# The properties of nuclear matter under the Bethe-Brueckner-Goldstone (BBG) Expansion

Verification of the convergence of the BBG expansion

Jia-Jing Lu 陆家靖 [1]

Advisor: Zeng-Hua Li 李增花[1]

Collaborators: H.-J Schulze[2], M. Baldo[2]

[1]Institute of Modern Physics, Fudan University
[2]INFN Sezione di Catania, Dipartimento di Fisica, Università di Catania

Aug 23, 2018

CNS, University of Tokyo

# Overview

# Introduction

## Properties of Nuclear Matter

- Saturation Density:
  $\rho = (0.17 \pm 0.01)\mathrm{fm}^{-3}$
- Binding Energy:
  $\frac{E_0}{A} = (16 \pm 0.5)\mathrm{MeV}$

The basic difficulty: traditional nucleon-nucleon forces feature
**a very strong repulsion at short distances.**

We mainly employ
**Bethe-Brueckner-Goldstone theory.**

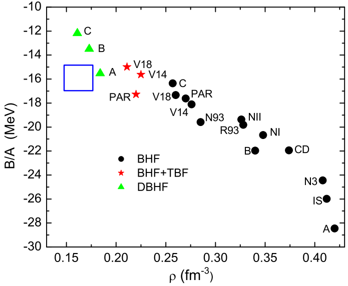



Fig 1: Saturation points obtained with different potentials and theoretical approaches.[1] The blue square indicates the empirical region.

[1] Z. H. Li et al. DOI: 10.1103/physrevc.74.047304

# Brueckner-Hartree-Fock Method

$$G(\rho,\beta;\omega) = v_{NN} + v_{NN} \sum_{k_1,k_2} \frac{|k_1 k_2\rangle\, Q(1,2)\, \langle k_1 k_2|}{\omega - \varepsilon(k_1) - \varepsilon(k_2)} G(\rho,\beta;\omega) \qquad (1)$$

- Nucleon-Nucleon interaction

$$V_{NN} = v_2 + V_3^{\text{eff}}$$

two-body interactions $v_2$: AV18, CDBONN, $N^3LO$, $N^4LO$

- Single Particle Energy:

$$\varepsilon(k) = \frac{\hbar^2 k^2}{2m} + U(k) \qquad (2)$$

- "Auxiliary" potential, cont. choice: for all $k$

$$U(k) = \sum_{k'<k_F} Re \langle kk' | G | kk' \rangle \qquad (3)$$

- Gap (or standard) choice:

$$U(k > k_F) \equiv 0 \qquad (4)$$

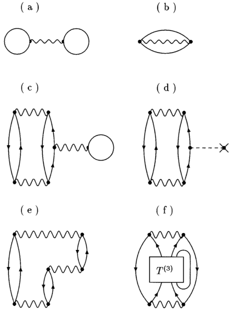



Fig 2: Different diagrams which contribute to the equation of state[1]. $B/A = T + E_2 + E_3 + \ldots$

[1] Song et al. DOI: 10.1103/physrevlett.81.1584

# Correlation Strength

## Definition

$$\kappa \equiv \sum_k n(k > k_F) / \sum_{k<k_F}$$

Coester et al. pointed out a close connection between the correlation parameter and the saturation properties of nuclear matter[1].

$$\kappa = \rho \int d^3r \langle |\eta(r)|^2 \rangle_{S,T} = N \frac{V_{core}}{V} = (\frac{c}{d})^3 \quad (5)$$

For a system with interaction range $c$ and average particle distance $d > c$, the probability of finding a cluster $n$ interacting particles is

$$(\frac{c}{d})^{3n} = \kappa^n \ll 1 \quad (6)$$

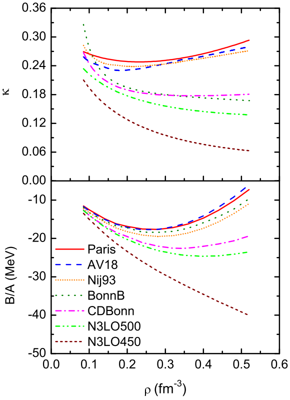



Fig 3: Top Panel: Average depletion of the momentum distribution. Bottom Panel: Saturation curves of SNM in the BHF approximation[2].

