Comparison study of transport models; DJBUU and SQMD

based on D.I. Kim et al. *J.Korean Phys.Soc.* 81 (2022) 12, 1204-1210

Dae Ik Kim[1]

in collaboration with C.-H. Lee[1], S. Jeon[2], Y. Kim[3] and K. Kim[4]

[1]Department of Physics, Pusan Nation University, South Korea
[2]Department of Physics, McGill University, Canada
[3]Center of Exotic Nuclear Study, Institute of Basic Science, South Korea
[4]Rare Isotope Science Project, Institute of Basic Science, South Korea

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Astro-Hadron Physics Group

Astrophysics and Nuclear Physics

Astrophysics

#Gravitational Wave

Laser Interferometer GW Observatory;LIGO

Nuclear Physics

#Rare Isotope Beam

RI Accelerator complex for ON-line experiment;RAON

Structure of Neutron Star

Group leader : Chang-Hwan Lee

Astro-Hadron Physics Group

Astrophysics and Nuclear Physics

Astrophysics

#Gravitational Wave

- Tidal deformability measurement error
- Neutron Star Equation of State
- R-process for Heavy elements

Nuclear Physics

#Rare Isotope Beam

- DFT model for Nuclear structure
- Transport model to study EoS
- Langevin model for Nuclear Reaction

Collaborate with S.Jeon at McGill Univ.
Y. Kim, K.Kim at IBS

Hee-Suk Cho
Research professor

Yong-Beom Choi
Post Doc

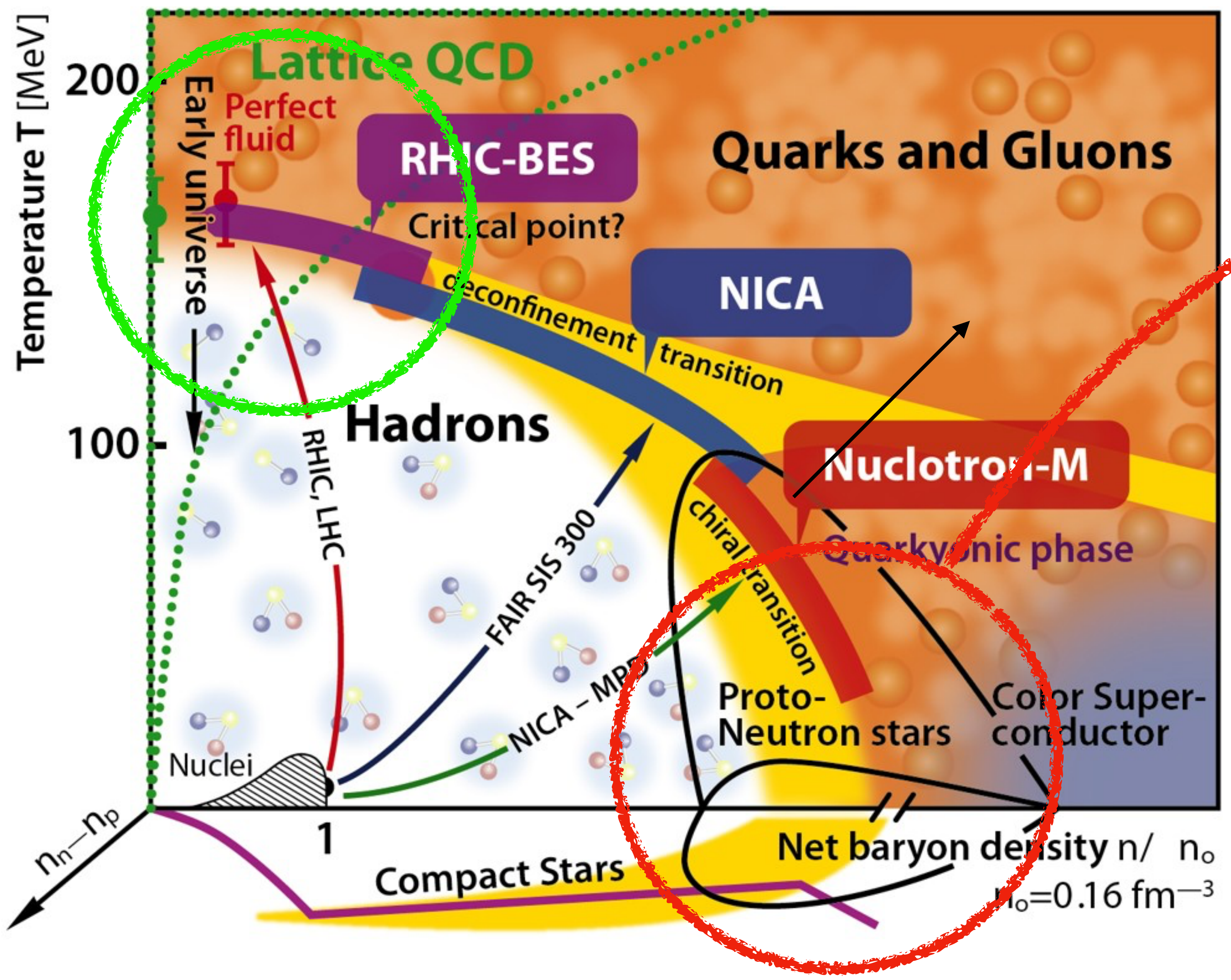
Chang-hoon Song
Ph.D course

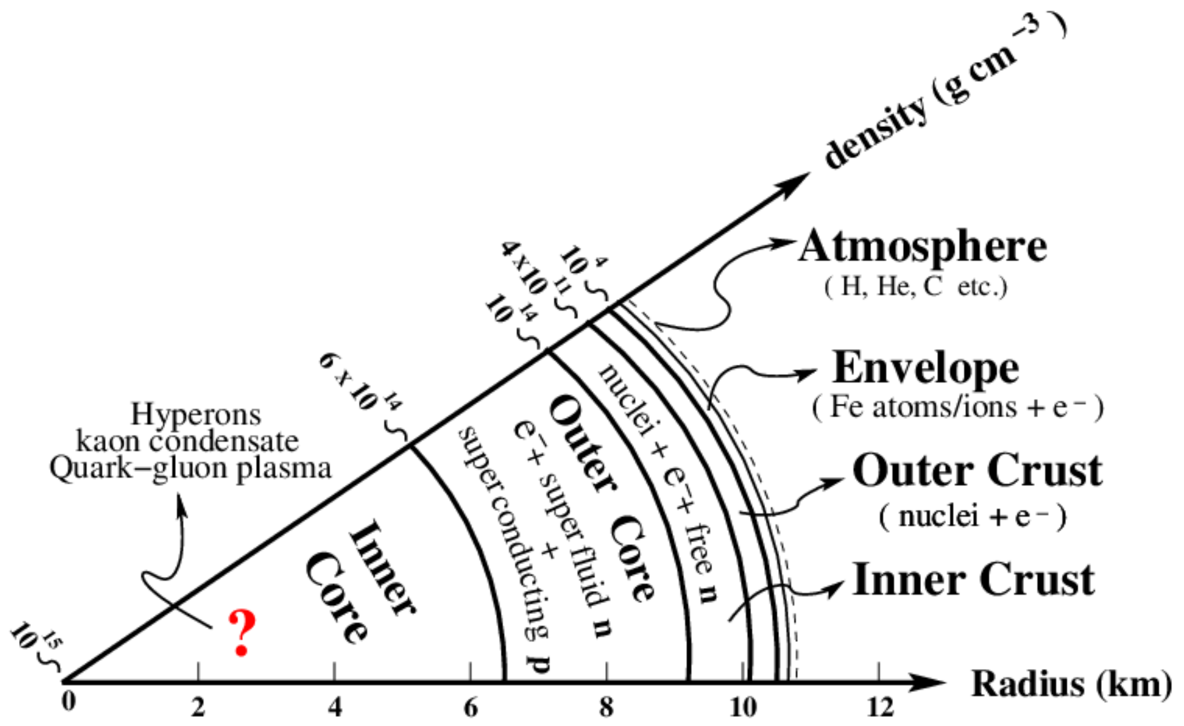
Dae Ik Kim
Master course

Group leader : Chang-Hwan Lee

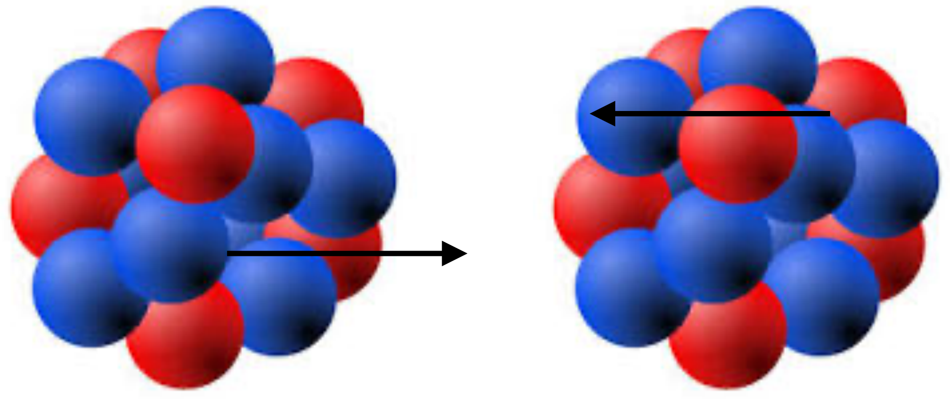
Motivation : Heavy ion collisions

Journeys to understand dense matter





Heavy Ion collision at Intermediate energy

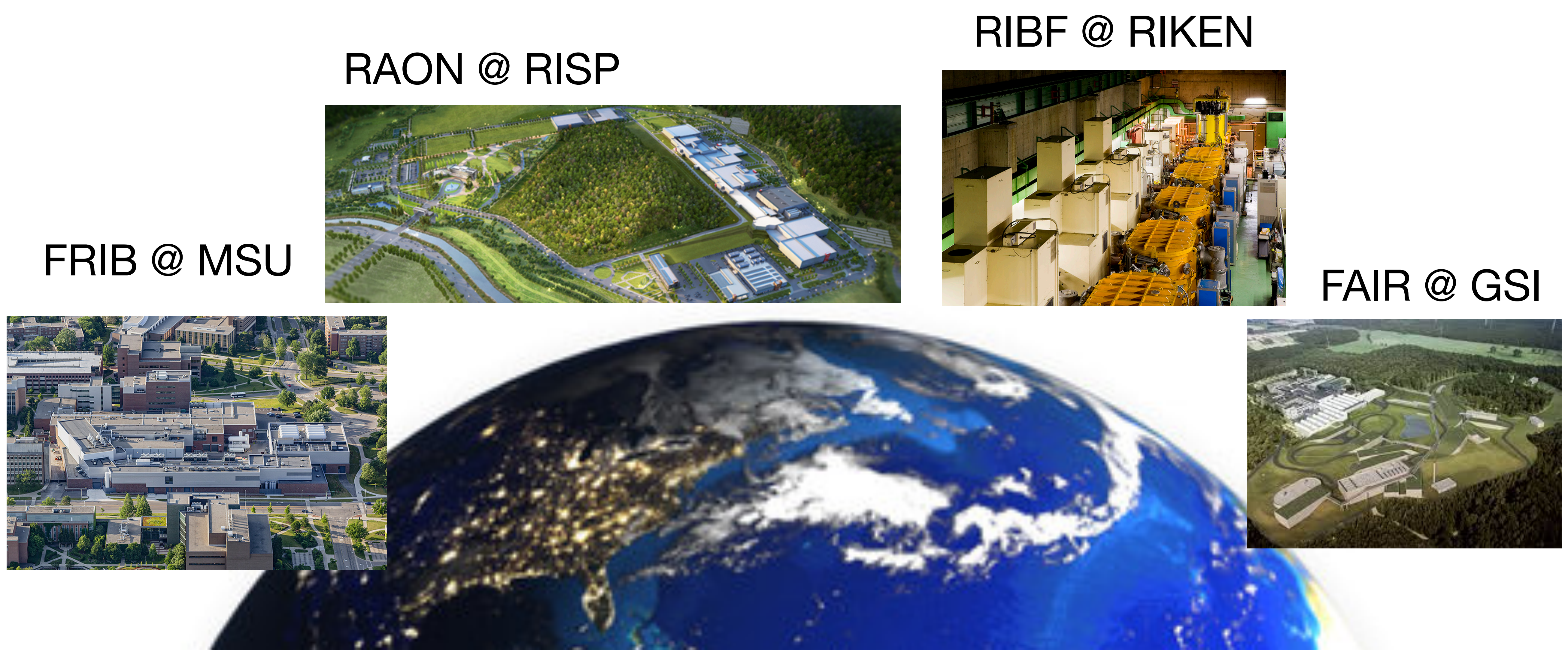


- Nucleon(proton, neutron)
- Nuclear matter, NS (EOS)

* One example; FOPI

$$E_{\mathrm{beam}} = 100\,A\,\mathrm{MeV}$$
$$\gamma = 1.1$$
$$v = 0.46c \quad (v_{cm} = 0.24c)$$
$$\lambda = \frac{h}{p} = 2.5\,\mathrm{fm}$$

Rare Isotope beam facilities



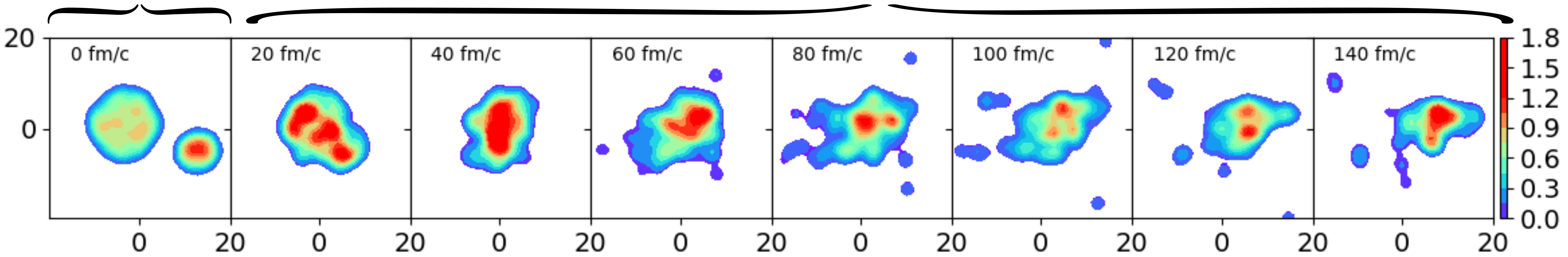
Transport model

Numerical models to describe HICs at Fermi/intermediate energy region

Initialization
Generate nuclei

Propagation
Motion of nucleon

Collisions
NN collision

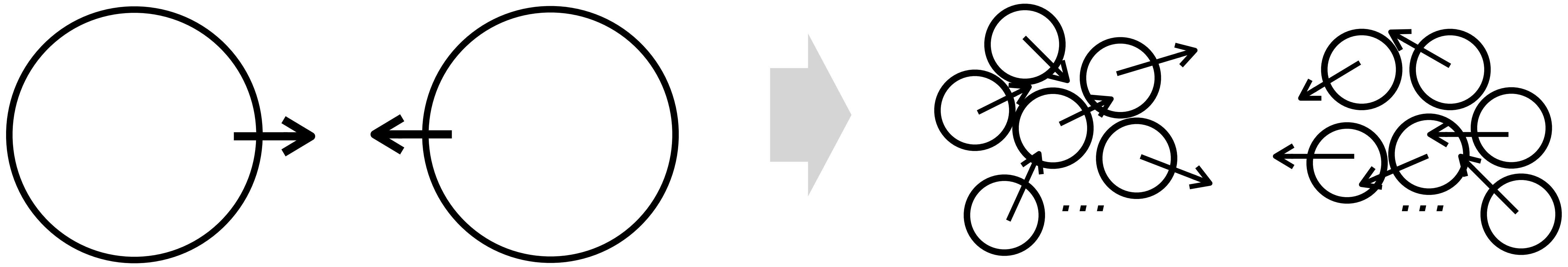


1 event Contour of Pb + Ca collision using SQMD model, Beam energy = 100 MeV /u, Impact parameter = 6 fm

