Nuclear reaction cross sections using Gamow shell model

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Introduction

Introduction

- Schematic diagram of various aspects of physics important in neutron-rich nuclei

Open Quantum System:
Weakly bound
& unbound nuclei

|2| MeV ≥ S_n
Weakly bound nuclei

Closed Quantum System:
Well bound nuclei

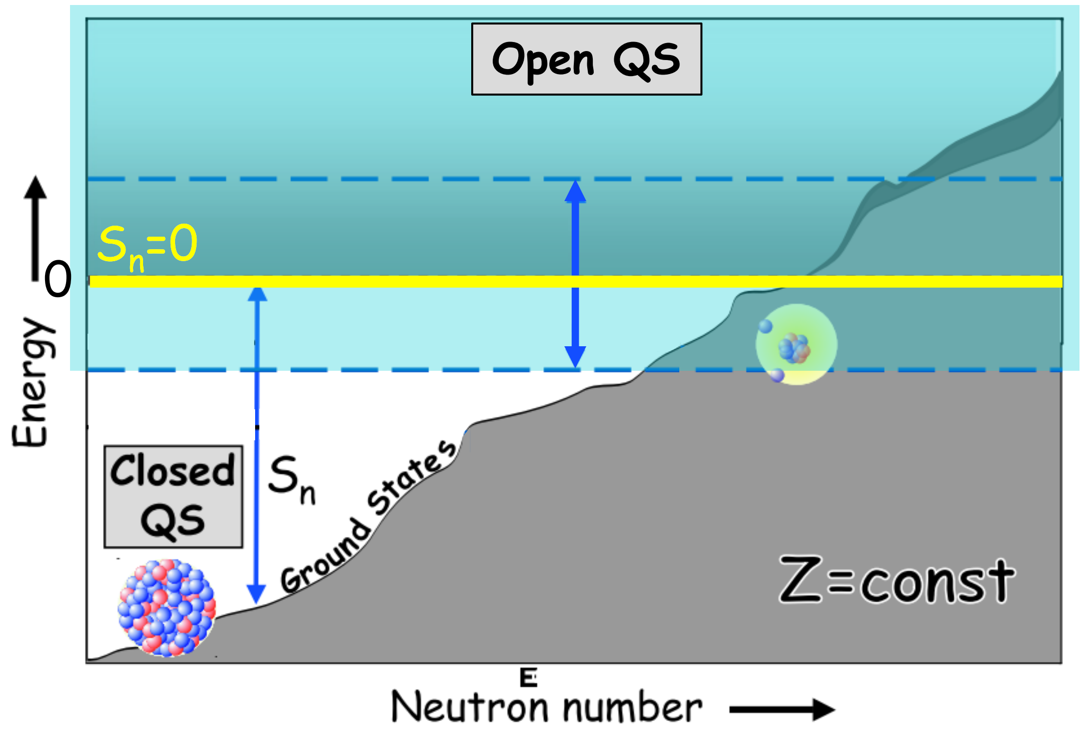


Fig.1

Gamow shell model (GSM)

: a particular realization of the continuum shell model (CSM) on the **Berggren ensemble.**

Gamow states and Berggren Completeness

Complex-energy eigenstates; $\tilde{E}_n = E_n - i\frac{\Gamma_n}{2}$

complex-momentum
$$\tilde{E} = \frac{\hbar^2}{2m}k^2$$

- The resonant states and bound states are identified with the poles of the complex-momentum plane.

$$k_n = \gamma_n - i\kappa_n \rightarrow e^{ik_n r} = e^{\kappa_n r}(where, Re[k] = 0)$$

$$\begin{cases} \text{bound states;} & Im[k] > 0 \\ \text{antibound states;} & Im[k] < 0 \end{cases}$$

- More generalized Berggren Completeness

$$\sum_{n\in(a,b,d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)|dk = 1$$

an integral over non-resonant scattering states from the contour L^+

sum runs over
antibound (weakly bound) states (a)
+ all bound (b)
+ resonant decaying (d) states