[1] Coester et al. DOI: 10.1103/PhysRevC.1.769

[2] Z.-H. Li and Schulze DOI: 10.1103/physrevc.94.024322

# Editors' Suggestion on Phys. Rev. C

PHYSICAL REVIEW C **96**, 044309 (2017)

## Convergence of the hole-line expansion with modern nucleon-nucleon potentials

Jia-Jing Lu (陆家靖), Zeng-Hua Li (李增花),* and Chong-Yang Chen (陈重阳)
*Department of Nuclear Science and Technology, Institute of Modern Physics, Fudan University, Shanghai 200433, People's Republic of China*

M. Baldo and H.-J. Schulze
*Dipartimento di Fisica, INFN Sezione di Catania, Universitá di Catania, Via Santa Sofia 64, 95123 Catania, Italy*

(Received 16 June 2017; published 6 October 2017)

We calculate the three-hole-line contributions to the binding energy of symmetric nuclear matter in the Brueckner-Bethe-Goldstone expansion using various modern nucleon-nucleon potentials of high precision. The relation with the correlation parameter $\kappa = \rho V_{\text{core}}$ is examined. In all cases the three-hole-line contributions turn out to be sufficiently small, but no satisfactory saturation is obtained. This means that three-nucleon forces are essential for all considered potentials.

DOI: 10.1103/PhysRevC.96.044309

### Motivations

1. In the absence of a hard core, the very justification of the HLE becomes doubtful.

   Explicitly compute the 3HL contributions.
2. Check the convergences of HLE with various modern NN potentials and different choices of aux. potentials.

# Binding Energy

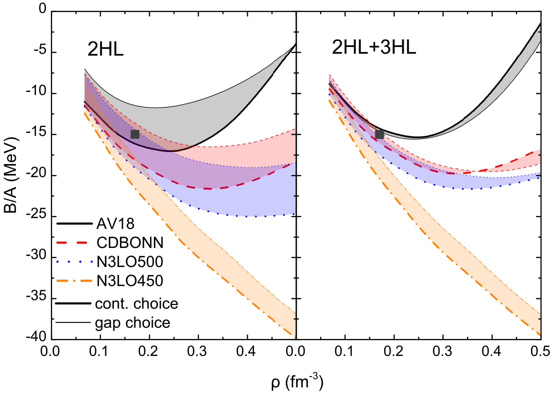
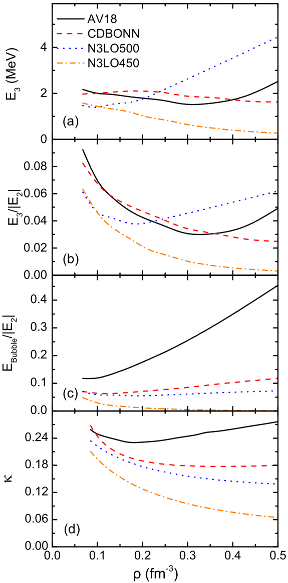



Fig 4: Saturation curves of symmetric nuclear matter for different NN potentials in the 2HL (left panel) and 2HL + 3HL (right panel) approximations with continuous (bold curves) or gap-choice (thin curves) s.p. potentials. The markers indicate the empirical saturation point.

1. Panel(a): From the density dependence of $E_3$ it becomes clear why no improvements: (Except $\mathrm{N^3LO500}$) all $E_3$ are decreasing function of density at $\rho_0$.
2. Panel(b) shows that for all potentials, the ratio $E_3/|E_2|$ remains well below $10\%$ at any density, show again good convergence.
3. Furthermore, no clear correspondence between the magnitudes of $\kappa$ and $E_3/|E_2|$ for in fact the total $E_3$ is the sum of the Bubble, Ring and Higher contributions.
4. Panel(c): However, the contributions of $E_{Bubble}/|E_2|$ are completely consistent with $\kappa$ and orders according to the hard-core interaction strength.