- Microscopic (Hadron degree of freedom)
- Semi-classical method
- Time steps from initial to final

Two nuclei collision

NN collisions



Particle wave function → Wigner function (phase space density)

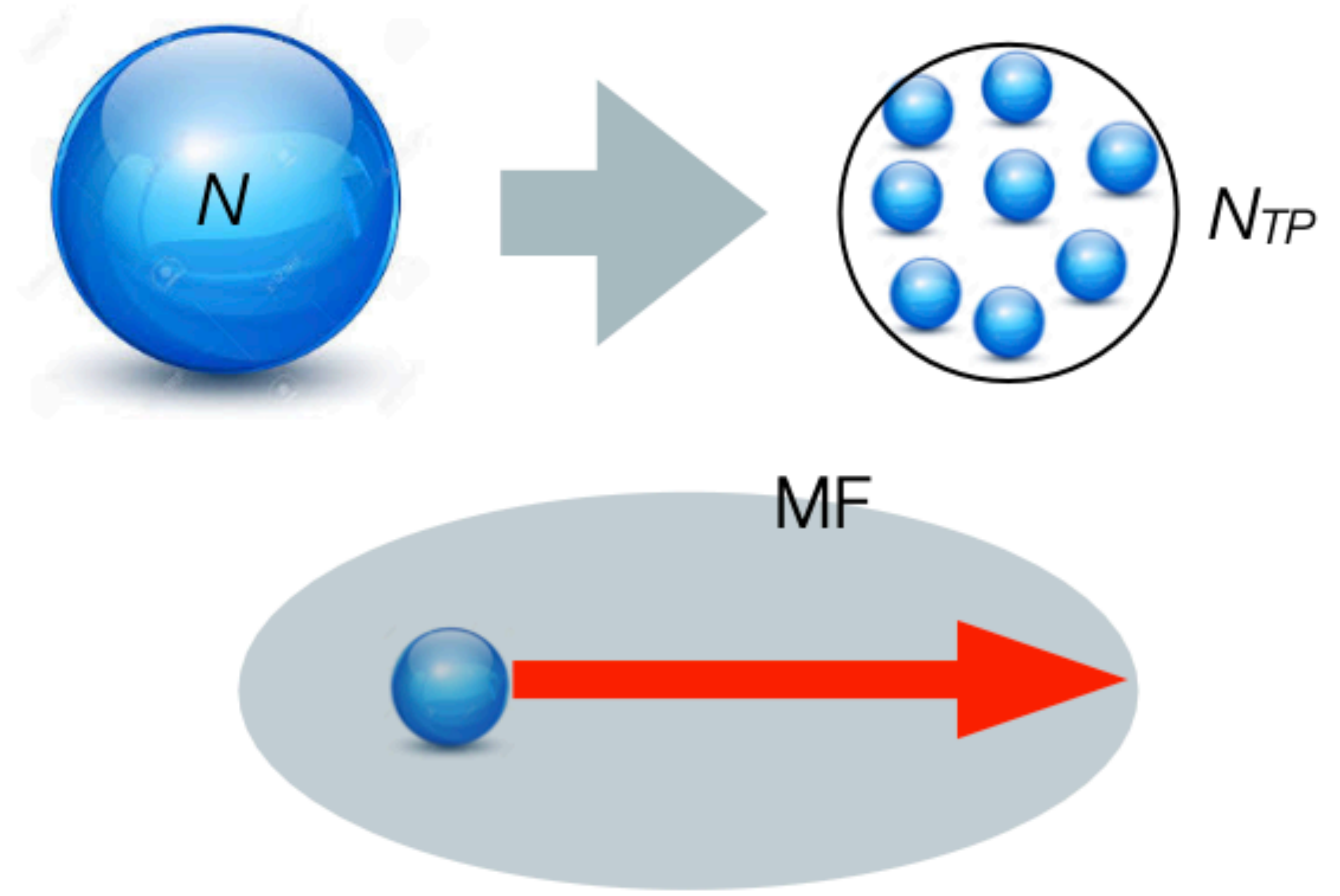
Wigner transformation

$$f(r,p;t) = \int d^4\zeta \exp(ip_\mu\zeta^\mu)\tilde{f}\left(r+\frac{\zeta}{2}, r-\frac{\zeta}{2}\right)$$

Transport model

Two types of approach, BUU and QMD

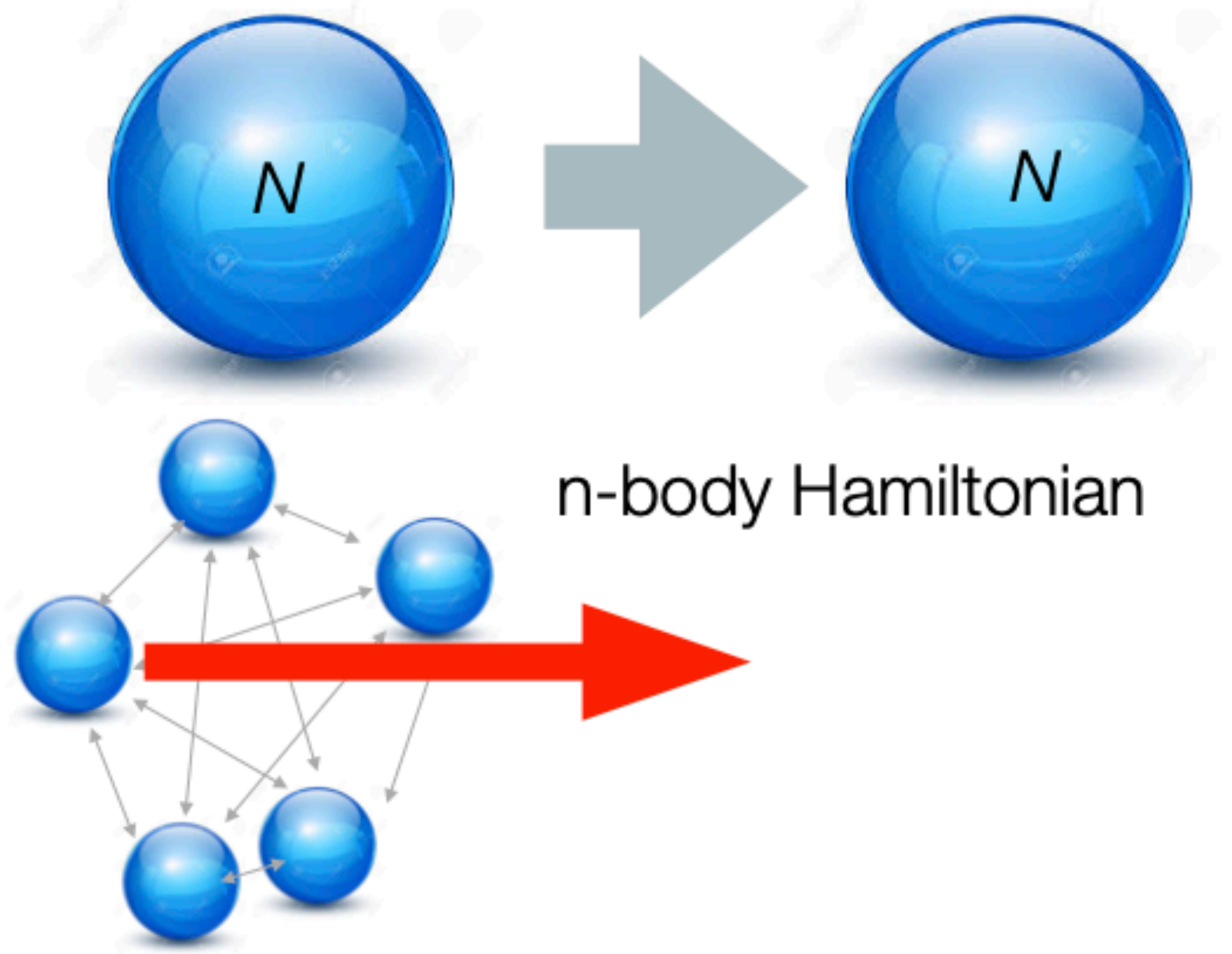
Boltzmann Uehling Uhlenbeck (BUU)



- nucleons divided by N_{TP}
  (infinite N_{TP} = exact solution)
- 1-body phase-space function under MF potential
- Point or finite size of particles

→ BLOB, GiBUU, pBUU, SMASH and **DJBUU** …

Quantum Molecular Dynamics (QMD)



- Gaussian wave packets (N_{TP} = 1)
- n-body Hamiltonian
- Correlation & fluctuations

courtesy of M. Kim

AMD, UrQMD, CoMD, ImQMD and **SQMD** …

DaeJeon Boltzmann-Uehling-Uhlenbeck (DJBUU)

BUU-like model written by Sangyong Jeon (McGill Univ.)

Phase space density of DJBUU is approximated by sum of function of test particles

C.Y. Wong, PRC25, 1460 (1982), G.F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1998)

$$f(\vec{x},\vec{p}) = \frac{(2\pi)^3}{N_{TP}} \sum_{i=1}^{AN_{TP}} g_x(\vec{x}-\vec{x}_i) g_p(\vec{p}-\vec{p}_i)$$, where $g(\mathbf{u}) = g(u) = \mathcal{N}_{m,n}(1-(u/a)^m)^n$ for $0 < u/a < 1$,

Boltzmann-Uehling-Uhlenbeck equation

$$(p^{*0})^{-1}\left[p^{*\mu} - \left(p^{*\mu}\mathcal{F}^{\mu i} - m^*\partial^i m^*(x)\right)\frac{\partial}{\partial p^{*i}}\right] f(\vec{x},\vec{p}) = C(\vec{x},\vec{p})$$

Equation of motion

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{p^{*0}}, \quad \frac{d\vec{p}}{dt} = -\nabla V^0 - \frac{m^{*0}\nabla S}{p^{*0}},$$

where $S = g_\sigma \sigma$,

$$V^{*0} = g_\omega \omega^0 + g_\rho \tau_3 \rho_3^0 + \frac{e}{2}(1+\tau_3)A^0$$

, so

$$V_p = V_\omega + V_\rho + V_{\text{EM}}$$
$$V_n = V_\omega - V_\rho$$

, where

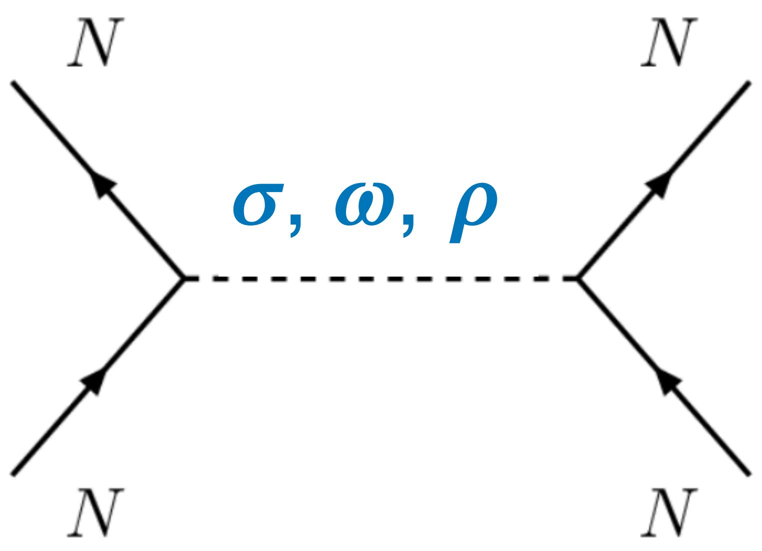
$$V_\omega = g_\omega \omega^0, \; V_\rho = g_\rho \rho^0, \; V_{\text{EM}} = eA^0$$

Lagrangian density with mean field approximation

$$\mathcal{L} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - g_\omega\gamma_0\omega^0 - g_\rho\gamma_0\tau_3\rho_3^3 - \frac{e}{2}\gamma_0(1+\tau^3)A^0 - (m_N + g_\sigma\sigma)\right]\psi$$

Meson equation

$$m_\sigma^2\sigma + a\sigma^2 + b\sigma^3 = -g_\sigma\rho_S$$
$$m_\omega^2\omega^0 = g_\omega\rho_B$$
$$m_\rho^2\rho_3^0 = g_\rho\rho_{B,I3}$$



scalar-Isoscalar σ → attractive
vector-Isoscalar ω → repulsive
vector-Isovector ρ → repulsive

J. D. Walecka, Annals Phys. 83 (1974) 491–529.

or Extend parity doublet model, another model (In progress)

Sindong Quantum Molecular Dynamics (SQMD)

QMD-like model written by Kyungil Kim (RISP, IBS)

Gaussian wave packet

$$\psi_i(\vec{r},t) = \frac{1}{(2\pi\sigma_r^2)^{3/4}} \exp\left(-\frac{(\vec{r}-\vec{r}_i)^2}{4\sigma_r^2} + \frac{i}{\hbar}(\vec{p}_i \cdot \vec{r})\right),$$

Wigner function

$$f(\vec{x}_i, \vec{p}_i) = \exp\left[-\frac{1}{2\sigma_r^2}(\vec{r}_i - \vec{R}_i)^2 + (-\frac{2\sigma_r^2}{\hbar^2})(\vec{p}_i - \vec{P}_i)^2\right]$$

σ_r = 1.3 fm (fixed) ← adopting time-dependent width is in progress

N-body hamiltonian

$$\mathcal{H}\{\mathbf{r}_n, \mathbf{p}_n\} = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j} V(|\mathbf{r}_i - \mathbf{r}_j|)$$