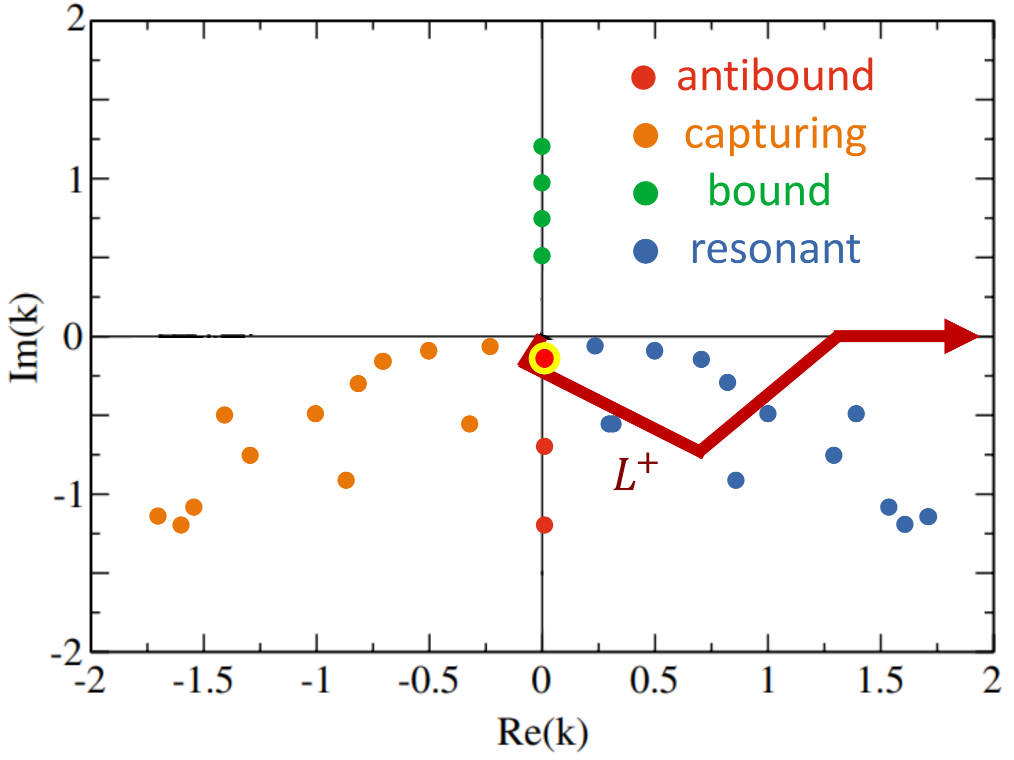


Fig.2

Resonant state in GSM

$$i\hbar \frac{\partial}{\partial t}\chi(\mathbf{r},t) = \hat{h}\chi(\mathbf{r},t) \;\rightarrow\; \chi(\mathbf{r},t) = \tau(t)\psi(\mathbf{r}),$$

time-dependent eigenfunction

$$\tau(t) = \mathrm{e}^{-i\frac{\tilde{E}_n}{\hbar}t} = \mathrm{e}^{-i\frac{E_n}{\hbar}t}\mathrm{e}^{-\frac{\Gamma_n}{2\hbar}t}$$

$$\rightarrow \mathrm{e}^{-i\frac{\tilde{E}_n}{\hbar}t} \propto \mathrm{e}^{-\frac{\Gamma_n}{2\hbar}t}$$

exponential temporal decrease

space-dependent eigenfunction

$$\psi_{\mathrm{n}} = \psi_{\mathrm{n}ljm}(\mathbf{r},k_n) = \frac{u_{nlj}(k_n,r)}{r}[Y_l(\hat{r})\chi_s]_{jm}$$

$$\rightarrow \mathrm{e}^{ik_n r} = \mathrm{e}^{i\gamma_n r}\mathrm{e}^{\kappa_n r} \propto \mathrm{e}^{\kappa_n r}$$

exponential spatial increase

- The divergence of the resonance wavefunction assures that the particle number is conserved.

Capture process
& resonance in unbound nucleus

Hamiltonian of the GSM

$$\hat{H} = \sum_{i=1}^{N_{val}} \left(\frac{\hat{\vec{p}}_i^2}{2\mu_i} + U_c(\hat{r}_i) \right) + \sum_{i<j}^{N_{val}} \left(V(\hat{\vec{r}}_i - \hat{\vec{r}}_j) + \frac{\hat{\vec{p}}_i \cdot \hat{\vec{p}}_j}{M_c} \right) = \hat{U}_{basis} + \hat{T} + \hat{V}_{res}$$

$$\hat{U}_{basis} = \sum_{i=1}^{N_{val}} \left(U_c(\hat{r}_i) + U(\hat{r}_i) \right)$$

$$\hat{V}_{res} = \sum_{i<j}^{N_{val}} \left(V(\hat{\vec{r}}_i - \hat{\vec{r}}_j) + \frac{\hat{\vec{p}}_i \cdot \hat{\vec{p}}_j}{M_c} \right) - \sum_{i<j}^{N_{val}} U(\hat{r}_i)$$

$$U_c(\hat{r}_i) = V_{Coulomb} + -V_O f(r) - V_{SO} 4\vec{l} \cdot \vec{s} \frac{1}{r} \frac{df(r)}{dr}$$

→ **s.p. potential of Woods-Saxon**

$U(\hat{r}_i)$: one-body mean-field

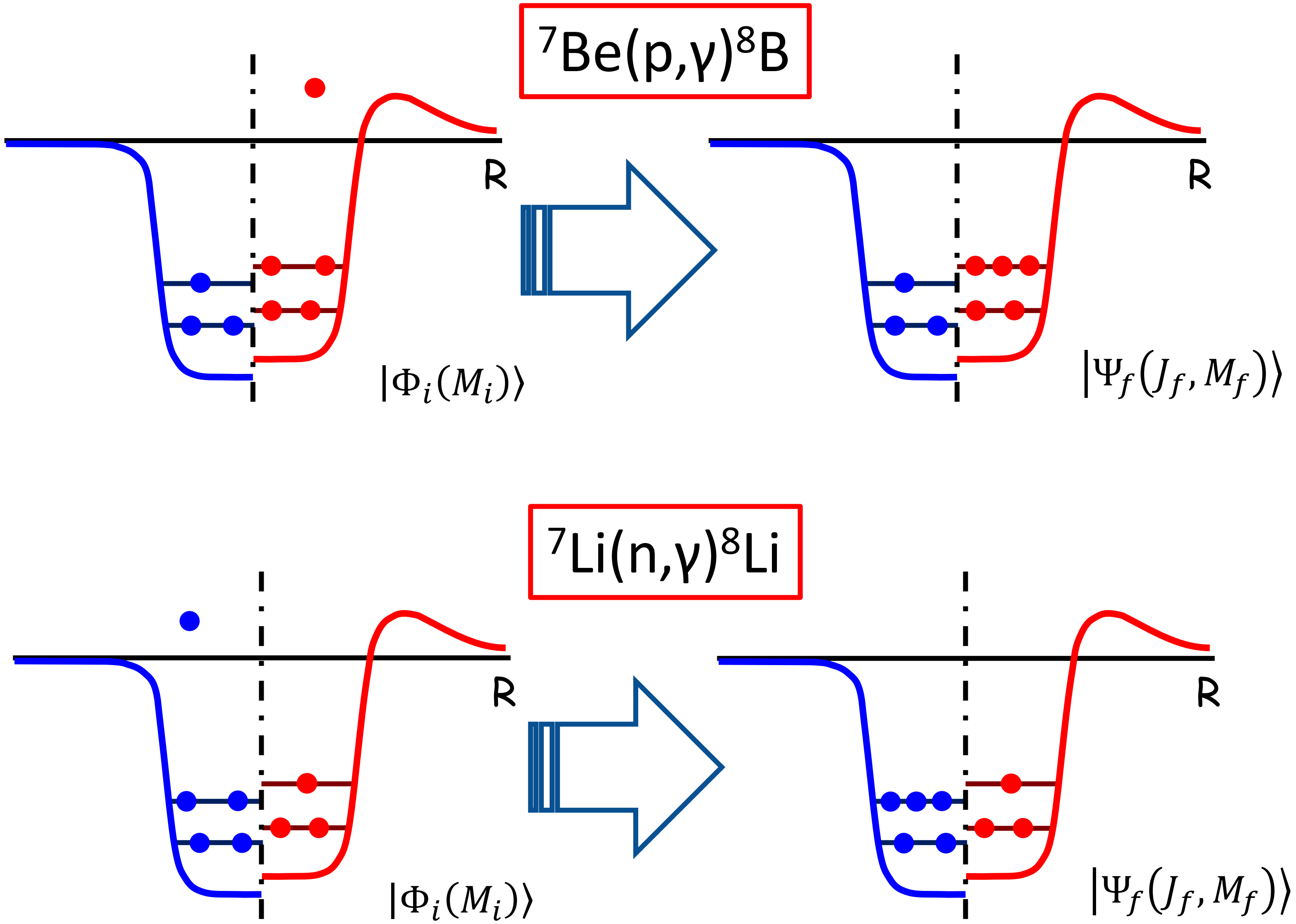
$$V(\hat{\vec{r}}_i - \hat{\vec{r}}_j) = V_{ij}^{C} + V_{ij}^{SO} + V_{ij}^{T} + V_{ij}^{Co}$$

