Calculation of radial moments of charge distribution compared to precision spectroscopy data

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Charge radius and moments

- Charge radius is obtained by charge density $\rho_c(r)$.

Charge radius : $\sqrt{\langle r^2 \rangle_c}$

2n th-order moments :

$$\langle r^{2n} \rangle_c = \frac{1}{Z} \int d^3 r r^{2n} \rho_c(r)$$

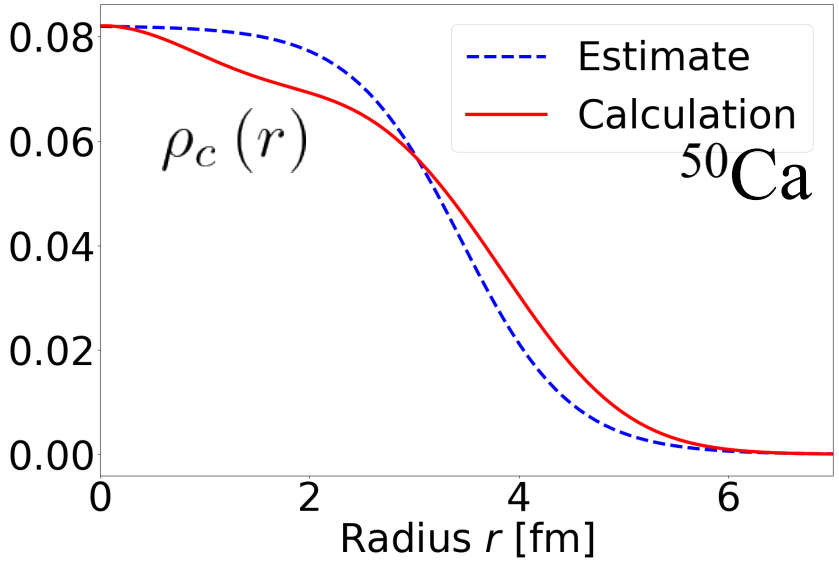
- Electron scattering measurements for charge density

[Hahn *et al.*, Phys. Rev. 101,1131(1956).]

$$\rho_c(r) = \frac{\rho_0}{\left[1 + \exp\left(\frac{r-R}{a}\right)\right]}$$

Constant parameters

ρ_0, R, a



More precise method for measuring relative charge radii → Isotope shift

Isotope shift

Relationship : Transition frequency & The charge moments

Transition frequency measured by laser spectroscopy

$$\delta\nu_{\mathrm{IS}}^{A',A} = \nu^{A'} - \nu^{A} \quad \text{(Isotope shift)}$$

$$= \underbrace{\delta\nu_{\mathrm{FS}}^{A',A}}_{\text{Field Shift term}} + \underbrace{\delta\nu_{\mathrm{MS}}^{A',A}}_{\text{Mass Shift term}}$$

(A,A') : Isotope set

Field Shift term in detail $\quad \delta\nu_{\mathrm{FS}}^{A',A} = F(Z)\,\underbrace{\lambda^{A',A}}_{\text{Selzer moment}}$

Selzer Moment $\quad \lambda^{A',A} = \delta\langle r^2\rangle_c^{A',A} + C\delta\langle r^4\rangle_c^{A',A} + \cdots$

Precision measurements of isotope shift provide the charge moments.

[X.F. Yang, *et al.,* Progress in Particle and Nuclear Physics, 129, 104005(2023).]