Skyrme parameterized potential

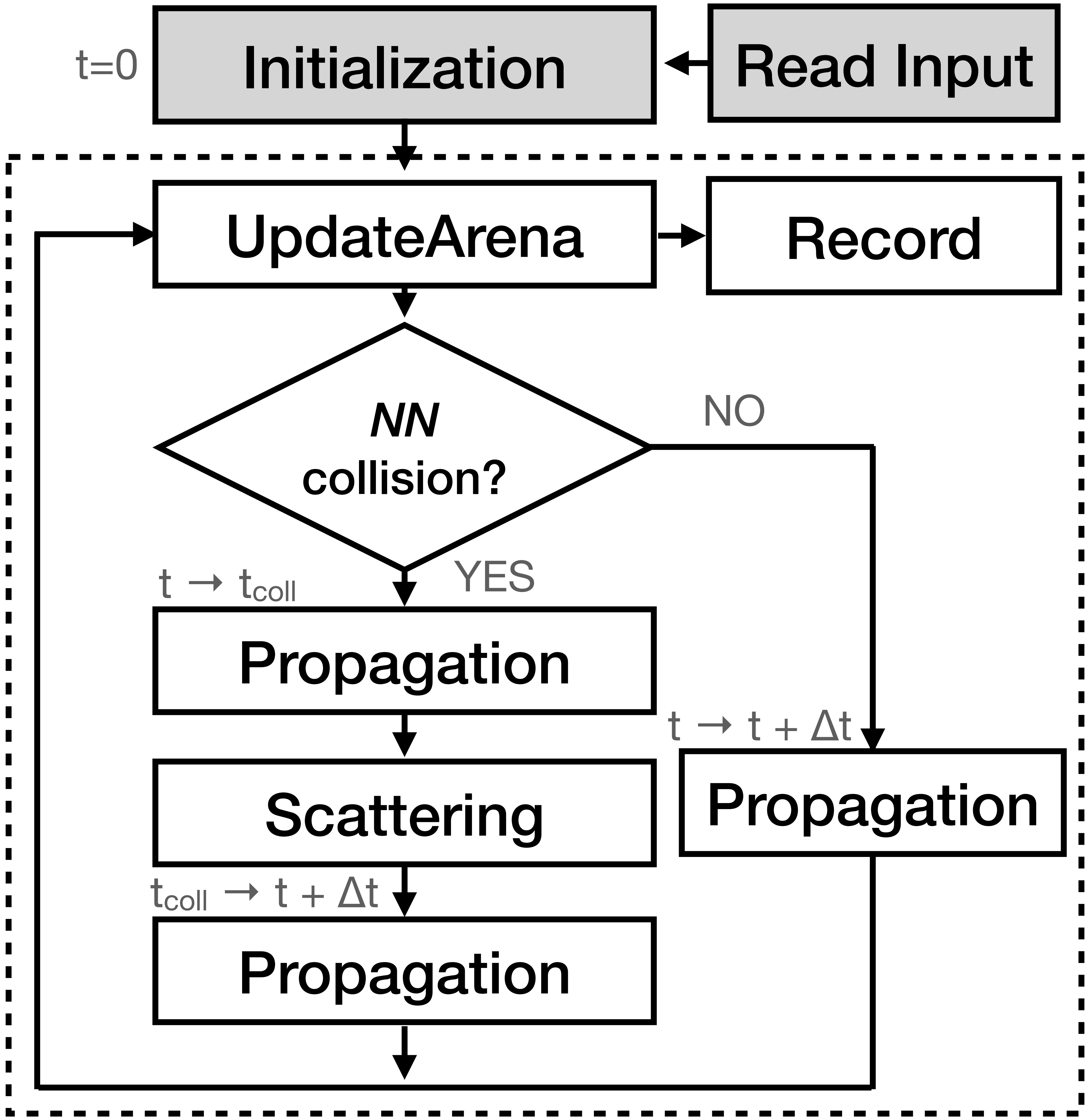
$$U_{Skyrme} = \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_0}\right)^{\gamma},$$

Equation of motion

$$\frac{d}{dt}\mathbf{r}_i = \{\mathbf{r}_i, \mathcal{H}\}, \quad \frac{d}{dt}\mathbf{p}_i = \{\mathbf{p}_i, \mathcal{H}\}$$

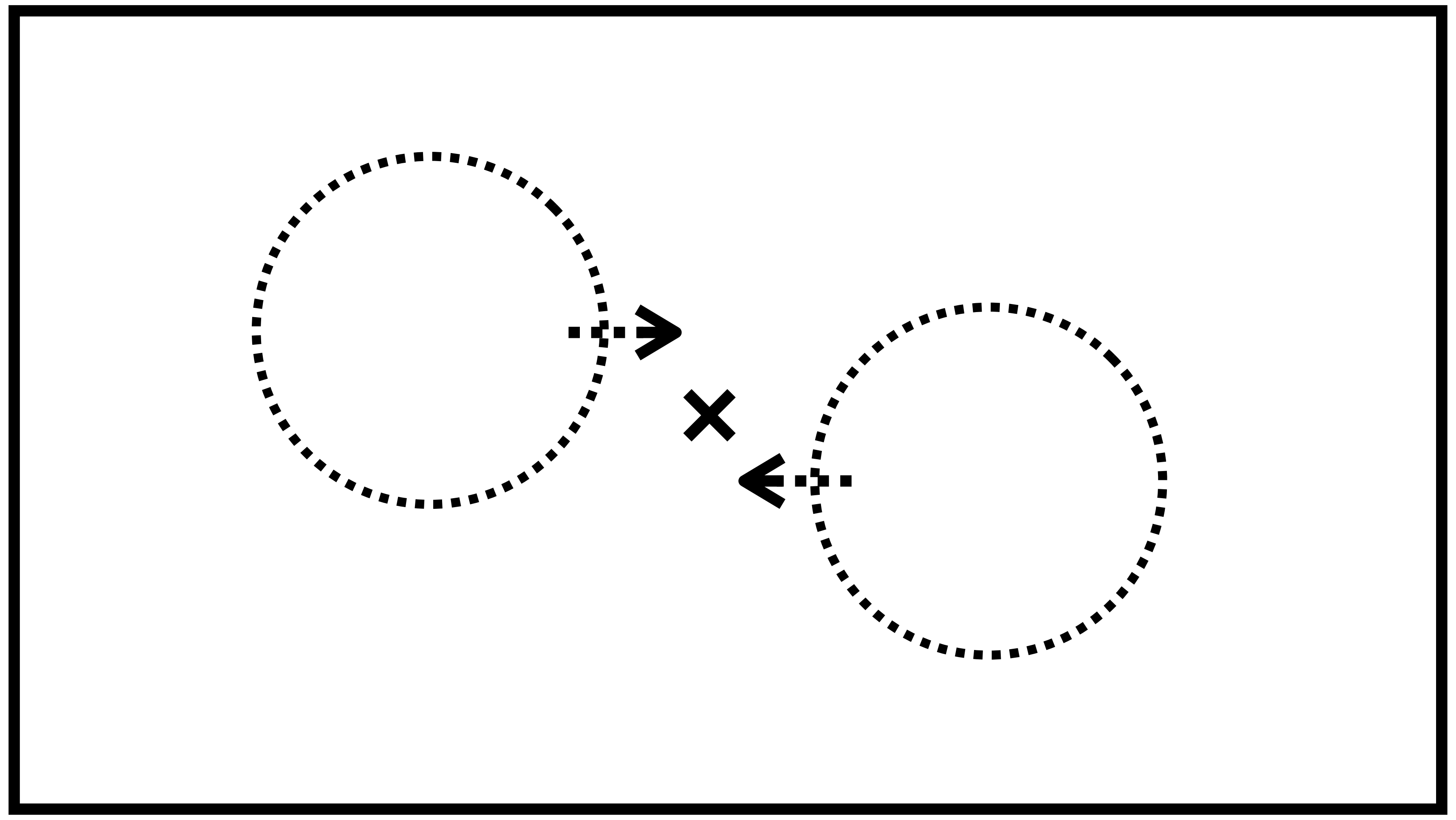
How transport model works

Flow chart of DJBUU ; Initialization



Iteration until t_{fin}

Read Input(jobcard) i.e. E_{beam}, b, $A_{p,t}$ $Z_{p,t}$ ...



1. Create particles corresponding on projectile and target nuclei

   AN_{TP} = number of nucleons * number of test particles

   $\vec{x}$ randomly according to nuclear profile *

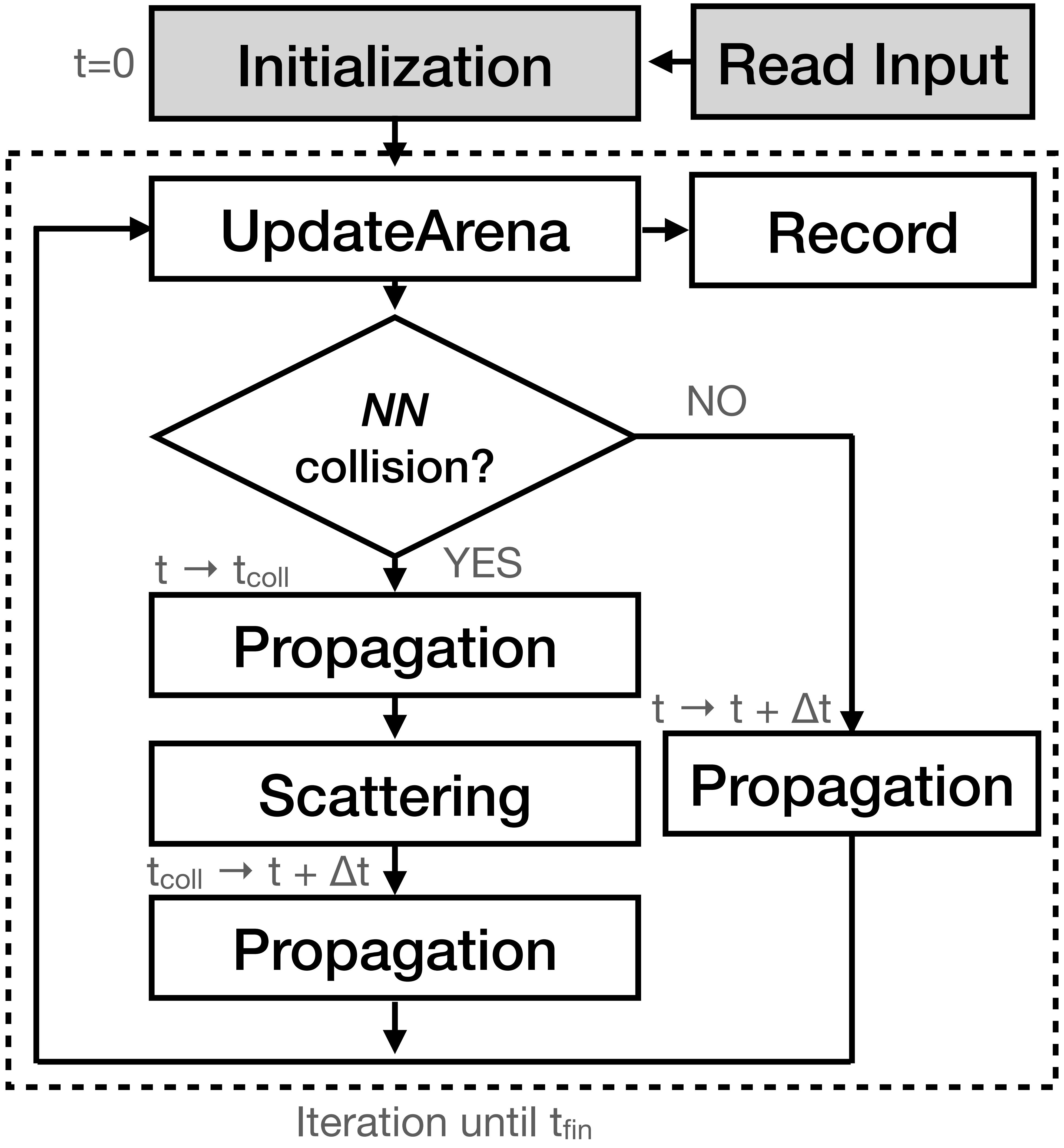
   $\vec{p}$ randomly in Fermi momentum

2. Put them in arena (cm frame)
3. Add momentum corresponding to Beam energy

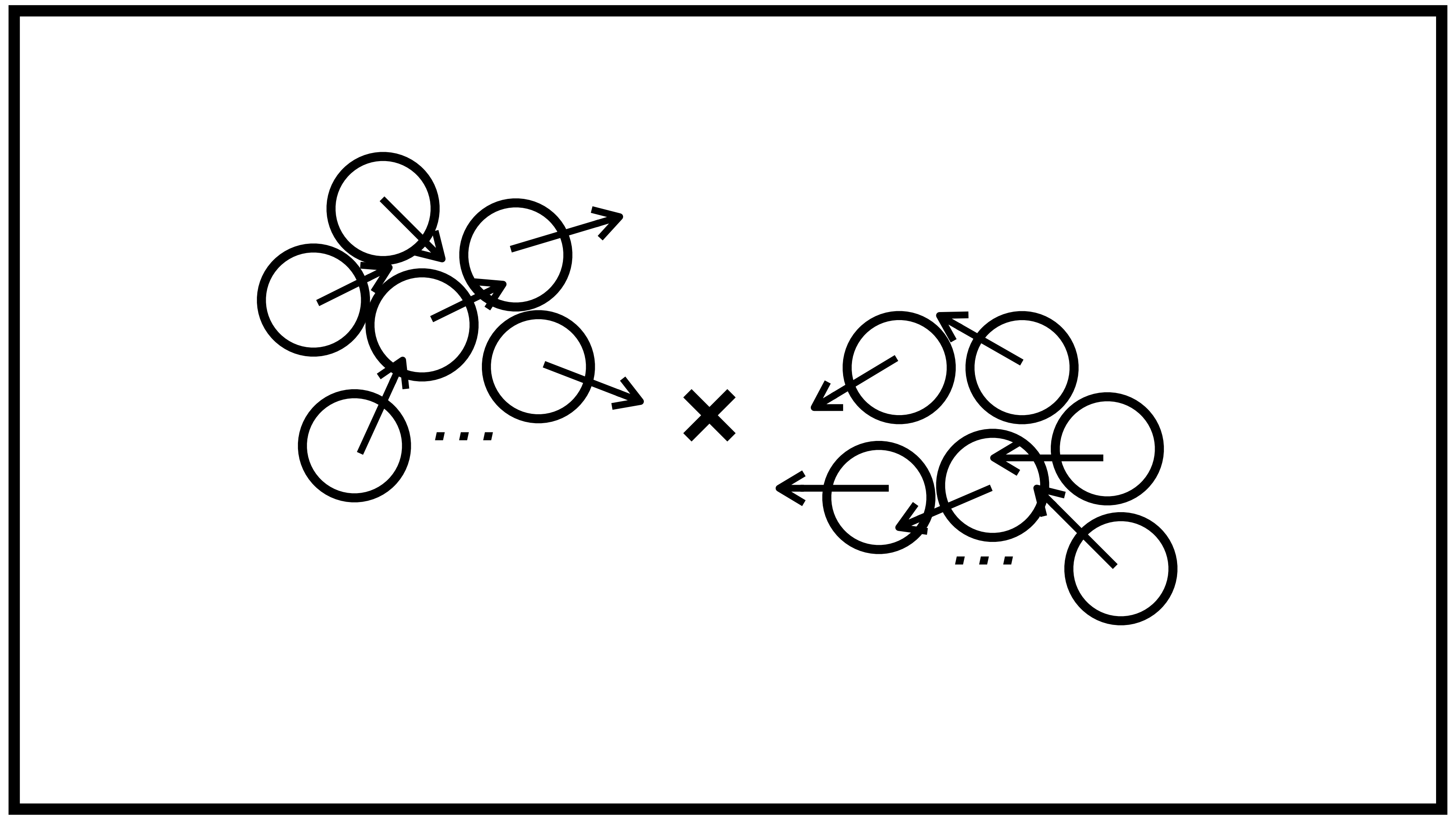
* SQMD use different treatments to decide initial position and check stability.

How transport model works

Flow chart of DJBUU ; Initialization



Read Input(jobcard) i.e. E_{beam}, b, $A_{\mathrm{p,t}}$ $Z_{\mathrm{p,t}}$ ...