→ **Furutani-Horiuchi-Tamagaki (FHT) finite-range two-body interaction**

V_{ij}^{C}: central V_{ij}^{SO}: spin-orbit

V_{ij}^{T}: tensor V_{ij}^{Co}: Coulomb

Radiative proton & neutron capture process



$^{3}He+^{4}He \rightarrow ^{7}Be+ \gamma$

$^{7}Be+p^{+} \rightarrow ^{8}B+\gamma$

$^{7}B \rightarrow ^{8}Be^{*}+e^{+}+\nu_e$

$^{8}Be^{*} \rightarrow ^{4}He+^{4}He$

ppIII

$^{7}Be+e^{-} \rightarrow ^{7}Li+\nu_e$

$^{7}Li+p^{+} \rightarrow ^{4}He+^{4}He$

ppII

Cross-sections of radiative capture using GSM

Differential cross section

$$\frac{d\sigma}{d\Omega_\gamma} = \frac{1}{8\pi}\left(\frac{k_\gamma}{k}\right)\left(\frac{e^2}{\hbar c}\right)\left(\frac{\mu_u c^2}{\hbar c}\right)\frac{1}{2s+1}\frac{1}{2J_{targ}+1}$$
$$\times \sum_{M_i,M_f,M_{targ},M_L,P,m_s}\left|\sum_L g^L_{M_L,P}(k,k_\gamma,\varphi_\gamma,\theta_\gamma)\langle\Psi_f(J_f,M_f)|\widehat{\mathcal{M}}_{L,M_L}|\Phi_i(M_i)\rangle\right|^2$$

[$g^L_{M_L,P}(k,k_\gamma,\varphi_\gamma,\theta_\gamma) = i^L\sqrt{2\pi(2L+1)}\left(\frac{k^L_\gamma}{k}\right)\sqrt{\frac{L+1}{L}}\frac{P}{(2L+1)!!}D^L_{M_L P}(\phi_\gamma,\theta_\gamma,0)$, $D^L_{M_L P}(\phi_\gamma,\theta_\gamma,0)$; Wigner D-matrix]

Total cross section

$$\sigma(E_{c.m.}) = \sum_{J_f}\int_0^{2\pi} d\phi_\gamma \int_0^{\pi}\sin\theta_\gamma\, d\theta_\gamma\, \frac{d\sigma_{J_f}(E_{c.m.},\theta_\gamma,\phi_\gamma)}{d\Omega_\gamma}$$

Astrophysical factor

$$S(E_{c.m.}) = \sigma(E_{c.m.})E_{c.m.}e^{2\pi\eta}$$

Channel states expansion in the Berggren basis

Target structure state

$$|c_{targ}\rangle = \sum_i \langle SD_i^{(A-1)} | c_{targ} \rangle \, |SD_i^{(A-1)}\rangle$$

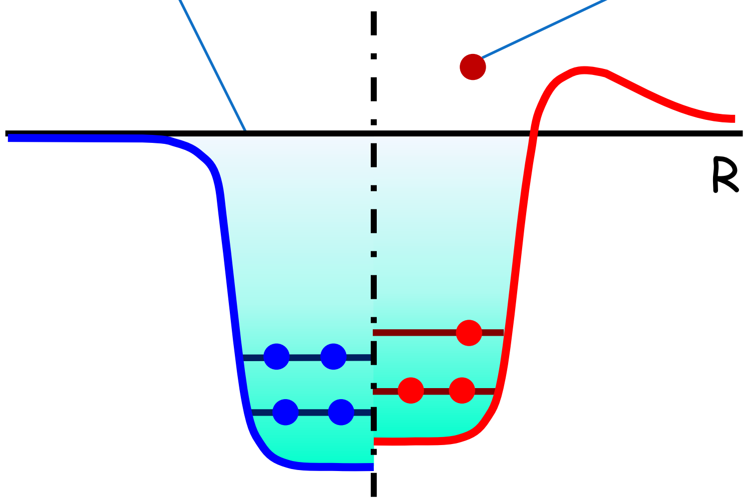
Projectile state

$|\phi_i^{rad}\rangle$: projectile radial part

$|c_{proj}\rangle$: projectile angular part

$$|\phi_{i;c_{proj}}\rangle = \hat{\mathcal{A}}\left(|\phi_i^{rad}\rangle \otimes |c_{proj}\rangle\right) = \hat{\mathcal{A}}\left(|\phi_i^{rad}\rangle \otimes |l,s;j,m_j\rangle\right)$$

$$\rightarrow |r, c_{proj}\rangle = \sum_i \frac{u_i(r)}{r} |\phi_{i;c_{proj}}\rangle$$