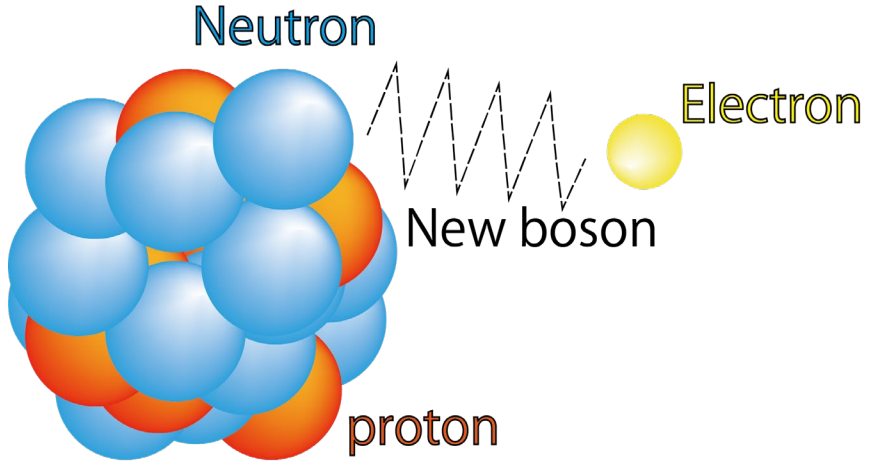
Calculation for precise moments

The main purpose is to calculate precise charge moments.

- Search for new boson in the final purpose
  [Koki Ono, *et al.*, Phys. Rev. X, 12:021033, 2022]

contribution

- Determination of second-order and fourth-order moments in nuclei



Next work

- Present stage : Need to evaluate calculation models to reproduce measurements

Purpose of this study : To calculate the charge moments
To find good calculation models

Mean-field calculation

- Hartree-Fock-Bogoliubov method

$$\begin{pmatrix} h(\vec{r})-\lambda & \tilde{h}(\vec{r}) \\ -\tilde{h}(\vec{r})^{*} & -h(\vec{r})^{*}+\lambda \end{pmatrix}\begin{pmatrix} U_i(\vec{r}) \\ V_i(\vec{r}) \end{pmatrix} = E_i \begin{pmatrix} U_i(\vec{r}) \\ V_i(\vec{r}) \end{pmatrix}$$

$h, \tilde{h}$: quasi-particle hamiltonian

λ : chemical potential

E_i : quasi-particle energy

- Energy Density Functionals(EDF)

$$E(\rho,\tilde{\rho}) = \int d\vec{r}\,\varepsilon(\rho,\tilde{\rho}) \qquad \varepsilon(\rho,\tilde{\rho}) = \varepsilon_{\text{kin+int}}(\rho) + \varepsilon_{\text{pair}}(\rho,\tilde{\rho})$$