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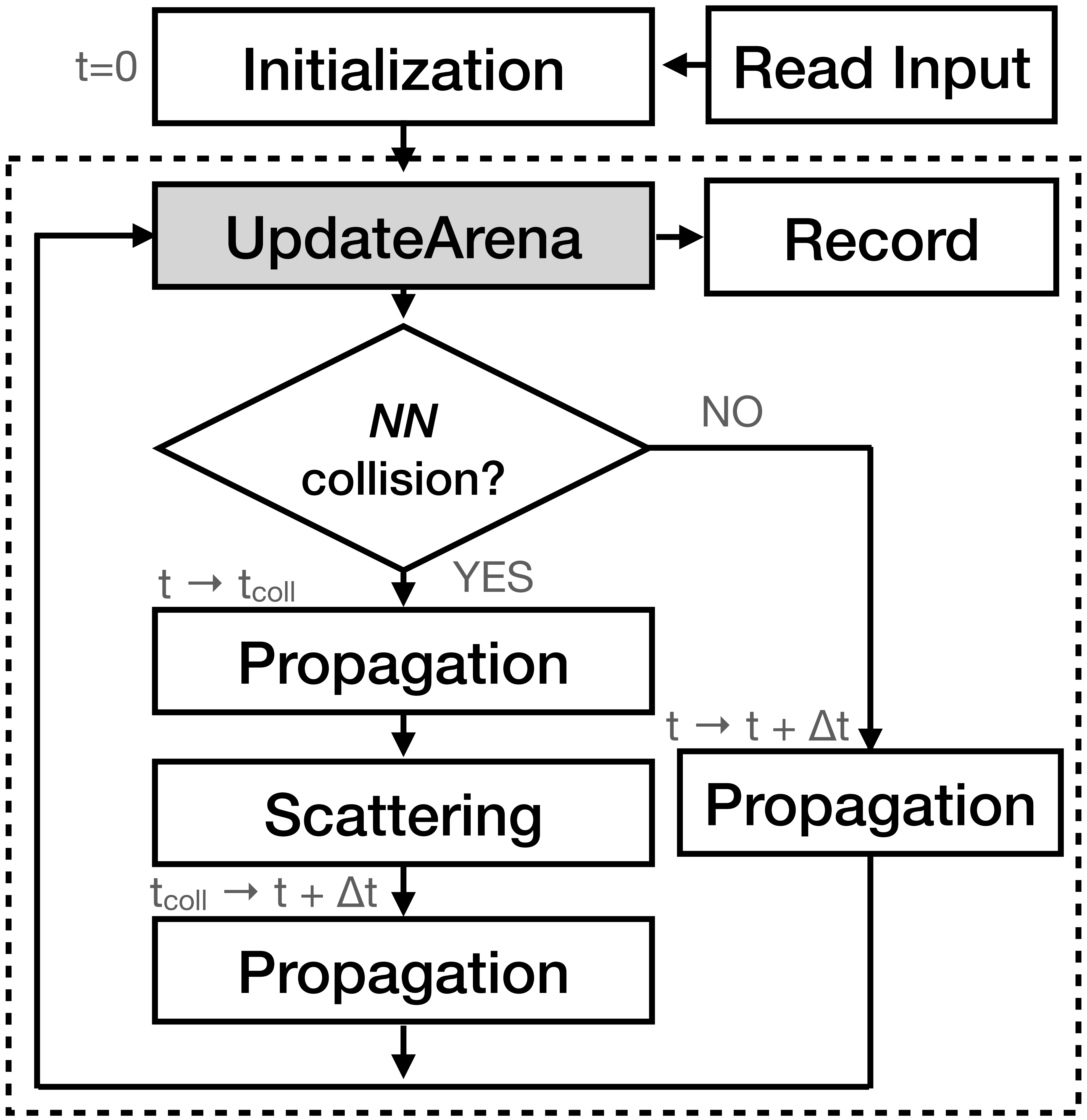
How transport model works

Flow chart of DJBUU



Iteration until t_{fin}

To get scalar, vector mean potential, we calculate meson field value at center of every unit cells in arena

Using openMP

Using meson equation

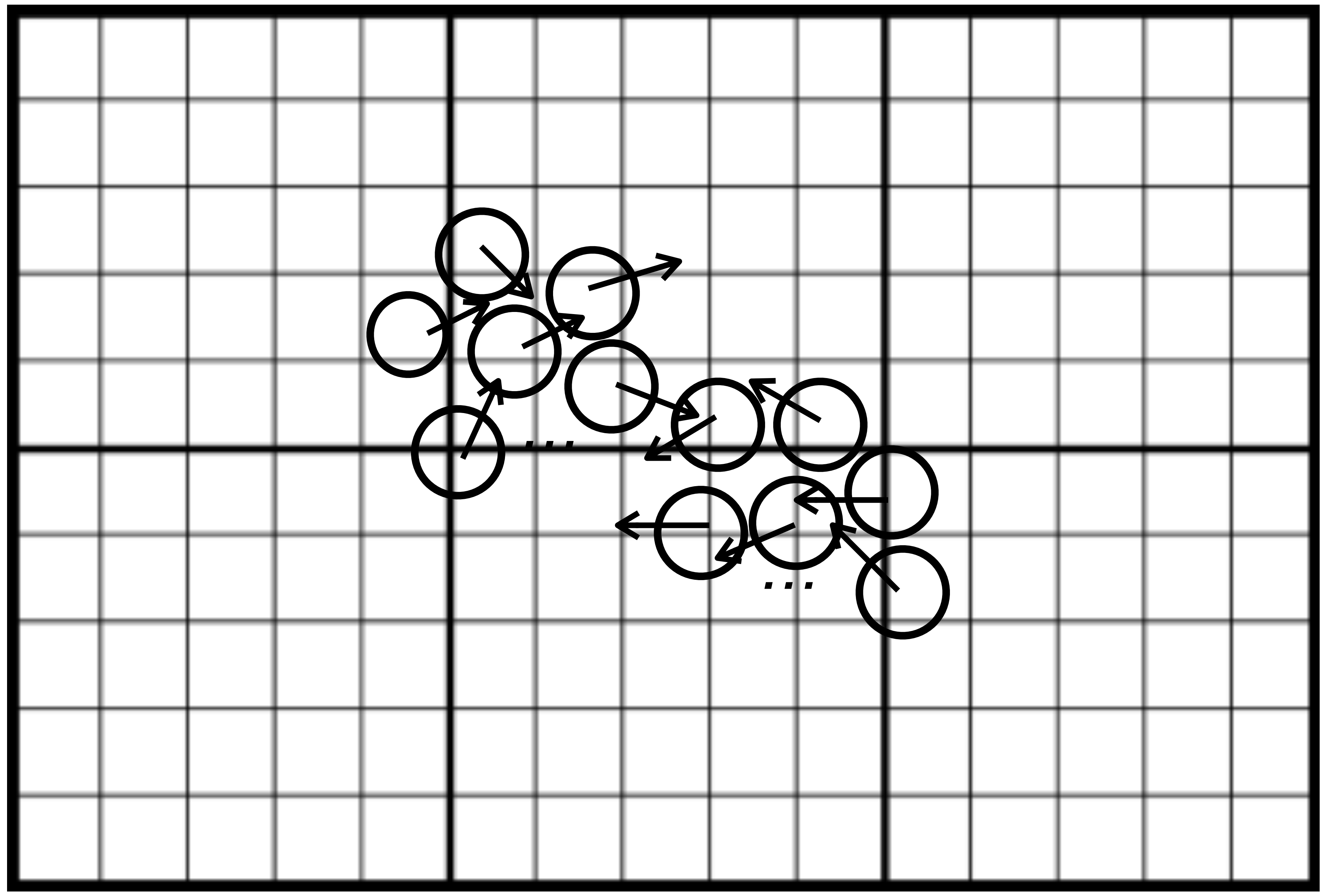
$$\rho_s \to \sigma_0 \to S, m^*,$$

$$\rho_{p,n} \to \omega_0, \rho_0 \to V_\omega^0, V_\rho^0$$

$$m_\sigma^2 \sigma + a\sigma^2 + b\sigma^3 = -g_\sigma \rho_S$$

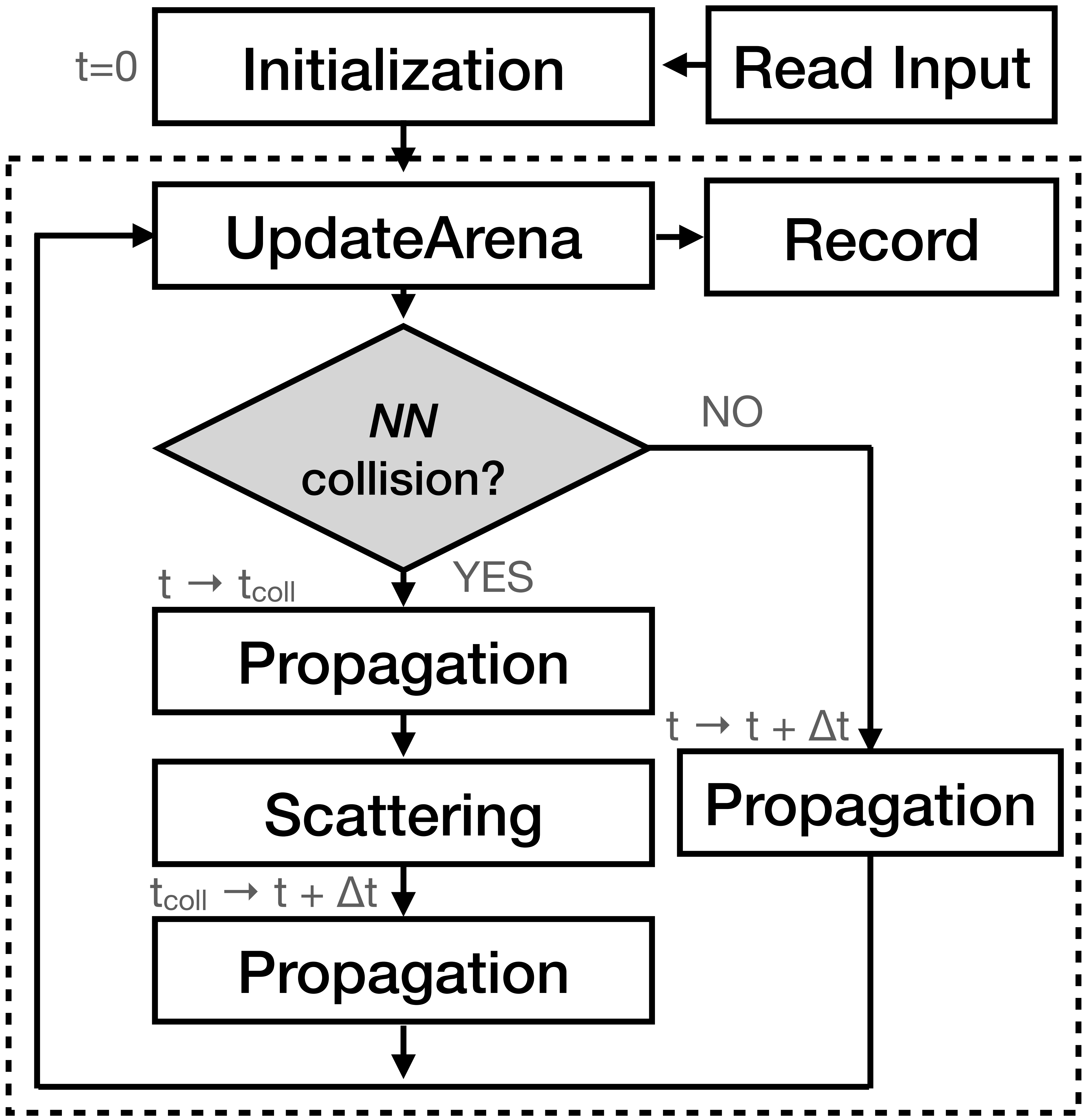
$$m_\omega^2 \omega^0 = g_\omega \rho_B$$

$$m_\rho^2 \rho_3^0 = g_\rho \rho_{B,I3}$$



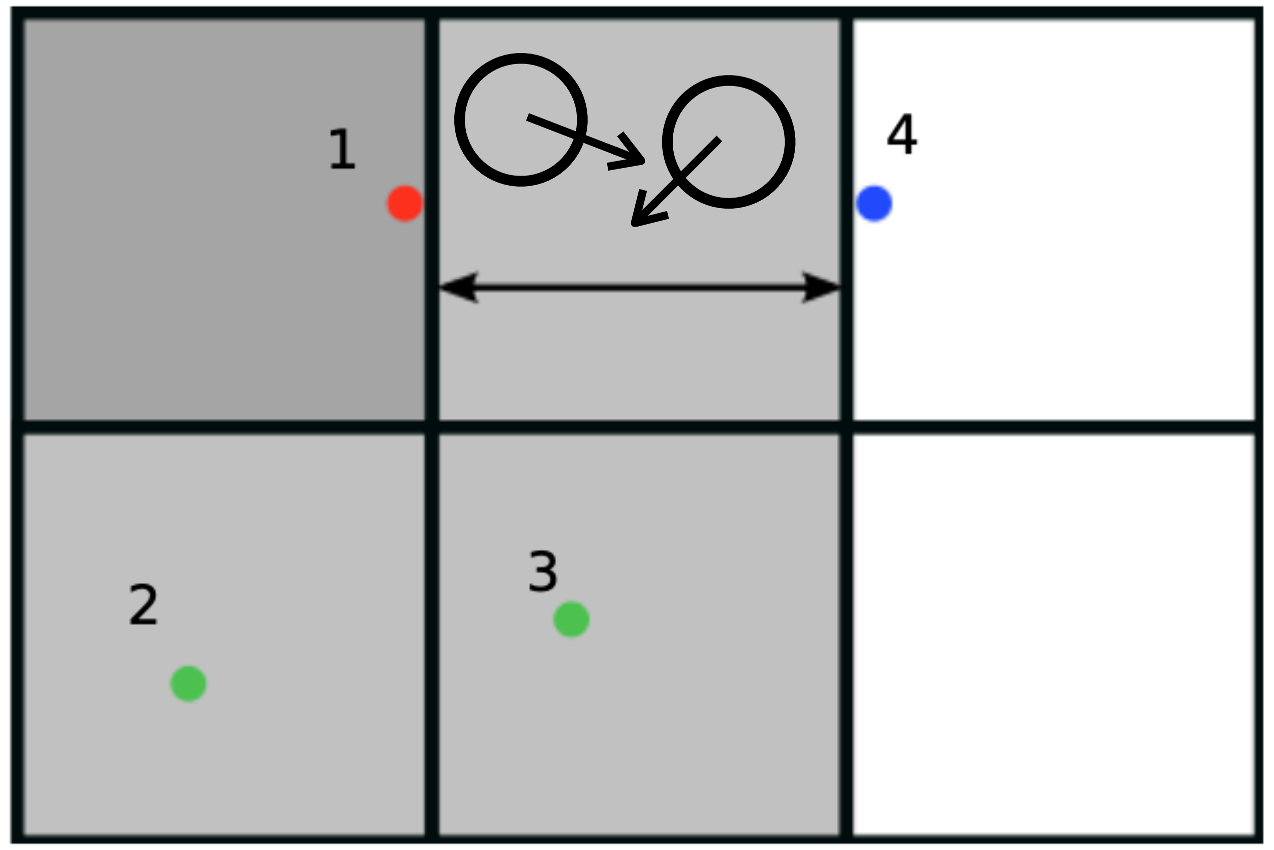
How transport model works

Flow chart of DJBUU



Iteration until t_{fin}

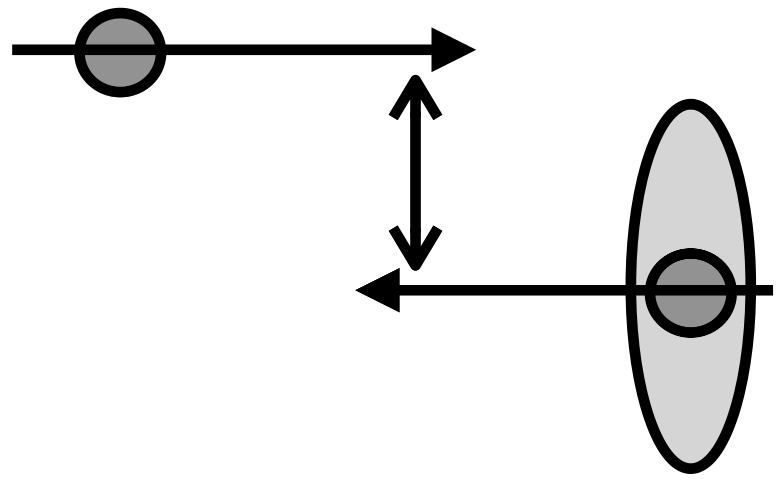
Find collision partner candidate



Using openMP

- In cell search
- face search
- edge search
- tip search

Detect collision



in the center of 2*N* c.m. frame

Bertsch collision criterion

$$d_{\mathrm{trans}} < \sqrt{\frac{\sigma}{\pi}\frac{1}{N_{TP}}} = \sqrt{\frac{\hat{\sigma}}{\pi}}$$