Target state $\otimes$ Projectile state

Channel basis state $\{|c\rangle\} \equiv \{|c_{proj}; c_{targ}\rangle\}$

$$|\phi_i^{rad}, c\rangle = \hat{\mathcal{A}}\left(|\phi_i^{rad}\rangle \otimes |c\rangle\right)$$

$$\rightarrow |\boldsymbol{r}, \boldsymbol{c}\rangle = \sum_i \frac{u_i(r)}{r} |\phi_i^{rad}, c\rangle = \sum_i \frac{u_i(r)}{r} \hat{\mathcal{A}}\left(|\phi_i^{rad}\rangle \otimes |c\rangle\right)$$

Astrophysical factor for $^{7}Be(p,\gamma)^{8}B$

- Two peaks: 1^{+}_{1} and 3^{+}_{1} unbound excited states of ^{8}B.

—— fully antisymmetrized CC GSM calculation

- - - - non antisymmetrized CC GSM calculation

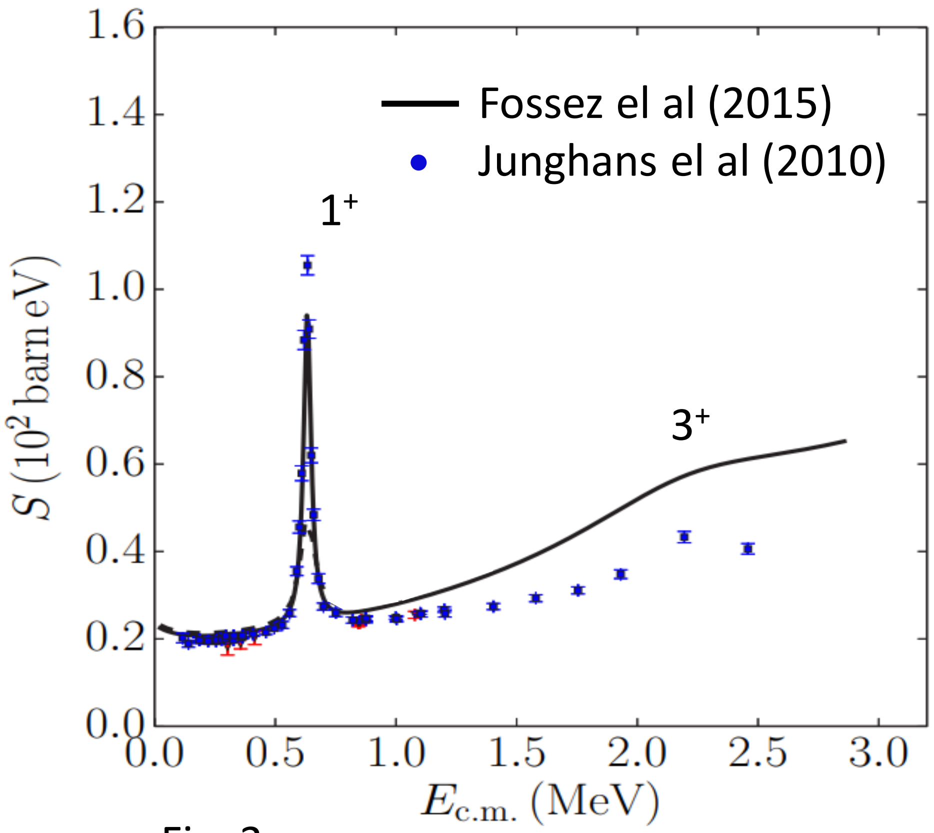
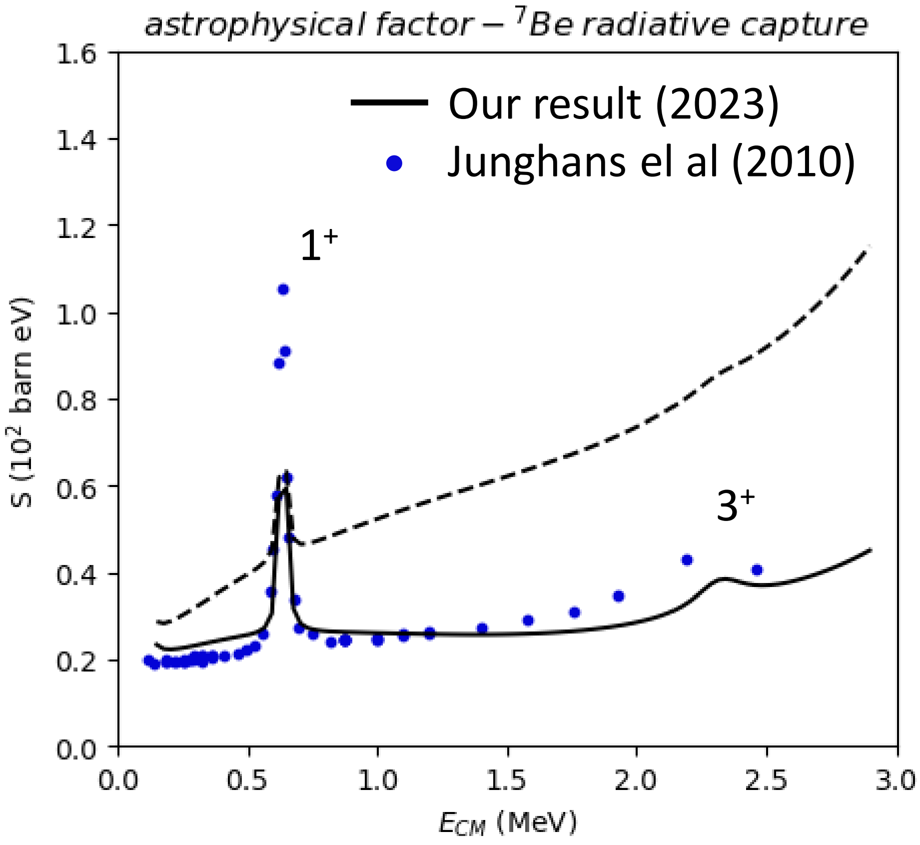