# Defect Functions for PNM

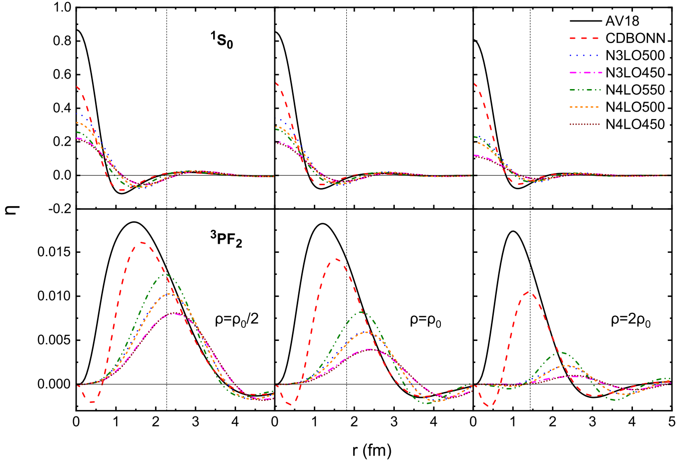



Fig 5: $^1S_0$ and $^3PF_2$ defect functions in PNM at different densities ($\rho_0 = 0.17 \text{ fm}^{-3}$) for all potentials. The vertical dashed lines indicate the interparticle distance $d \equiv \rho^{-1/3}$ .

# Defect Functions for SNM[1]

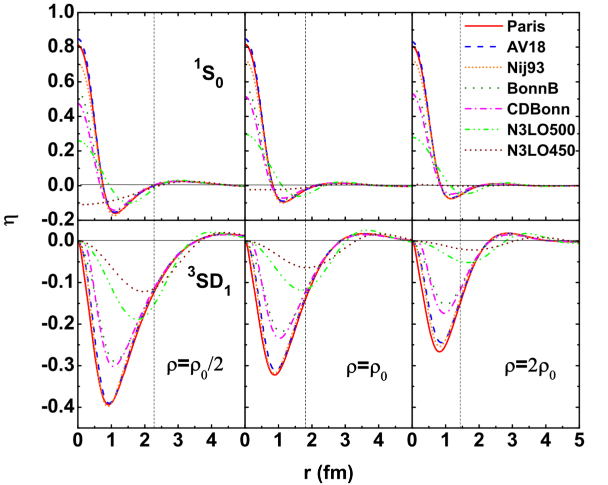



Fig 6: r-space $^1S_0$ and $^3SD_1$ defect functions at different densities ($\rho_0 = 0.17\ \mathrm{fm}^{-3}$) for different potentials.

[1] Z.-H. Li and Schulze DOI: 10.1103/physrevc.94.024322

# Binding Energy of PNM

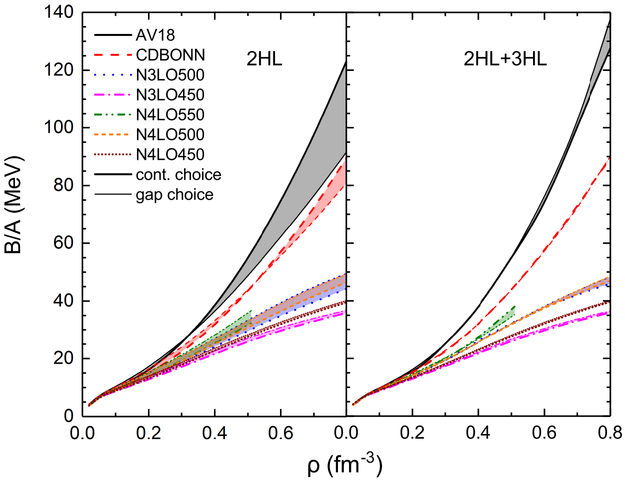



Fig 7: Energy per particle of PNM for different NN potentials in 2HL (left panel) and 2HL+3HL (right panel) approximation with continuous (thick curves) or gap-choice (thin curves) s.p. potentials. The $\mathrm{N^4LO550}$ calculations do not converge well for $\rho \gtrsim 0.5\ \mathrm{fm}^{-3}$.

# Conclusions

## HLE are well converged

1. With all frequently used potentials, the 3HL contributions are sufficiently small.
2. Results with gap- and continuous- choice show better agreement after including 3HL results.

For SNM, the empirical saturation properties of nuclear matter are **not reproduced for any potentials** which means that **very strong nuclear three-body forces are required** in order to achieve satisfactory saturation properties of nuclear matter.

The relation with the correlation parameter $\kappa = \rho V_{core}$ is examined.

We analyze the potentials in terms of the strength of their hard core.

# Thanks for your attention!

ご静聴ありがとうございます