+ Pauli Blocking (1 - *f*)

Distance of closest approach

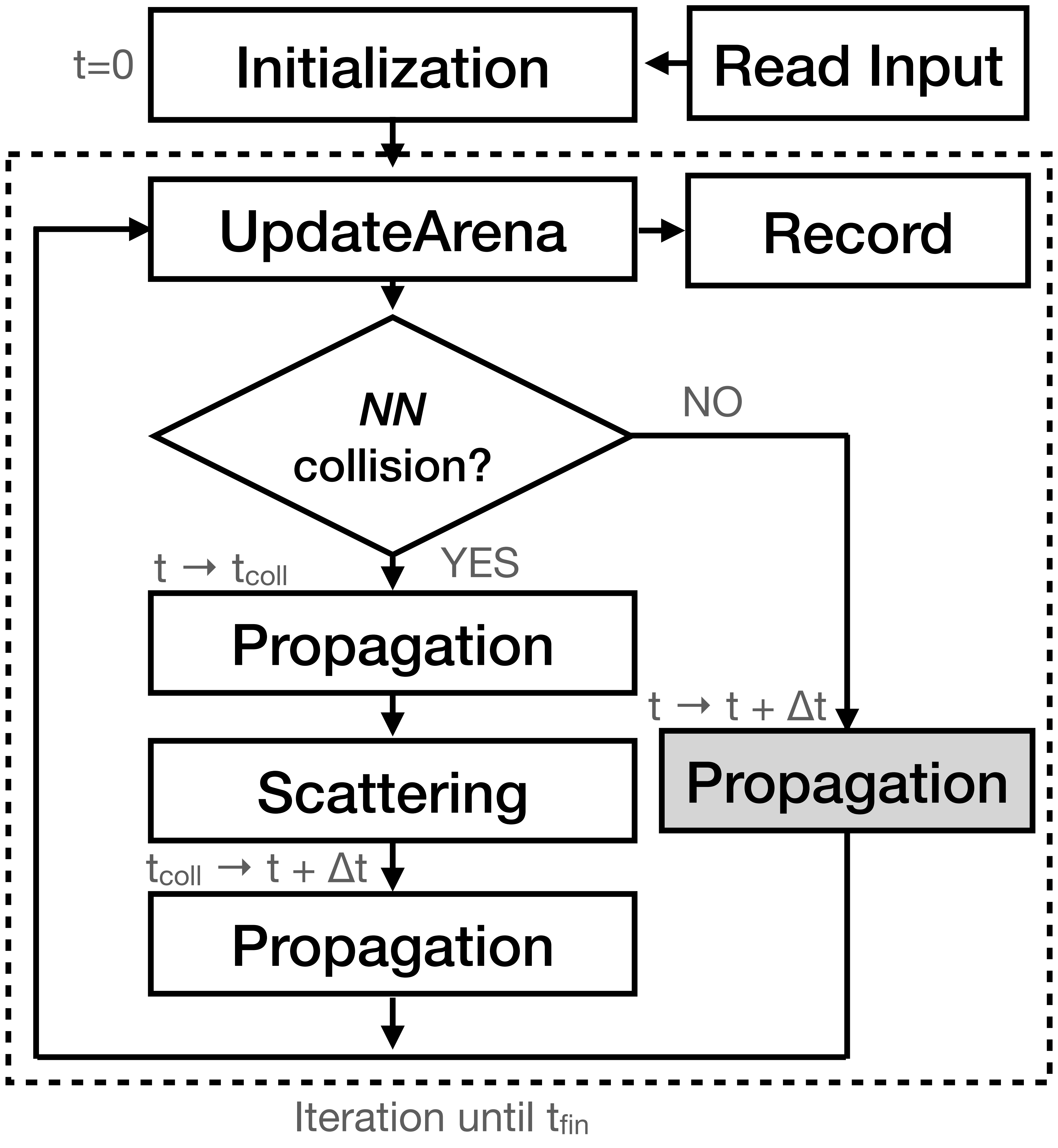
$$d^2_{\mathrm{trans}} = \left| \Delta \vec{x}_{\mathrm{c.m.}} \times \hat{n}_{\mathrm{c.m.}} \right|^2$$

Collision time

$$\Delta t_{\mathrm{coll}} = \frac{\Delta \vec{x} \cdot \Delta \vec{v}}{|\vec{v}|^2}$$

How transport model works

Flow chart of DJBUU



Propagation ~~and collision~~

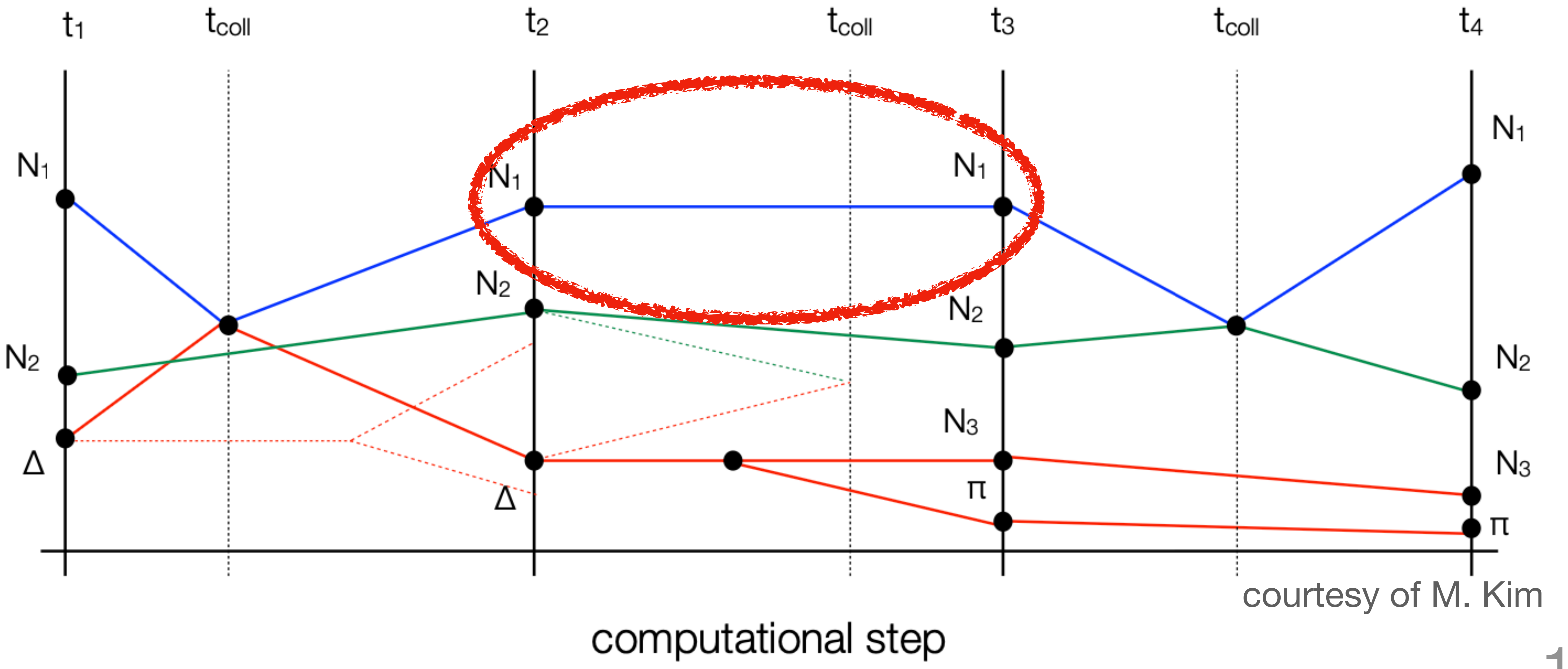
No collision case : (1) solve eq of motion and (2) freely move Δt

Equation of motion in DJBUU

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{p^{*0}}, \quad \frac{d\vec{p}}{dt} = -\nabla V^0 - \frac{m^{*0}\nabla S}{p^{*0}},$$

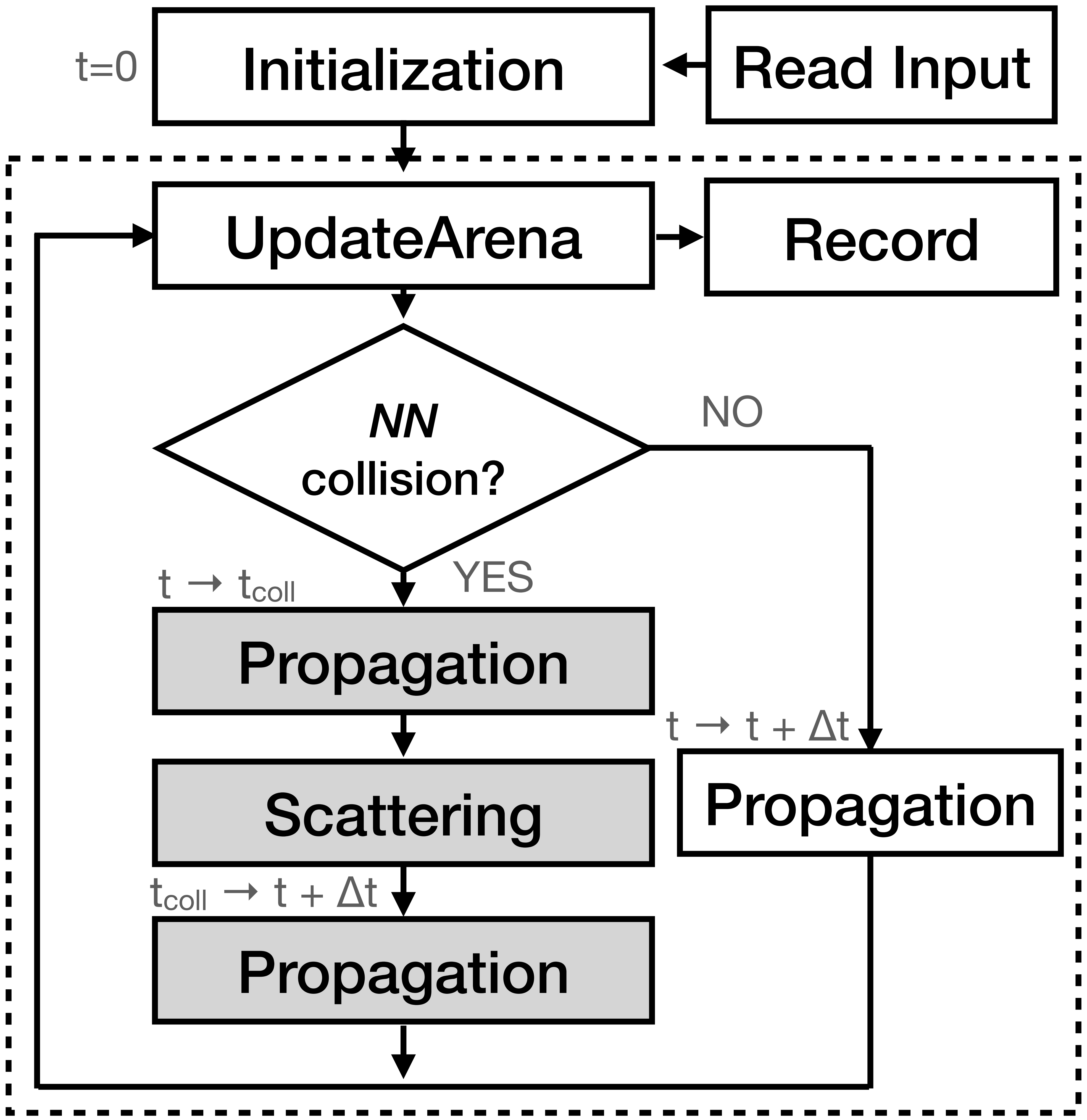
$$\overrightarrow{x_f} = \overrightarrow{x_i} + \overrightarrow{v}\Delta t$$

* SQMD use different equation of motion, but step is almost same.



How transport model works

Flow chart of DJBUU



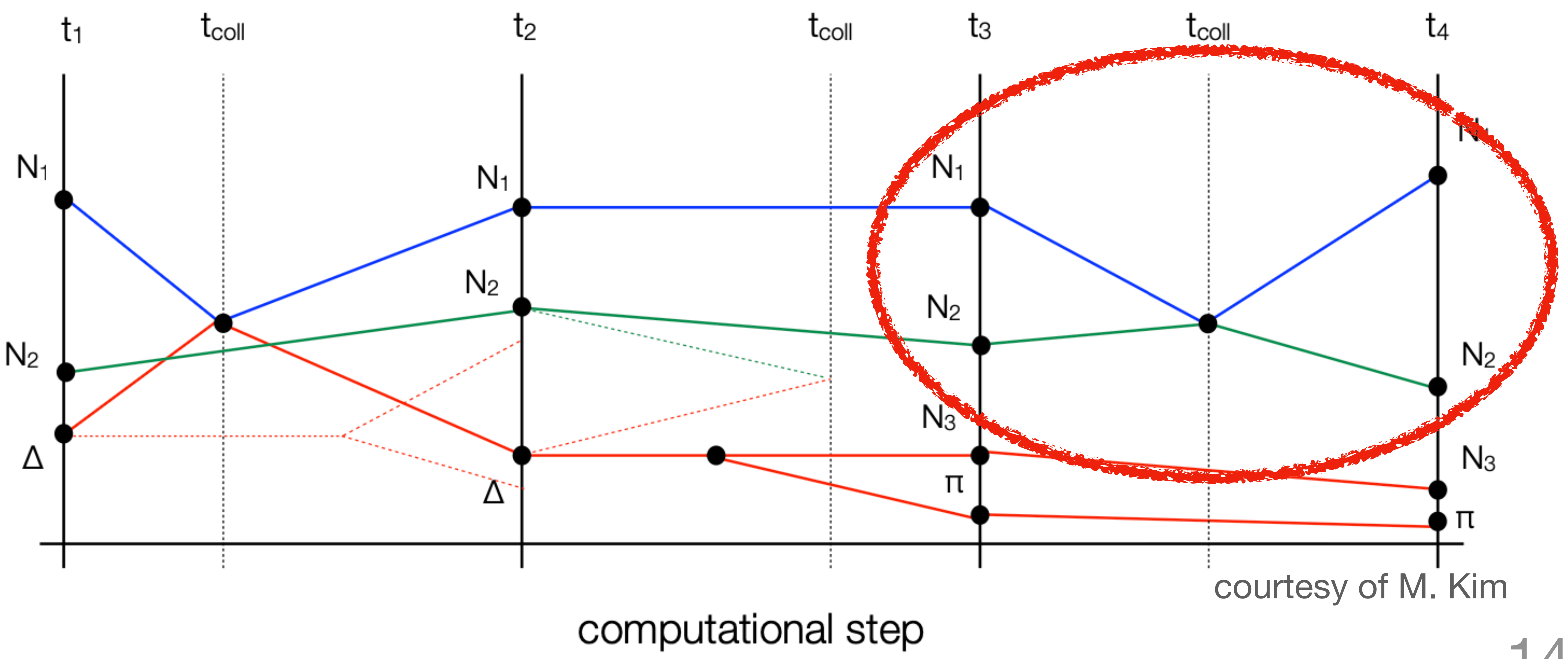
Iteration until t_{fin}

Propagation and collision

YES collision case :

1. Propagate until collision time
2. Give them random momentum (Scattering, *NN* collision) with Energy conservation.
3. Propagate until t + Δt

We explained only elastic scattering in this slid.

