

Burnier-Kharzeev-Liao-Yee (2011)

Looks promising, so far so good...
Summary

Quark-Gluon Plasma

Fluctuations

Hadronic Phase

Quarkyonic Matter

Color Superconductors

CME/CMW

Temperature $T$

Baryon Chemical Potential $\mu_B$