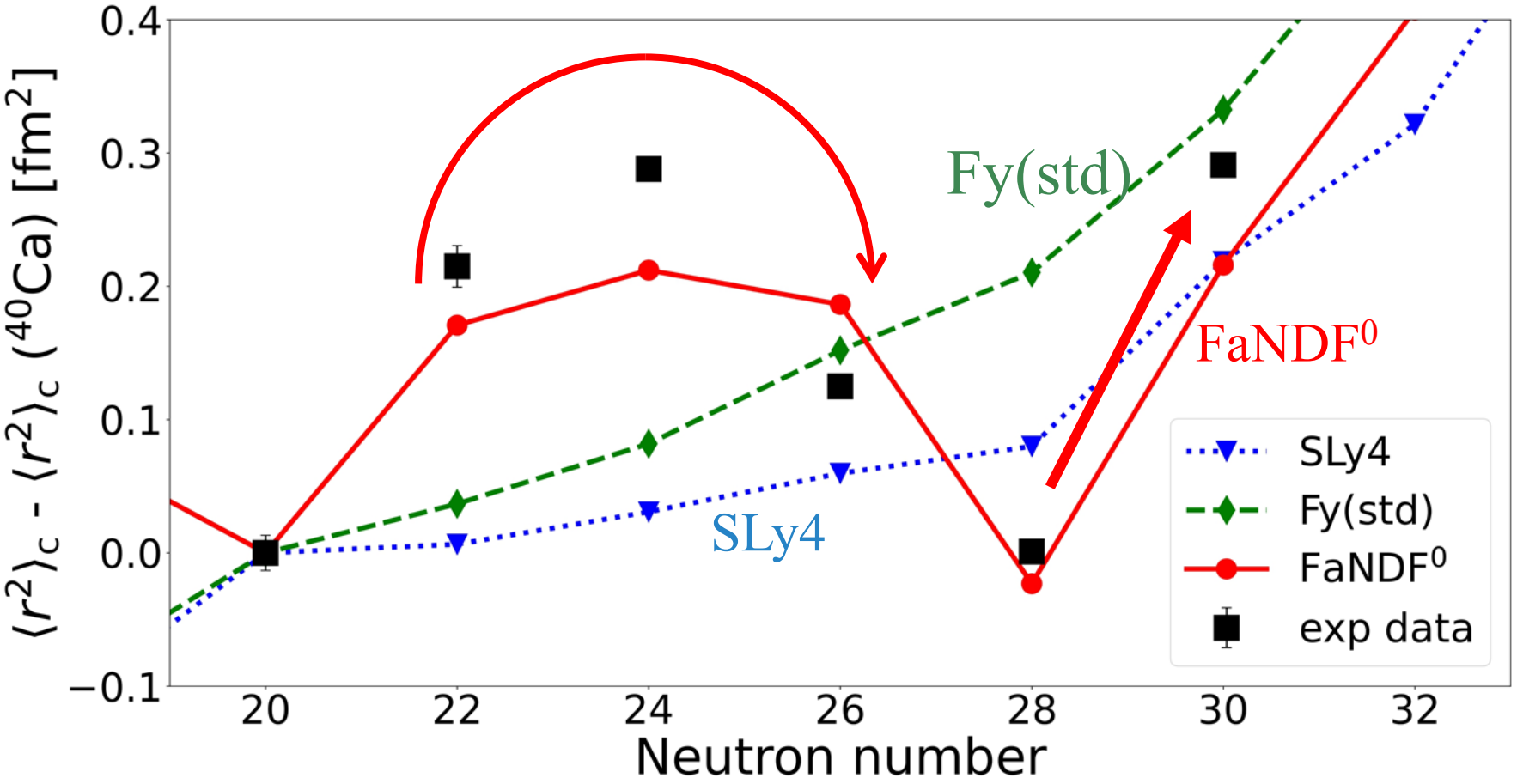
We used EDF models for calculating charge radius.

- Skyrme type(SLy4, SkM*)
- Fayans type(FaNDF^0, Fy(std))

Fayans type has the differential term for pairing density functionals.

Ca isotopes

Fayans(FaNDF^0) reproduces the isotope dependence on Ca isotopes.