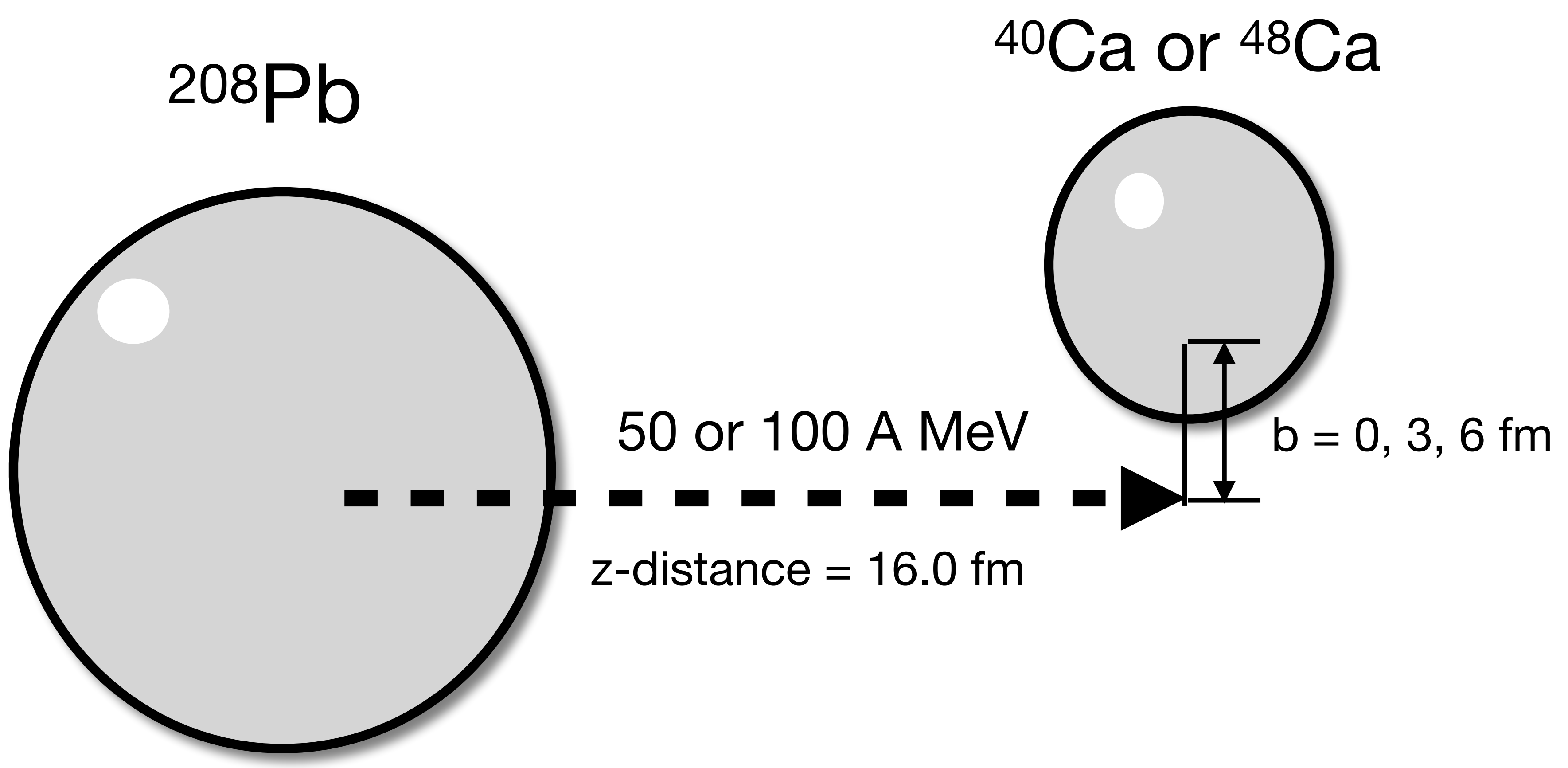


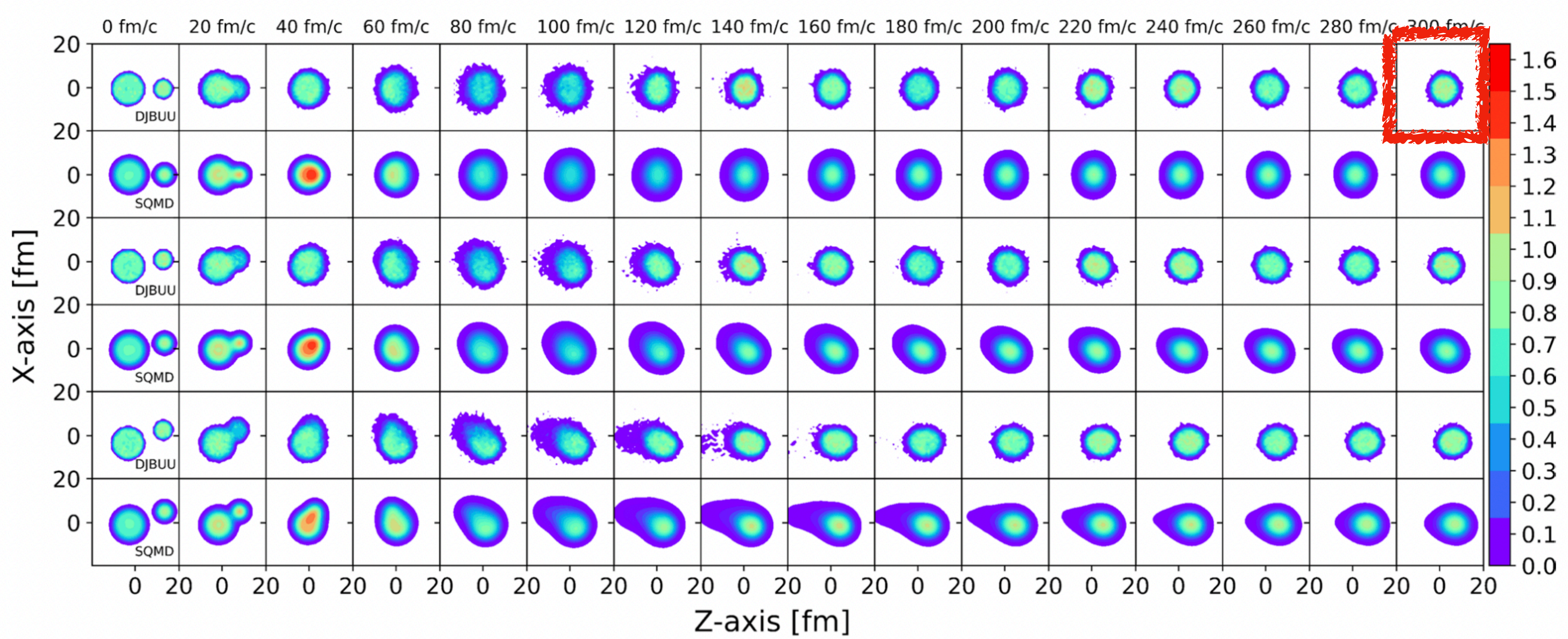
courtesy of M. Kim

* SQMD use different treatments to collision criterion and *NN* collision dynamics during Δt

Comparison study; DJBUU & SQMD

Code comparison : Same Initial conditions using both models





Density distribution in the collision plane. For comparison, the results of DJBUU and SQMD are shown alternatively. From top to bottom, the systems are 208 Pb + 40 Ca at E beam = 50 AMeV .

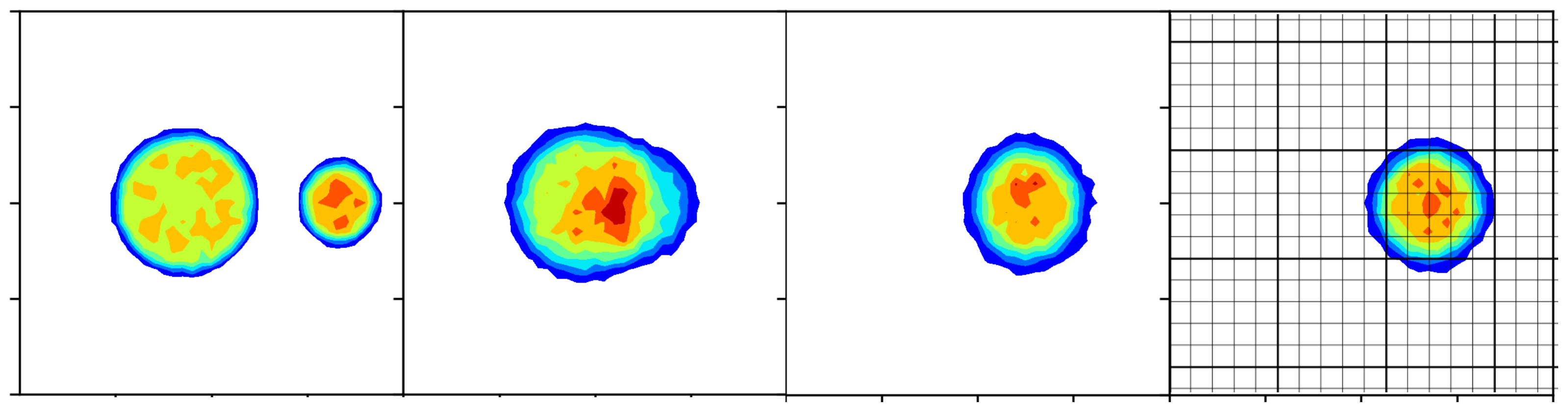
- Comparison DJBUU with SQMD
- (pair of projectile and target) × (beam energy) × (impact parameter) = 12
  Ca40 , Ca48   50, 100 A MeV   0, 3, 6. fm
- # of events required ⇒ DJBUU : 10 (100)
  SQMD : 10,000

Clustering and Fragment comparison

we focused on the Biggest Fragment(BF)

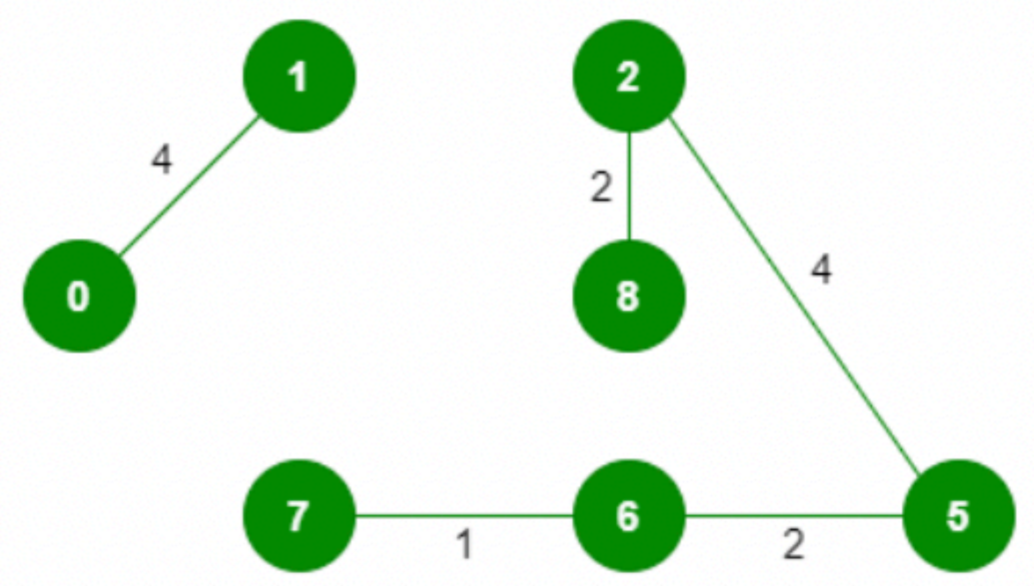
Clustering

DJBUU : Counting *N* in cells beyond 0.1 ρ_0



divide configuration space into 1fm^3 unit cells, and regard collection of cells belong 0.1 ρ_0 as the cluster.

SQMD : Minimum spanning tree (MST)



If the minimum distance of a nucleon pair is longer than 3.5 fm, then the two nucleons are not connected.

Fragment Comparison

- Primary fragment (no De-excitation)
- The Biggest Fragment (BF)
  (different definition in each models)

= the cluster searched at final time (DJBUU)
= the largest one in fragments at final time (SQMD)

Target	E_{beam} (AMeV)	b (fm)	DJBUU	SQMD
${}^{40}\mathrm{Ca}$	50	0	${}^{163}_{73}\mathrm{Ta}$, ${}^{162}_{73}\mathrm{Ta}$, ${}^{164}_{73}\mathrm{Ta}$, ${}^{163}_{74}\mathrm{W}$	${}^{163}_{69}\mathrm{Tm}$, ${}^{173}_{74}\mathrm{W}$, ${}^{169}_{72}\mathrm{Hf}$
		3	${}^{163}_{73}\mathrm{Ta}$, ${}^{165}_{74}\mathrm{W}$, ${}^{164}_{73}\mathrm{Ta}$,	${}^{169}_{72}\mathrm{Hf}$, ${}^{173}_{74}\mathrm{W}$, ${}^{172}_{74}\mathrm{W}$
		6	${}^{167}_{74}\mathrm{W}$, ${}^{169}_{75}\mathrm{Re}$, ${}^{165}_{73}\mathrm{Ta}$, ${}^{168}_{75}\mathrm{Re}$	${}^{168}_{72}\mathrm{Hf}$, ${}^{164}_{70}\mathrm{Yb}$, ${}^{169}_{72}\mathrm{Hf}$
	100	0	${}^{123}_{56}\mathrm{Ba}$, ${}^{121}_{55}\mathrm{Cs}$, ${}^{124}_{57}\mathrm{La}$, ${}^{122}_{56}\mathrm{Ba}$, ${}^{124}_{56}\mathrm{Ba}$	${}^{78}_{33}\mathrm{As}$, ${}^{114}_{50}\mathrm{Sn}$, ${}^{124}_{54}\mathrm{Xe}$
		3	${}^{130}_{59}\mathrm{Pr}$, ${}^{130}_{58}\mathrm{Ce}$, ${}^{128}_{57}\mathrm{La}$, ${}^{128}_{58}\mathrm{Ce}$, ${}^{129}_{58}\mathrm{Ce}$, ${}^{127}_{58}\mathrm{Ce}$, ${}^{127}_{57}\mathrm{La}$	${}^{125}_{53}\mathrm{I}$, ${}^{128}_{56}\mathrm{Ba}$, ${}^{132}_{57}\mathrm{La}$
		6	${}^{145}_{64}\mathrm{Gd}$, ${}^{144}_{64}\mathrm{Gd}$, ${}^{146}_{65}\mathrm{Tb}$, ${}^{147}_{65}\mathrm{Tb}$	${}^{151}_{64}\mathrm{Gd}$, ${}^{149}_{63}\mathrm{Eu}$, ${}^{154}_{66}\mathrm{Dy}$
${}^{48}\mathrm{Ca}$	50	0	${}^{161}_{72}\mathrm{Hf}$, ${}^{162}_{72}\mathrm{Hf}$, ${}^{160}_{71}\mathrm{Lu}$, ${}^{159}_{71}\mathrm{Lu}$	${}^{167}_{70}\mathrm{Yb}$, ${}^{167}_{71}\mathrm{Lu}$, ${}^{170}_{71}\mathrm{Lu}$
		3	${}^{162}_{72}\mathrm{Hf}$, ${}^{164}_{73}\mathrm{Ta}$	${}^{165}_{70}\mathrm{Yb}$, ${}^{167}_{70}\mathrm{Yb}$, ${}^{167}_{71}\mathrm{Lu}$
		6	${}^{164}_{72}\mathrm{Hf}$, ${}^{163}_{72}\mathrm{Hf}$, ${}^{166}_{73}\mathrm{Ta}$, ${}^{165}_{72}\mathrm{Hf}$	${}^{165}_{69}\mathrm{Tm}$, ${}^{159}_{68}\mathrm{Er}$, ${}^{164}_{69}\mathrm{Tm}$
	100	0	${}^{113}_{51}\mathrm{Sb}$, ${}^{115}_{52}\mathrm{Te}$, ${}^{114}_{51}\mathrm{Sb}$, ${}^{116}_{52}\mathrm{Te}$, ${}^{112}_{51}\mathrm{Sb}$	${}^{58}_{25}\mathrm{Mn}$, ${}^{74}_{32}\mathrm{Ge}$, ${}^{107}_{48}\mathrm{Pd}$
		3	${}^{121}_{54}\mathrm{Xe}$, ${}^{122}_{55}\mathrm{Cs}$, ${}^{120}_{54}\mathrm{Xe}$, ${}^{123}_{55}\mathrm{Cs}$, ${}^{121}_{55}\mathrm{Cs}$	${}^{120}_{52}\mathrm{Te}$, ${}^{106}_{45}\mathrm{Rh}$, ${}^{113}_{48}\mathrm{Cd}$
		6	${}^{140}_{62}\mathrm{Sm}$, ${}^{139}_{62}\mathrm{Sm}$, ${}^{138}_{61}\mathrm{Pm}$, ${}^{137}_{61}\mathrm{Pm}$, ${}^{137}_{60}\mathrm{Nd}$	${}^{147}_{62}\mathrm{Sm}$, ${}^{153}_{64}\mathrm{Gd}$, ${}^{148}_{62}\mathrm{Sm}$