Fig. 3



Cross sections for $^7Li(n,\gamma)^8Li$

- The peak: 3^+_1 resonance of ^8Li.

—— fully antisymmetrized CC GSM calculation
- - - - non antisymmetrized CC GSM calculation

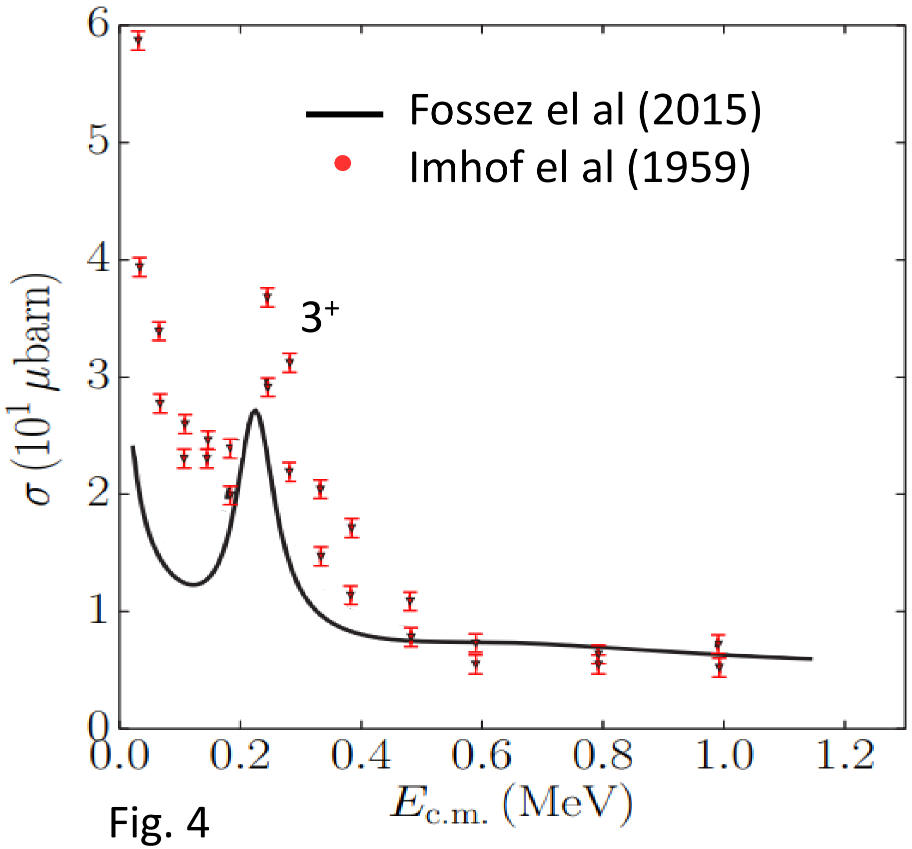
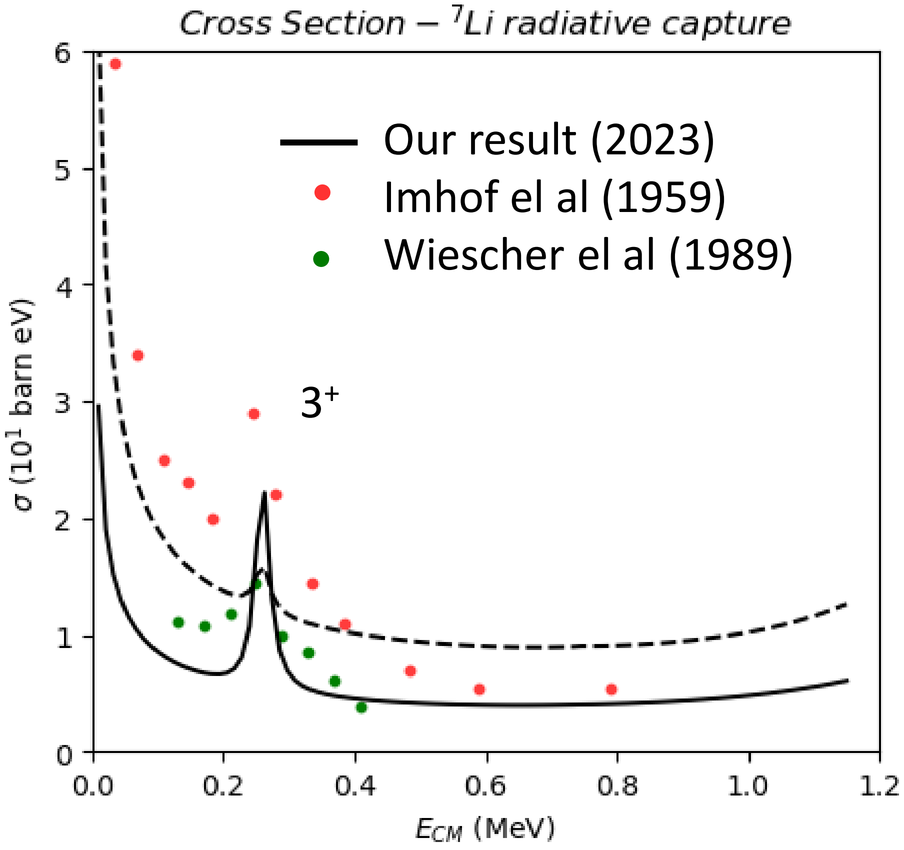
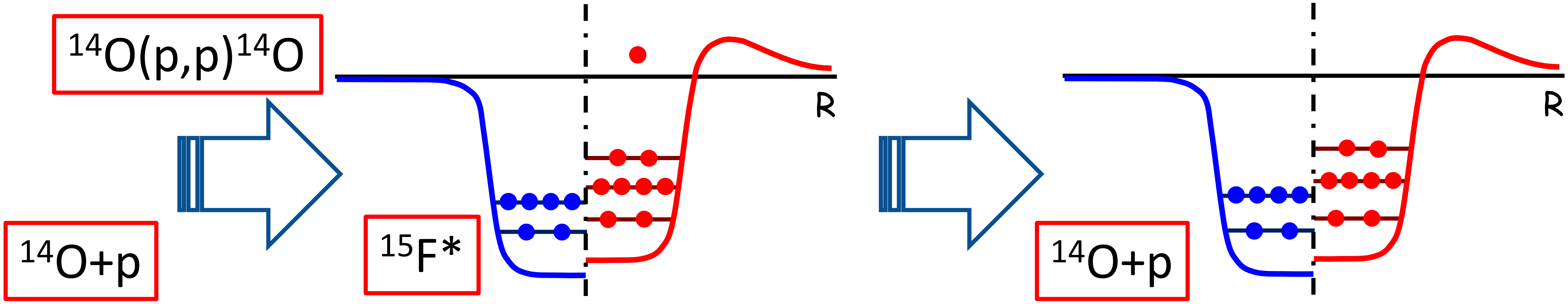
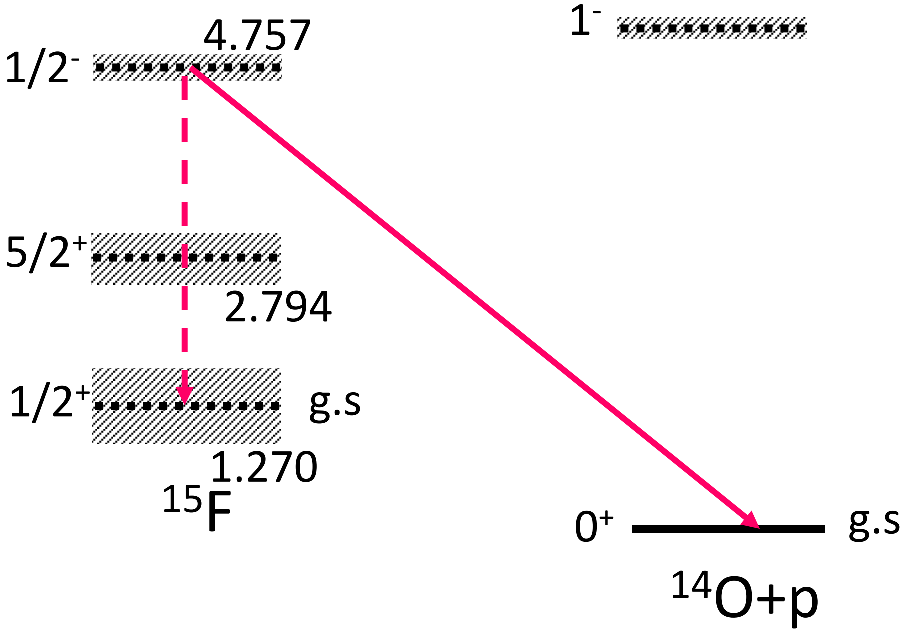


