Young Scientist Session, CNS Summer School, August 7th, 2023

Cooper quartet correlations in infinite symmetric nuclear matter

YG, Tajima, and Liang, Phys. Rev. C **105** (2022) 024317
YG, Tajima, and Liang, Phys. Rev. Res. **4** (2022) 023152

Yixin Guo (郭一昕)

Department of Physics, The University of Tokyo, Japan

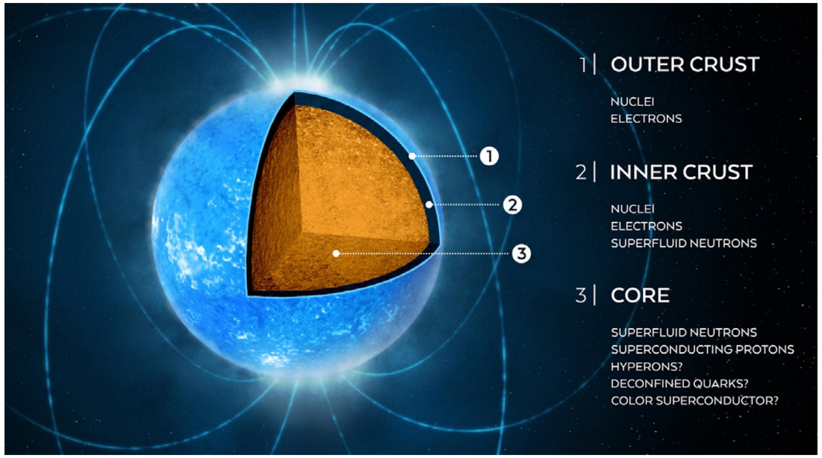
RIKEN iTHEMS, Japan

Collaborators: Hiroyuki Tajima (UTokyo)
Haozhao Liang (UTokyo)

Introduction

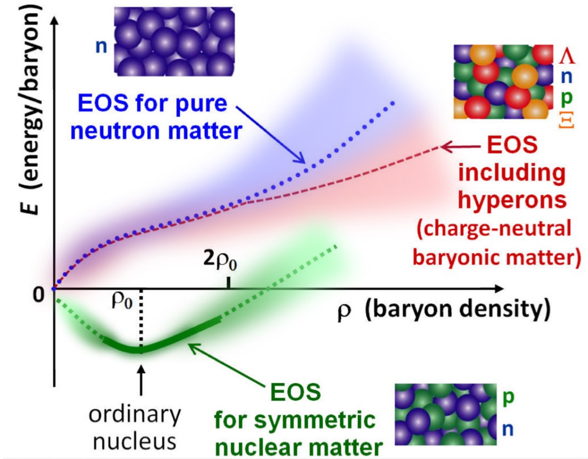
Nuclear matter is useful for the correct description of neutron stars

Neutron star interior



Rev. Mod. Phys. 88 (2016), 021001

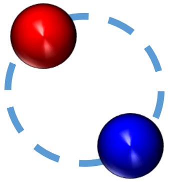
Nuclear equation of state (EOS)



Tamura, JPS Conf. Proc. 1 (2014) 011003

Introduction

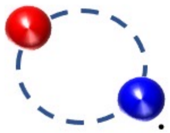
Bardeen-Cooper-Schrieffer (BCS) theory: the Fermi-surface instability.



Cooper pair

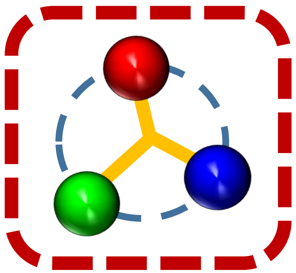
Bardeen, Cooper, and Schrieffer, Phys. Rev. 108 (1957) 1175