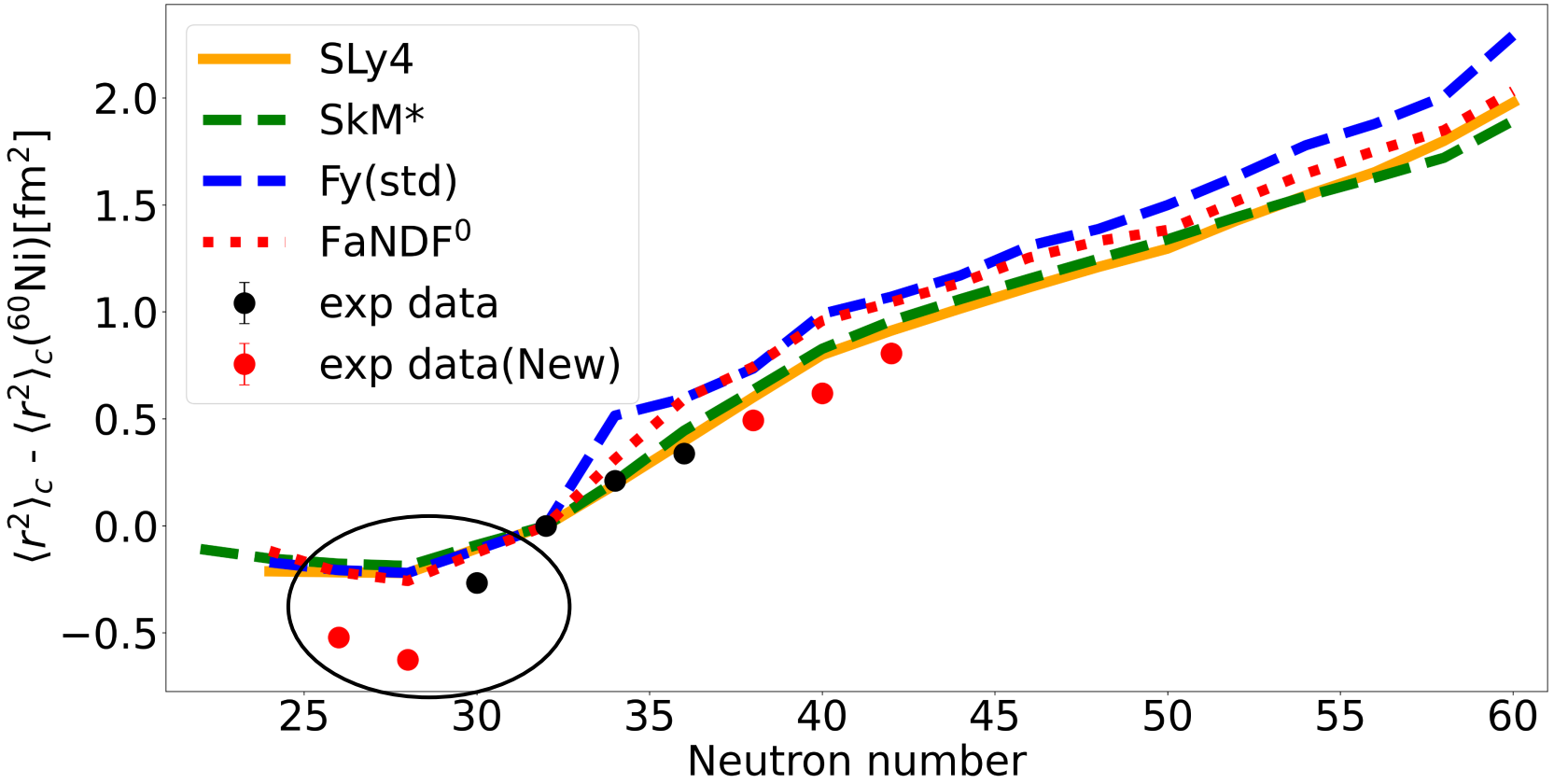


- ✖ SLy4
- ✖ Fy(std)
- ✓ FaNDF^0

The difference term for pairing density functionals plays important role.