Fig. 4



Resonance in ^{15}F





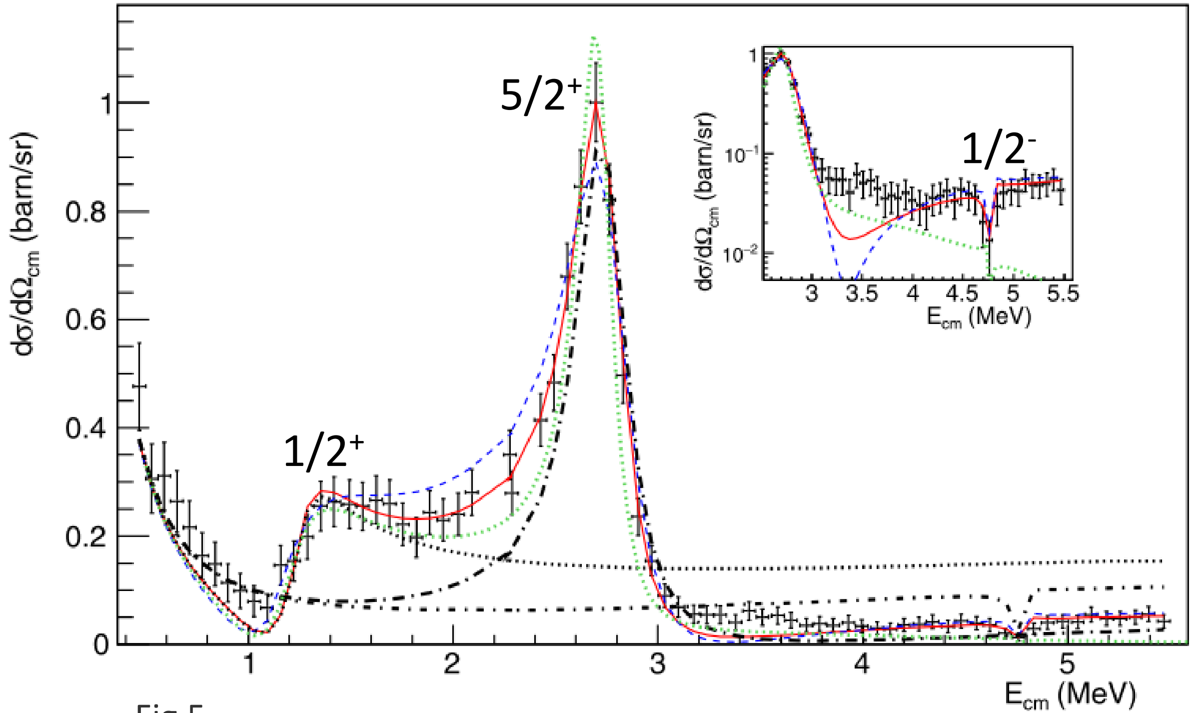


Fig.5

Resonance in ^{15}F : ^{14}O(p,p')^{14}O*

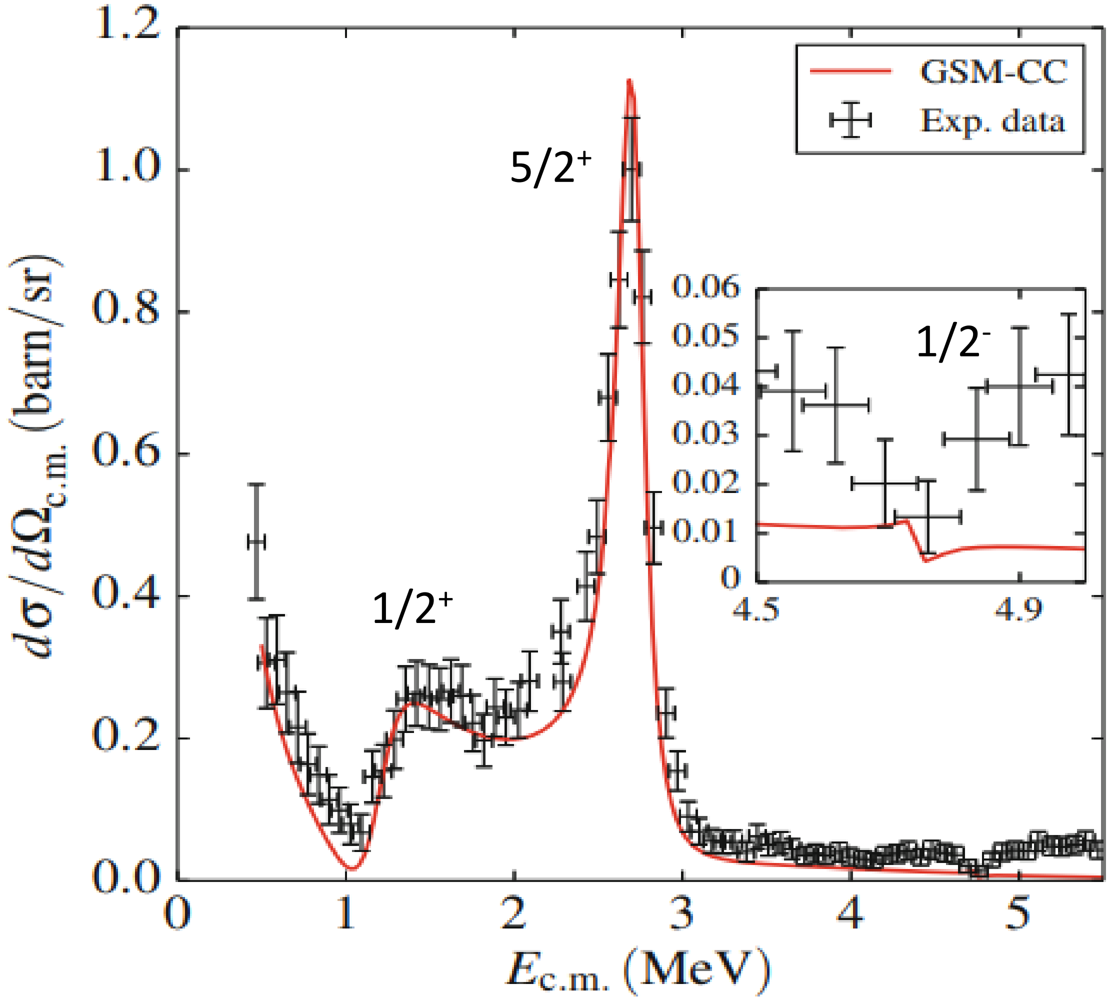
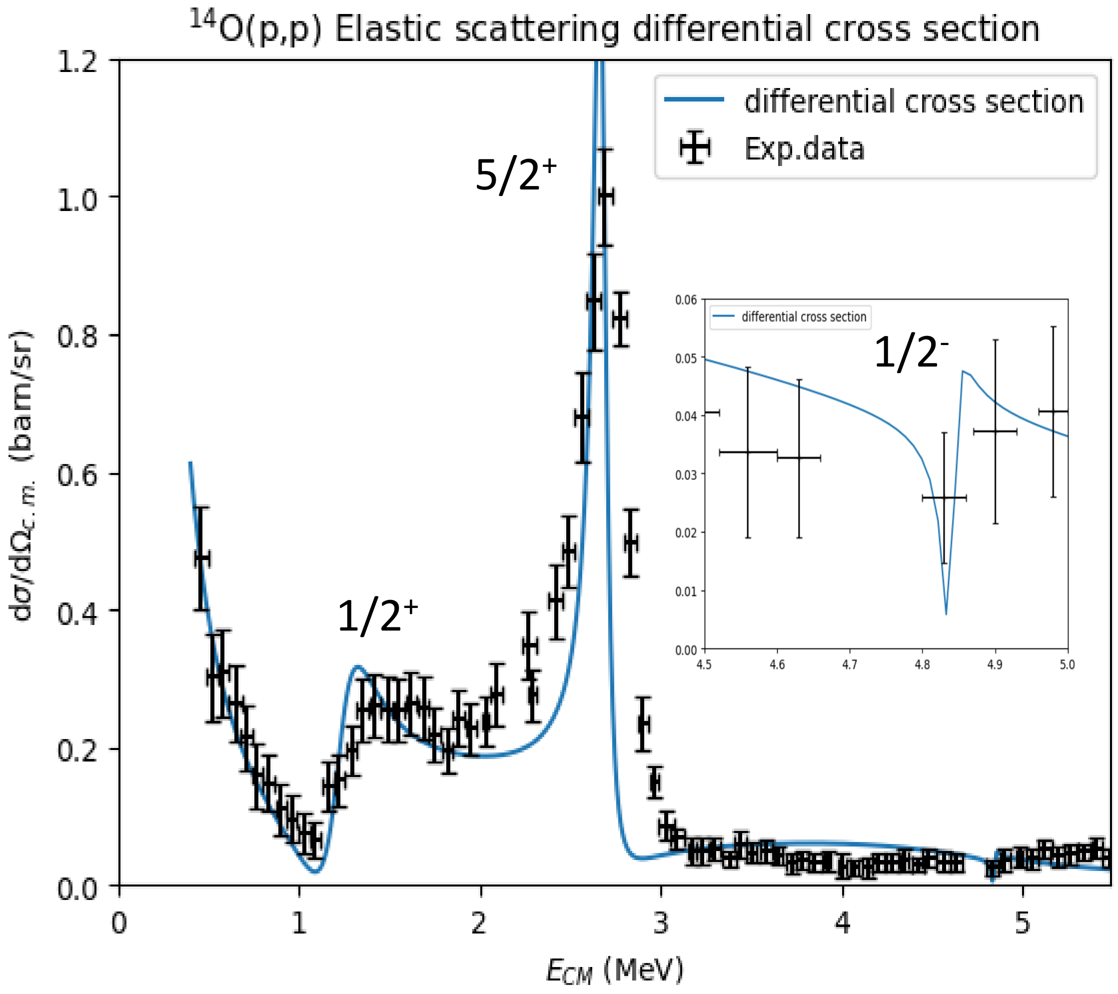