The BFs in DJBUU and SQMD; the BFs from the ten runs of DJBUU and the most abundantly produced three BFs from SQMD runs

Comparison study; DJBUU & SQMD

Difference depending on models

Target	E_{beam} (AMeV)	b (fm)	DJBUU	SQMD
^{40}Ca	50	0	${}^{163}_{73}$Ta, ${}^{162}_{73}$Ta, ${}^{164}_{73}$Ta, ${}^{163}_{74}$W	${}^{163}_{69}$Tm, ${}^{173}_{74}$W, ${}^{169}_{72}$Hf
		3	${}^{163}_{73}$Ta, ${}^{165}_{74}$W, ${}^{164}_{73}$Ta,	${}^{169}_{72}$Hf, ${}^{173}_{74}$W, ${}^{172}_{74}$W
		6	${}^{167}_{74}$W, ${}^{169}_{75}$Re, ${}^{165}_{73}$Ta, ${}^{168}_{75}$Re	${}^{168}_{72}$Hf, ${}^{164}_{70}$Yb, ${}^{169}_{72}$Hf
	100	0	${}^{123}_{56}$Ba, ${}^{121}_{55}$Cs, ${}^{124}_{57}$La, ${}^{122}_{56}$Ba, ${}^{124}_{56}$Ba	${}^{78}_{33}$As, ${}^{114}_{50}$Sn, ${}^{124}_{54}$Xe
		3	${}^{130}_{59}$Pr, ${}^{130}_{58}$Ce, ${}^{128}_{57}$La, ${}^{128}_{58}$Ce, ${}^{129}_{58}$Ce, ${}^{127}_{58}$Ce, ${}^{127}_{57}$La	${}^{125}_{53}$I, ${}^{128}_{56}$Ba, ${}^{132}_{57}$La
		6	${}^{145}_{64}$Gd, ${}^{144}_{64}$Gd, ${}^{146}_{65}$Tb, ${}^{147}_{65}$Tb	${}^{151}_{64}$Gd, ${}^{149}_{63}$Eu, ${}^{154}_{66}$Dy
^{48}Ca	50	0	${}^{161}_{72}$Hf, ${}^{162}_{72}$Hf, ${}^{160}_{71}$Lu, ${}^{159}_{71}$Lu	${}^{167}_{70}$Yb, ${}^{167}_{71}$Lu, ${}^{170}_{71}$Lu
		3	${}^{162}_{72}$Hf, ${}^{164}_{73}$Ta	${}^{165}_{70}$Yb, ${}^{167}_{70}$Yb, ${}^{167}_{71}$Lu
		6	${}^{164}_{72}$Hf, ${}^{163}_{72}$Hf, ${}^{166}_{73}$Ta, ${}^{165}_{72}$Hf	${}^{165}_{69}$Tm, ${}^{159}_{68}$Er, ${}^{164}_{69}$Tm
	100	0	${}^{113}_{51}$Sb, ${}^{115}_{52}$Te, ${}^{114}_{51}$Sb, ${}^{116}_{52}$Te, ${}^{112}_{51}$Sb	${}^{58}_{25}$Mn, ${}^{74}_{32}$Ge, ${}^{107}_{48}$Pd
		3	${}^{121}_{54}$Xe, ${}^{122}_{55}$Cs, ${}^{120}_{54}$Xe, ${}^{123}_{55}$Cs, ${}^{121}_{55}$Cs	${}^{120}_{52}$Te, ${}^{106}_{45}$Rh, ${}^{113}_{48}$Cd
		6	${}^{140}_{62}$Sm, ${}^{139}_{62}$Sm, ${}^{138}_{61}$Pm, ${}^{137}_{61}$Pm, ${}^{137}_{60}$Nd	${}^{147}_{62}$Sm, ${}^{153}_{64}$Gd, ${}^{148}_{62}$Sm

The BFs in DJBUU and SQMD; the BFs from the ten runs of DJBUU and the most abundantly produced three BFs from SQMD runs

The higher beam energy
→ break nuclei into small piece
→ more pronounced different definition of BF

The smaller impact parameter
→ more nucleons participate in collision
→ feel equation of state adopted

For example, Incompressibility *K* is 240 (236) MeV for DJBUU (SQMD)

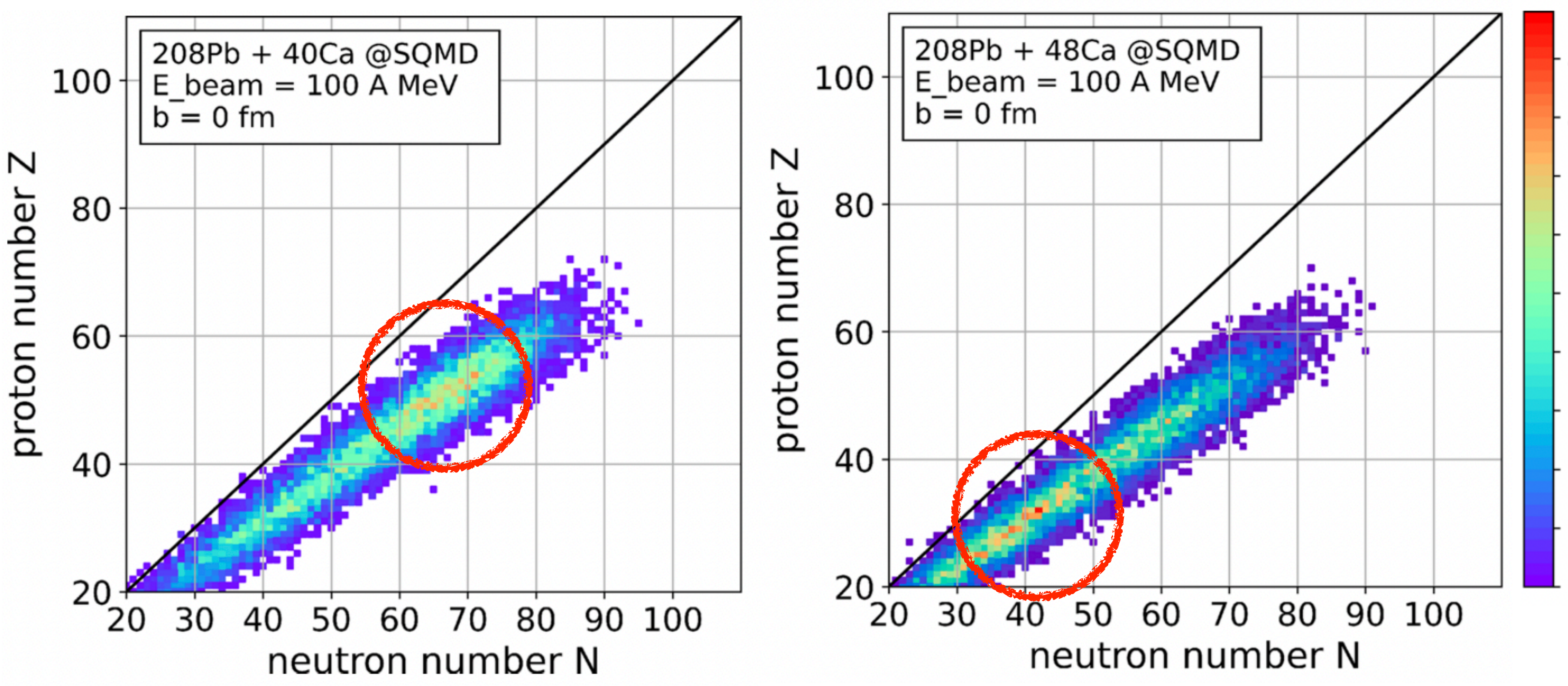
The bigger model dependences

Comparison study; DJBUU & SQMD

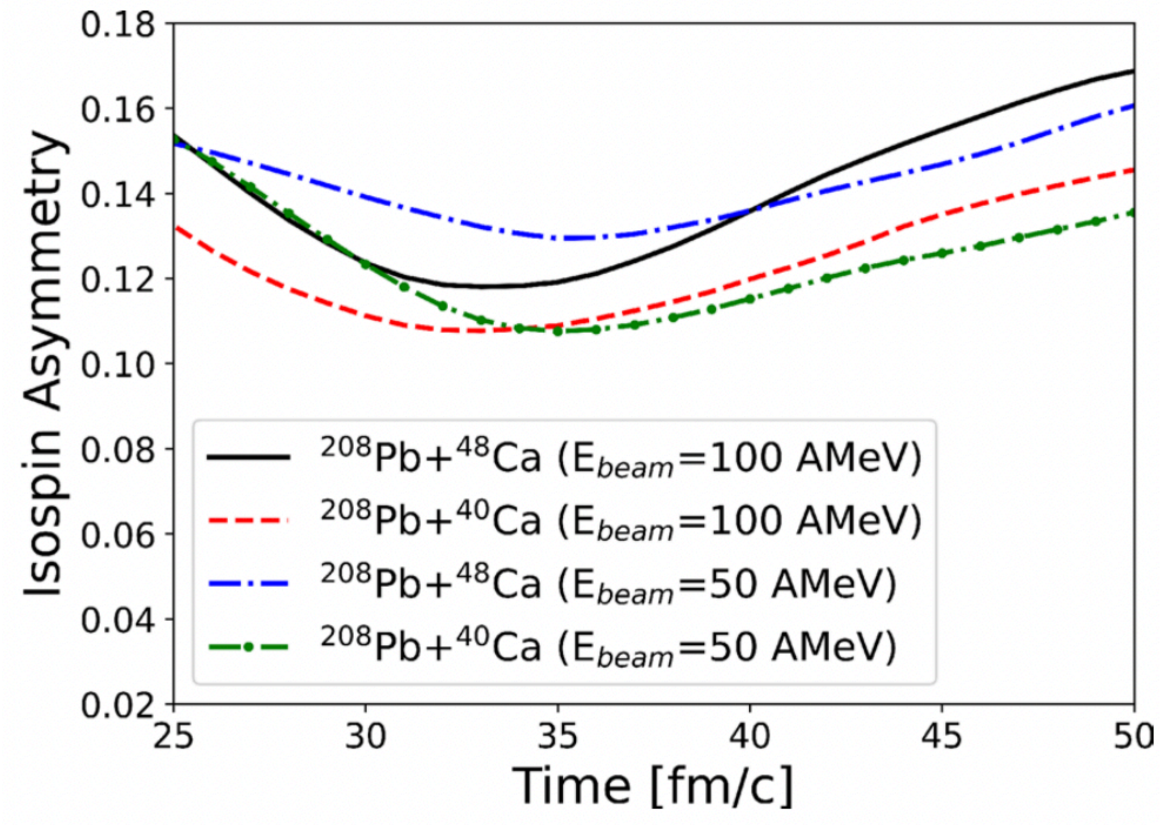
Difference depending on neutron number of target in SQMD model

Target	$E_{\rm beam}$ (AMeV)	b (fm)	DJBUU	SQMD
^{40}Ca	50	0	$^{163}_{73}$Ta, $^{162}_{73}$Ta, $^{164}_{73}$Ta, $^{163}_{74}$W	$^{163}_{69}$Tm, $^{173}_{74}$W, $^{169}_{72}$Hf
		3	$^{163}_{73}$Ta, $^{165}_{74}$W, $^{164}_{73}$Ta,	$^{169}_{72}$Hf, $^{173}_{74}$W, $^{172}_{74}$W
		6	$^{167}_{74}$W, $^{169}_{75}$Re, $^{165}_{73}$Ta, $^{168}_{75}$Re	$^{168}_{72}$Hf, $^{164}_{70}$Yb, $^{169}_{72}$Hf
	100	0	$^{123}_{56}$Ba, $^{121}_{55}$Cs, $^{124}_{57}$La, $^{122}_{56}$Ba, $^{124}_{56}$Ba	$^{78}_{33}$As, $^{114}_{50}$Sn, $^{124}_{54}$Xe
		3	$^{130}_{59}$Pr, $^{130}_{58}$Ce, $^{128}_{57}$La, $^{128}_{58}$Ce, $^{129}_{58}$Ce, $^{127}_{58}$Ce, $^{127}_{57}$La	$^{125}_{53}$I, $^{128}_{56}$Ba, $^{132}_{57}$La
		6	$^{145}_{64}$Gd, $^{144}_{64}$Gd, $^{146}_{65}$Tb, $^{147}_{65}$Tb	$^{151}_{64}$Gd, $^{149}_{63}$Eu, $^{154}_{66}$Dy
^{48}Ca	50	0	$^{161}_{72}$Hf, $^{162}_{72}$Hf, $^{160}_{71}$Lu, $^{159}_{71}$Lu	$^{167}_{70}$Yb, $^{167}_{71}$Lu, $^{170}_{71}$Lu
		3	$^{162}_{72}$Hf, $^{164}_{73}$Ta	$^{165}_{70}$Yb, $^{167}_{70}$Yb, $^{167}_{71}$Lu
		6	$^{164}_{72}$Hf, $^{163}_{72}$Hf, $^{166}_{73}$Ta, $^{165}_{72}$Hf	$^{165}_{69}$Tm, $^{159}_{68}$Er, $^{164}_{69}$Tm
	100	0	$^{113}_{51}$Sb, $^{115}_{52}$Te, $^{114}_{51}$Sb, $^{116}_{52}$Te, $^{112}_{51}$Sb	$^{58}_{25}$Mn, $^{74}_{32}$Ge, $^{107}_{48}$Pd
		3	$^{121}_{54}$Xe, $^{122}_{55}$Cs, $^{120}_{54}$Xe, $^{123}_{55}$Cs, $^{121}_{55}$Cs	$^{120}_{52}$Te, $^{106}_{45}$Rh, $^{113}_{48}$Cd
		6	$^{140}_{62}$Sm, $^{139}_{62}$Sm, $^{138}_{61}$Pm, $^{137}_{61}$Pm, $^{137}_{60}$Nd	$^{147}_{62}$Sm, $^{153}_{64}$Gd, $^{148}_{62}$Sm

The BFs in DJBUU and SQMD; the BFs from the ten runs of DJBUU and the most abundantly produced three BFs from SQMD runs



Proton and neutron distributions of the BFs in SQMD



$$E(\rho, \alpha_I) = E(\alpha_I = 0) + E_{\rm sym}\alpha_I^2 + \cdots ,$$

$$E_{\rm sym} = g_{\rm sym}\left(\frac{\rho}{\rho_0}\right)^{\sigma},$$

the symmetry energy pushes out the neutrons and so disturbs the formation of large fragments.