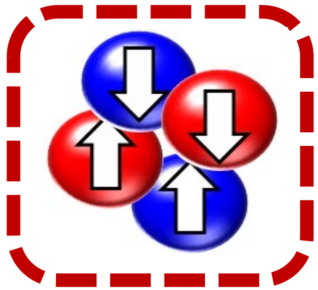
Generalized Cooper problems



Cooper triple

PRA 86 (2012) 013628





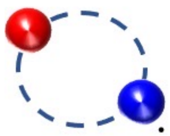
Alpha-like particle

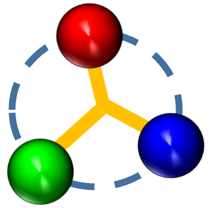
Cooper quartet

PRL 80 (1998) 3177
PRC 85 (2012) 061303
PLB 805 (2020) 135462

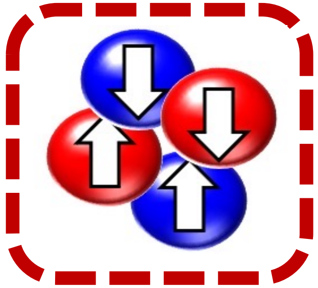
Introduction

Generalized Cooper problems





Usually treated as “extra” particles, but microscopically they should be treated as in-medium correlations

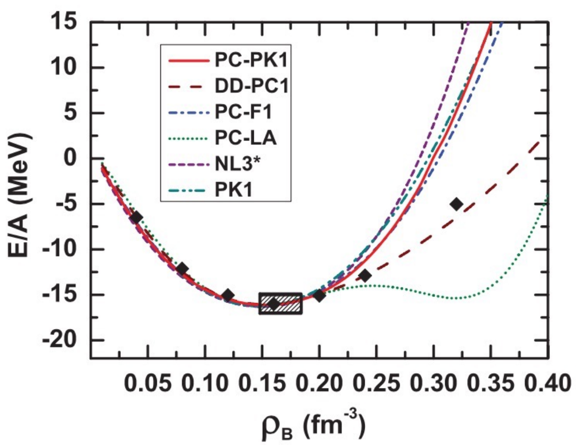


Alpha-like particle

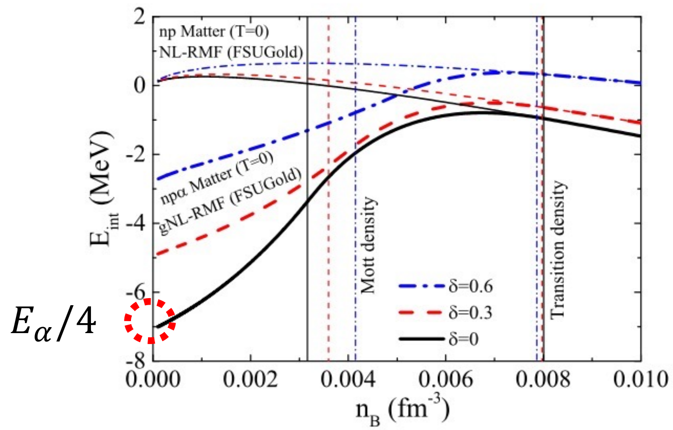
Cooper quartet

PRL 80 (1998) 3177
PRC 85 (2012) 061303
PLB 805 (2020) 135462

Nuclear EOS (Relativistic mean-field calculations)



PRC 82 (2010) 054319



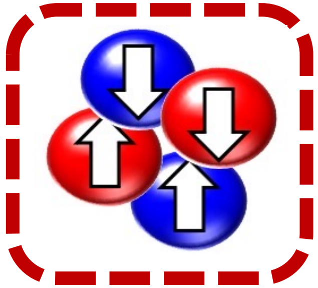
PRC 100 (2019) 054304

$$\mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_\alpha + \mathcal{L}_{\rm meson}$$

$$\mathcal{L}_\alpha = \frac{1}{2}\left(iD^\mu_\alpha \phi_\alpha\right)^* \left(iD_{\mu\alpha}\phi_\alpha\right) - \frac{1}{2}\phi^*_\alpha (M^*_\alpha)^2 \phi_\alpha$$