Fig. 6



Our calculation

Summary

- **Gamow shell model (GSM)**
  - Introduce **complex-energy eigenstates** to treat antibound, bound, resonant, and scattering states on the same footing.
  - All states belong to Berggren completeness relation, so we can write GSM formulism in rigged Hilbert space.
- We used the GSM to explain **^{7}Be(p,γ)^{8}B and ^{7}Li(n,γ)^{8}Li, and ^{14}O(p,p)^{14}O reaction cross sections** and our results reproduce the peaks of the experimental data overall well.
- The **GSM** explains not only nuclear reactions but also nuclear structures, simultaneously.

Thank you for your attention

reference

thank you
for
your attention

Fig. 1 N Michel *et al* 2009 *J. Phys. G: Nucl. Part. Phys.* **36** 013101

Fig. 2 N Michel *et al* 2009 *J. Phys. G: Nucl. Part. Phys.* **36** 013101

Fig. 3 K. Fossez, N. Michel, M. Płoszajczak, Y. Jaganathen, R.M. Id Betan, Phys. Rev. C 91, 034609 (2015)

Fig. 4 Fig. 6 N Michel and M Płoszajczak, *Gamow Shell Model*(Springer, 2021)

Fig. 5 F. de Grancey *et al* Phys. Lett. B 758, 26 (2016)

PHYSICAL REVIEW C **91**, 034609 (2015)

Description of the proton and neutron radiative capture reactions in the Gamow shell model

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We formulate the Gamow shell model (GSM) in coupled-channel (CC) representation for the description of proton/neutron radiative capture reactions and present the first application of this new formalism for the calculation of cross sections in mirror reactions ^{7}Be$(p,\gamma)^8$B and ^{7}Li$(n,\gamma)^8$Li. The GSM-CC formalism is applied to a translationally invariant Hamiltonian with an effective finite-range two-body interaction. Reactions channels are built by GSM wave functions for the ground state $3/2^-$ and the first excited state $1/2^-$ of ^{7}Be/^{7}Li and the proton/neutron wave function expanded in different partial waves.

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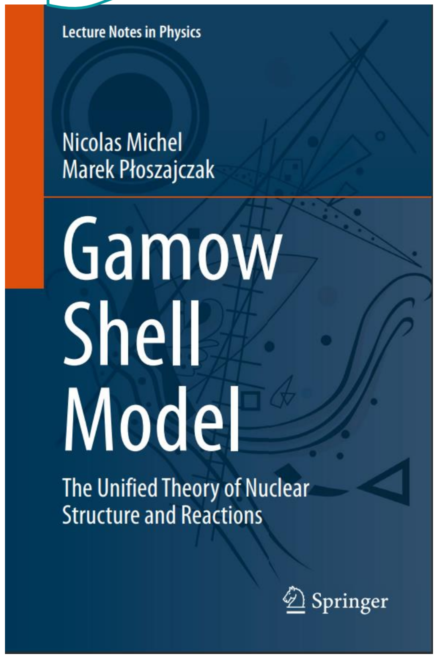

IOP PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS
J. Phys. G: Nucl. Part. Phys. **36** (2009) 013101 (40pp) doi:10.1088/0954-3899/36/1/013101

TOPICAL REVIEW

Shell model in the complex energy plane

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Abstract
This work reviews foundations and applications of the complex-energy continuum shell model that provides a consistent many-body description of bound states, resonances and scattering states. The model can be considered a quasi-stationary open quantum system extension of the standard configuration interaction approach for well-bound (closed) systems.