Ni isotopes

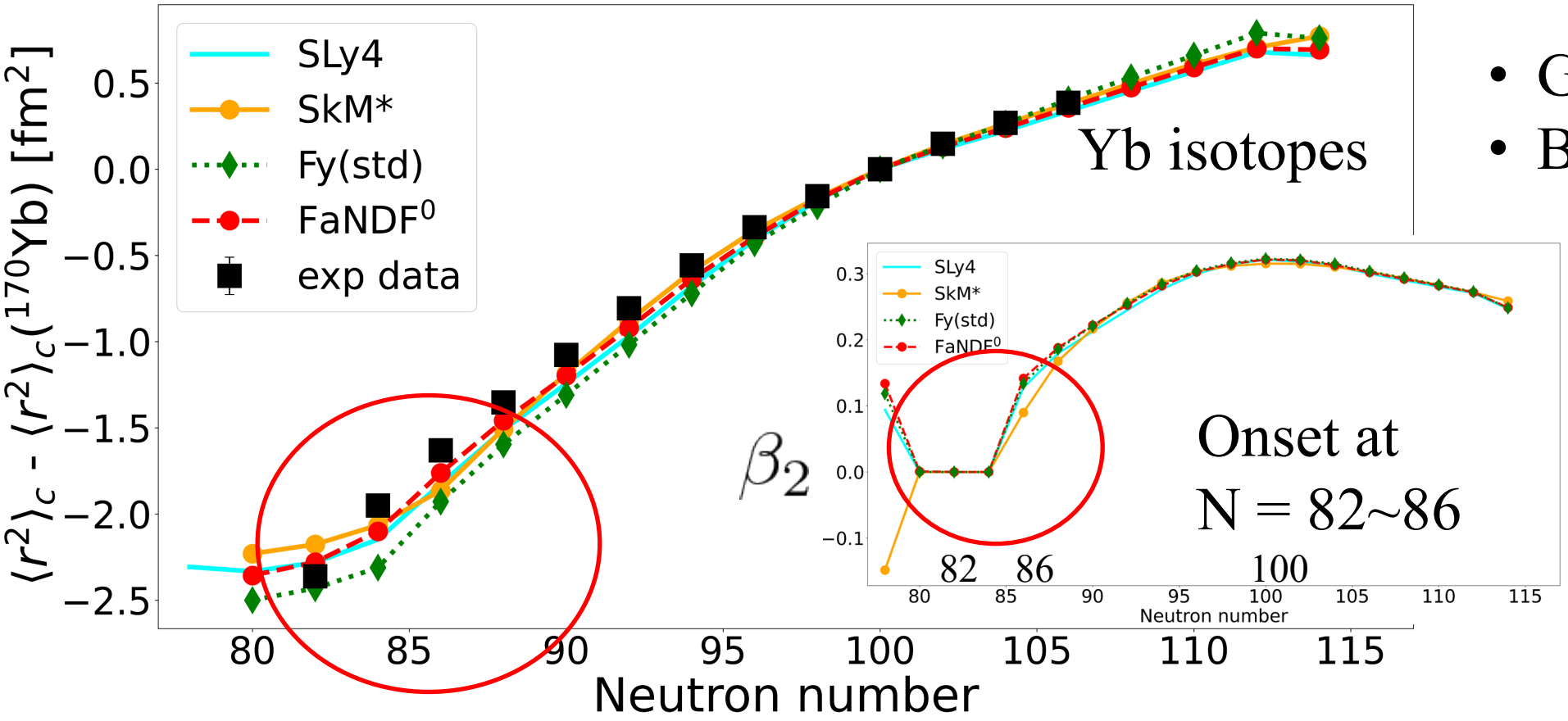
Recently, experimental results of $^{54\text{-}70}$Ni are obtained by laser spectroscopy.
[F. Sommeret al., PRL 129, 132501 (2022).] [S. Malbrunot-Ettenauer *et al.*, Phys. Rev. Lett. 128.022502(2022).]



Calculation cannot reproduce isotope dependence near 28 unlike Ca isotopes.