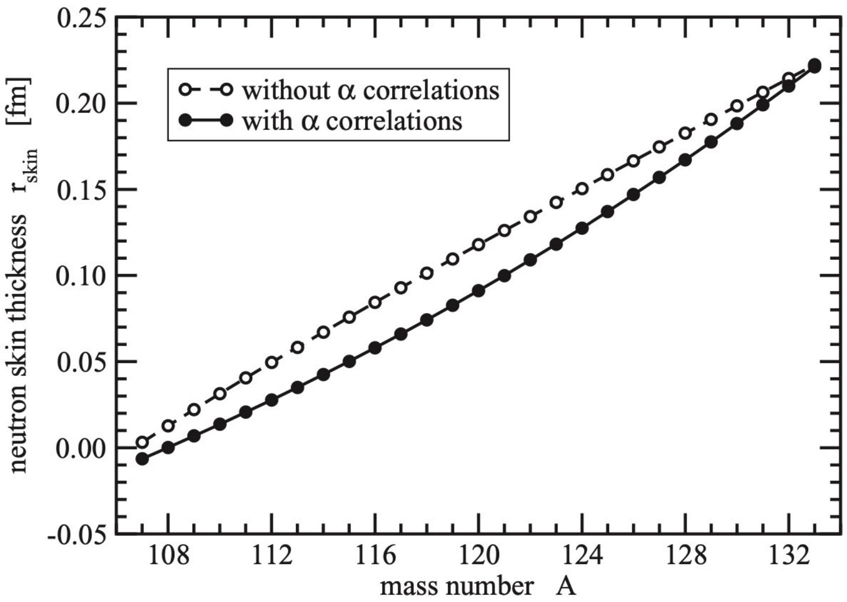
Introduction

Alpha-like Cooper quartet correlations

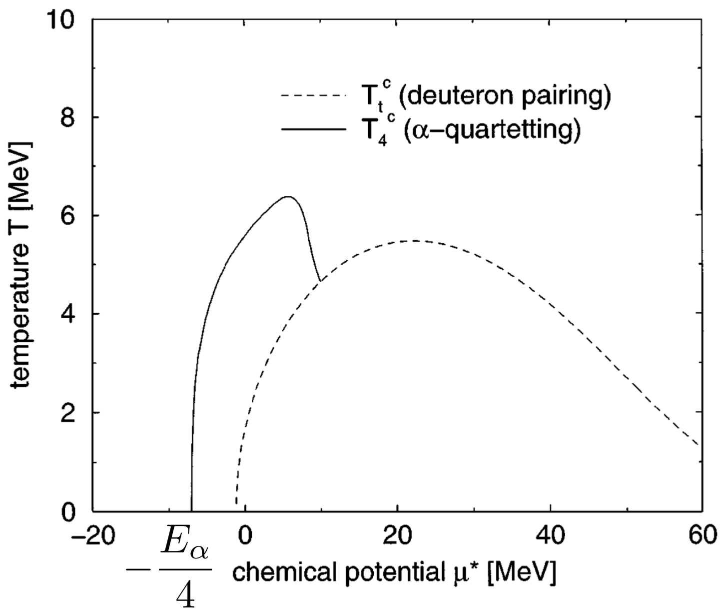


$$r_{\text{skin}} = r_{\text{n}} - r_{\text{p}}$$

Critical temperature in symmetric nuclear matter



Typel, Phys. Rev. C 89 (2014) 064321



G. Röpke, et al., Phys. Rev. Lett. 80 (1998) 3177

Hamiltonian

$$H = \sum_{\boldsymbol{p}, s_z} \left(\varepsilon_{\nu,\boldsymbol{p}} \nu^\dagger_{\boldsymbol{p},s_z} \nu_{\boldsymbol{p},s_z} + \varepsilon_{\pi,\boldsymbol{p}} \pi^\dagger_{\boldsymbol{p},s_z} \pi_{\boldsymbol{p},s_z} \right)$$

$$+ \frac{1}{2} \sum_{\boldsymbol{P},\boldsymbol{q},\boldsymbol{q}'} \sum_{T_3=-1}^{+1} P^\dagger_{1,T_3}(\boldsymbol{P},\boldsymbol{q}) V_s(\boldsymbol{q},\boldsymbol{q}') P_{1,T_3}(\boldsymbol{P},\boldsymbol{q}')$$