4. Summary

- Transport models DJBUU and SQMD have been developed for RAON experiment.
- We compare BFs in DJBUU and SQMD with Pb208 + Ca40,48 system.
- With higher beam energy, smaller impact parameter, there were more significant differences btw models.
- At 100 A MeV, b = 0 fm, Ca48 collisions make bigger BFs than Ca40 collsions, We tried to understand this using symmetry energy.

Acknowledgement
This work was supported by the National Supercomputing Center with supercomputing resources including technical support (KSC-2023-CRE-0170)

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Thank you for your attention.

Back-up slice

DJBUU .. Result etc

Motivation : Heavy ion collisions

Transport model can describe time evolution of dynamics in HICs

Symmetry energy; Dense matter EoS

$$\mathcal{E}(\rho,\delta) = E(\rho) + S(\rho)\delta^2 + O(\delta^4),$$

$$E(\rho) = E_0 + \frac{1}{2}K_0 x^2 + \frac{1}{6}Q_0 x^3 + \cdots,$$

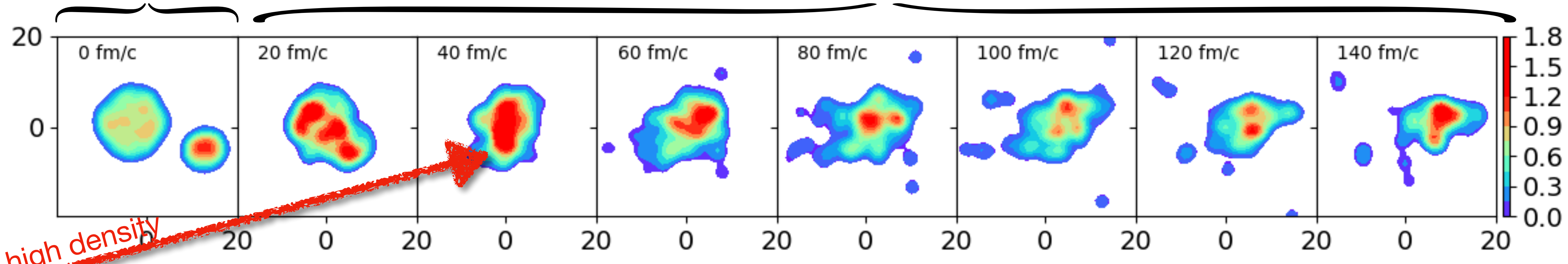
$$S(\rho) = J + Lx + \frac{1}{2}K_{\rm sym}x^2 + \frac{1}{6}Q_{\rm sym}x^3 + \cdots.$$

How to describe HiC (microscopic)

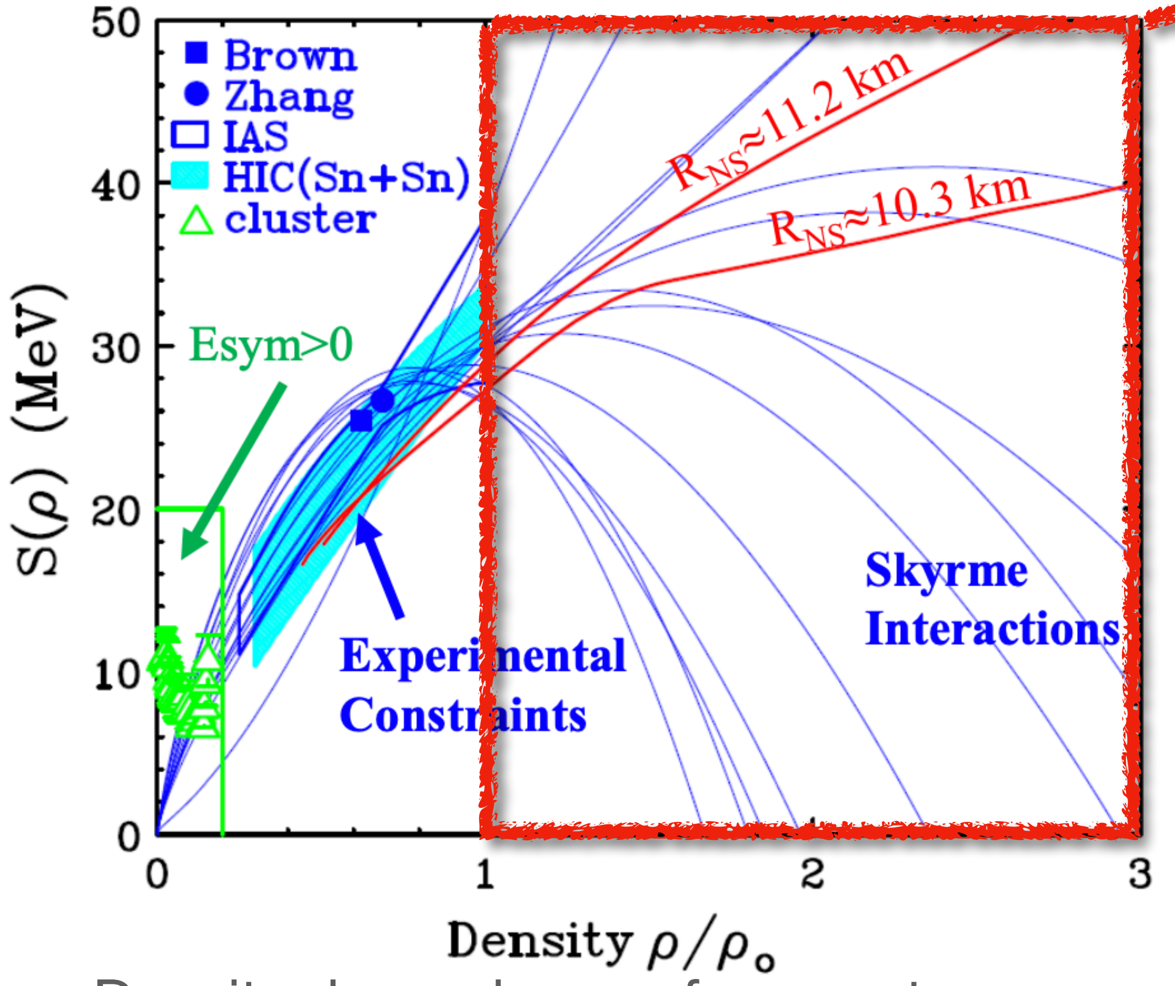
Initialization Generate nuclei

Propagation Motion of constituent

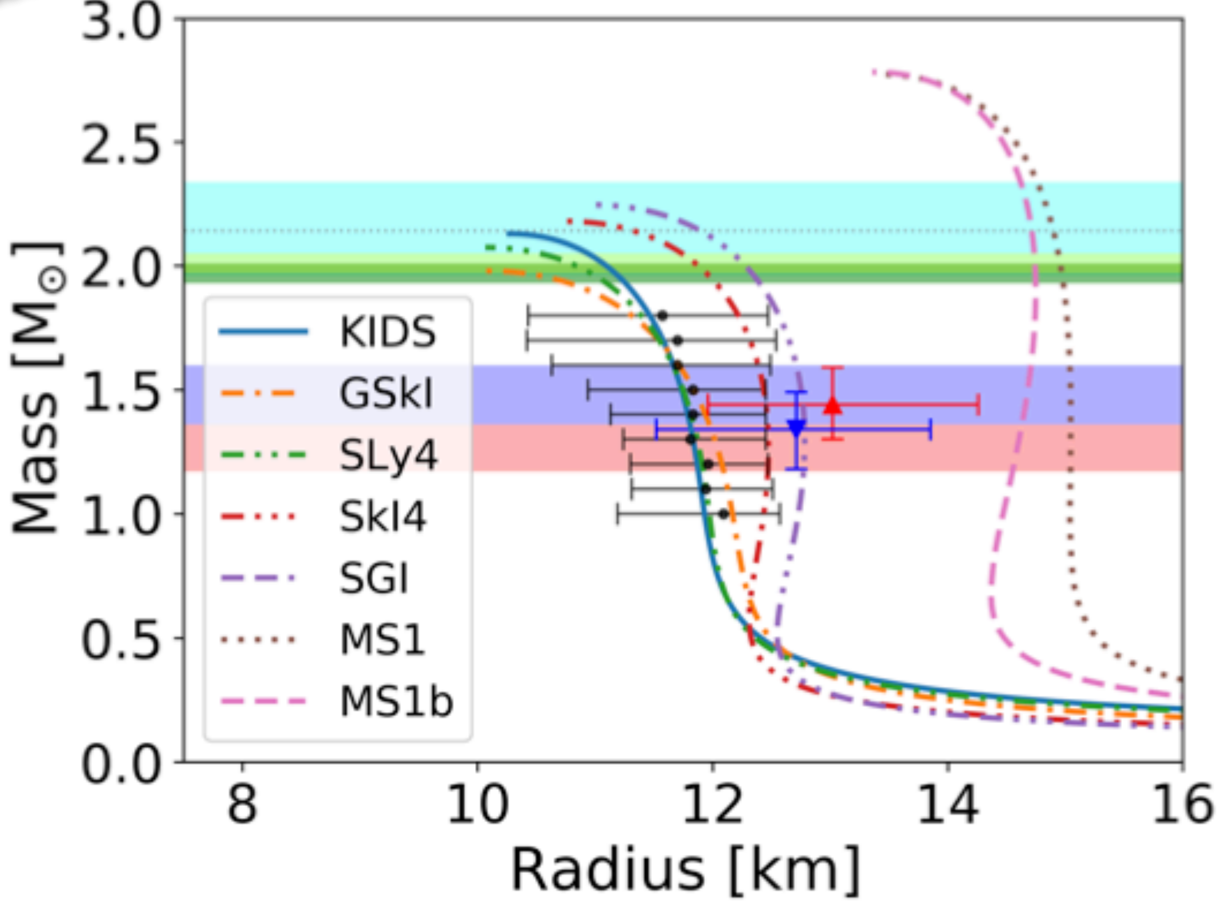
Collisions *NN* collision



1 event Contour of Pb + Ca collision using SQMD model, Beam energy = 100 MeV /u, Impact parameter = 6 fm



Density dependence of symmetry energy



Neutron Star Radius vs Mass

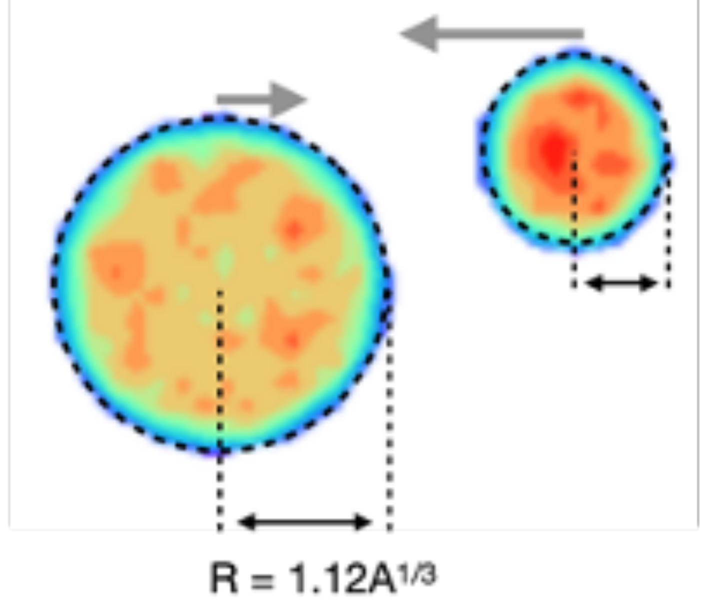
- Semi-classical method
- Microscopic point of view
- Time steps from initial to final
- Numerical simulation

Wigner transformation

$$f(r,p;t) = \int d^4\zeta \exp(ip_\mu\zeta^\mu)\tilde{f}\left(r+\frac{\zeta}{2}, r-\frac{\zeta}{2}\right)$$

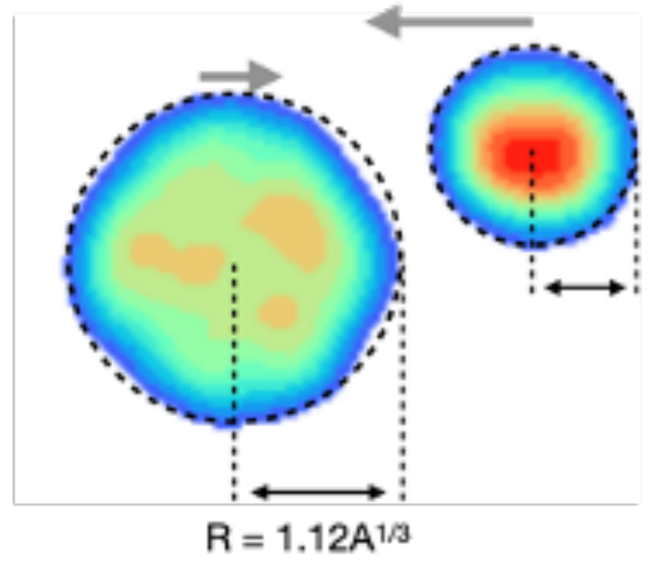
Boltzmann Uehling Uhlenbeck (BUU)

DJBUU



Quantum Molecular Dynamics (QMD)

SQMD



Parameter set

DJBUU (DaeJeon Boltzmann Uehling Uhlenbeck)

Parameter	f_σ	f_ω	f_ρ	f_δ	A (fm^{-1})	B
Set I	10.33	5.42	0.95	0.00	0.033	-0.0048
Set II	same	same	3.15	2.50	same	same
NL3	15.73	10.53	1.34	0.00	-0.01	-0.003

Parameter sets used in mean-field potentials. The existence of δ meson field is the only difference in Set I and Set II. NL3 parameters are widely used in nuclear structure calculation [62]. Dimension of all coupling constant fi is [fm2] and B is dimensionless. Taken from Ref. [61].

$f_i = (g_i^2/m_i^2), i = \sigma, \omega, \rho \qquad A = a/g_\sigma^3,\ B = b/g_\sigma^4$

three fixed meson masses mσ =0.5082, mω =0.783, and mρ =0.763 GeV

saturation density ρ0 =0.16 fm3, binding energy E/A = 16 MeV,
nucleon effective mass m* =0.75mN where mN =0.939 GeV,
and incompressibility K = 240 MeV at saturation density.

$$eA^0 = \begin{cases} \left(\frac{a_r^2}{8} - \frac{x^2}{6} + \frac{3x^4}{20a_r^2} - \frac{x^6}{14a_r^4} + \frac{x^8}{72a_r^6}\right)\frac{315e}{64\pi a_r^3} & (0 < x \le a_r)\,, \\ \frac{e}{4\pi x} & (a_r < x \le 2a_r)\,, \end{cases}$$

Elastic
Only p, x
parameterized by Cugnon (1996) 215-220

In-elastic
(N, Δ, π)*
parametrized inelastic cross-sections from Huber and Aichelin NPA 573 (1994) 587

SQMD (Sindong Quantum Molecular Dynamics)

$$U_{\text{Skyrme}} = \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_0}\right)^\gamma, \qquad (5)$$

where ρ is the baryon number density. We take $\alpha = -218$ MeV, $\beta = 164$ MeV and $\gamma = 4/3$. With these parameters we obtain the incompressibility K=236 MeV and the binding energy -16 MeV at the saturation density. For initialization, propagation and collision, SQMD adopts the standard methods used in transport models [9].

parameterized by Li and Machleidt