Yb isotopes

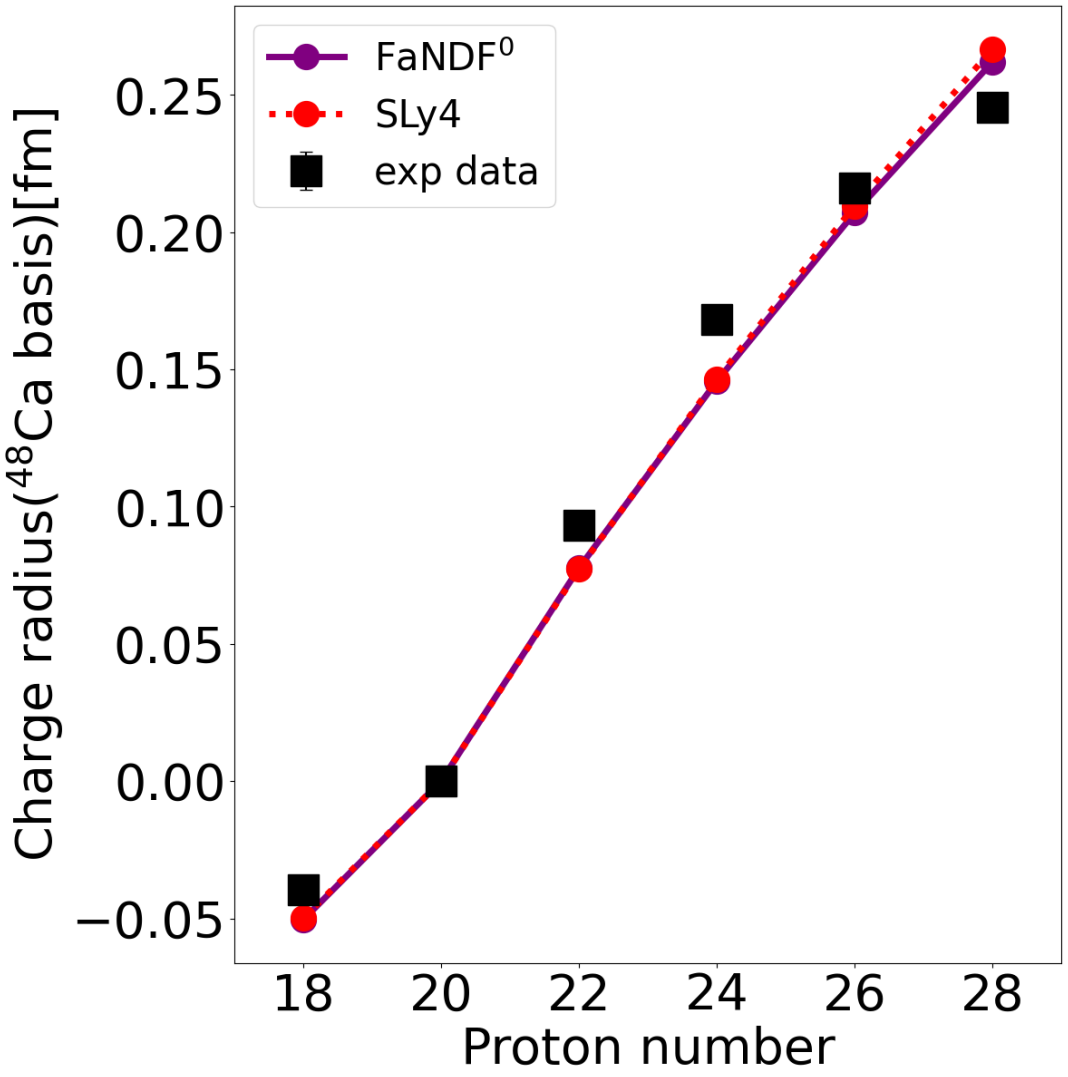
The charge moments depend on deformation with neutron number.



- Good at N~100
- Bad at N = 82~86

The rapid change happens by mean-field calculation.

N = 28 Isotones



Fayans(FaNDF^0) can reproduce isotone dependence.

Nuclear shape is spherical by mean-field calculation.

Deformation effect beyond mean-field approximation is required.

[B. A. Brown and K. Minamisono Phys. Rev. C 106, L011304 (2022).]

Conclusion

We calculated the charge moments in Ca, Ni, Yb isotopes and N = 28 isotones used for mean-field theory.

- This results :
  - Ca isotopes : FaNDF^0 reproduce isotope dependence.
  - Ni isotopes : Calculation cannot reproduce kink near magic number 28.
  - Yb isotopes : The development of deformation has influence on charge radius.
  - N = 28 isotones : The isotones have a spherical shape within mean-field approximation.

    → Deformation effect may be required like Yb isotopes.
- Future work :

  It is necessary to develop a method beyond mean-field approximation.

  We will contribute to the search of new boson.