Isovector interaction (T=1, S=0)

$$+ \frac{1}{2} \sum_{\boldsymbol{P},\boldsymbol{q},\boldsymbol{q}'} \sum_{S_z=-1}^{+1} D^\dagger_{1,S_z}(\boldsymbol{P},\boldsymbol{q}) V_t(\boldsymbol{q},\boldsymbol{q}') D_{1,S_z}(\boldsymbol{P},\boldsymbol{q}')$$

Isoscalar interaction (T=0, S=1)

$\varepsilon_{\nu(\pi),\boldsymbol{p}} = \frac{p^2}{2M} - \mu_{\nu(\pi)}$: nucleon kinetic energy

$\nu^{(\dagger)}_{\boldsymbol{p},s_z}$: neutron annihilation (creation) operator

$\pi^{(\dagger)}_{\boldsymbol{p},s_z}$: proton annihilation (creation) operator

Interactions

Isovector interaction (T=1, S=0)

$$\frac{1}{2}\sum_{\boldsymbol{P},\boldsymbol{q},\boldsymbol{q}'}\sum_{T_3=-1}^{+1} P^{\dagger}_{1,T_3}(\boldsymbol{P},\boldsymbol{q})V_s(\boldsymbol{q},\boldsymbol{q}')P_{1,T_3}(\boldsymbol{P},\boldsymbol{q}')$$

Isoscalar interaction (T=0, S=1)

$$\frac{1}{2}\sum_{\boldsymbol{P},\boldsymbol{q},\boldsymbol{q}'}\sum_{S_z=-1}^{+1} D^{\dagger}_{1,S_z}(\boldsymbol{P},\boldsymbol{q})V_t(\boldsymbol{q},\boldsymbol{q}')D_{1,S_z}(\boldsymbol{P},\boldsymbol{q}')$$

<u>Spin-singlet *NN* pair operator</u>

$$P_{1,+1}(\boldsymbol{P},\boldsymbol{q}) = \frac{\sqrt{2}}{2}\left(\nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}} - \nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}}\right),$$

$$P_{1,0}(\boldsymbol{P},\boldsymbol{q}) = \frac{1}{2}\left(\nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}} - \pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}} + \pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}} - \nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}}\right),$$

$$P_{1,-1}(\boldsymbol{P},\boldsymbol{q}) = \frac{\sqrt{2}}{2}\left(\pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}} - \pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}}\right)$$

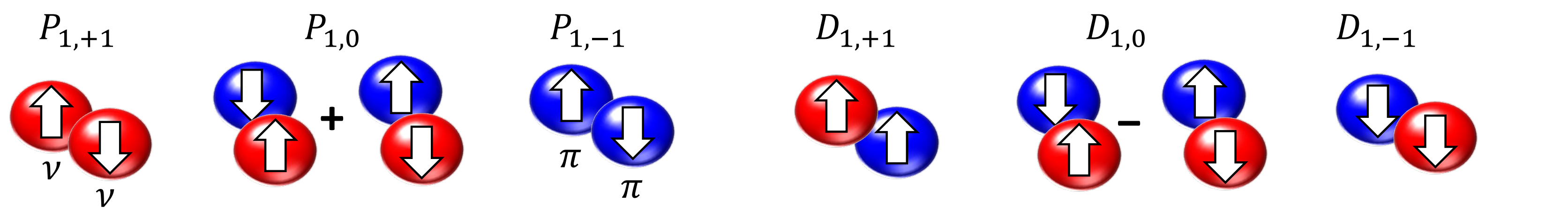
<u>Spin-1 deuteron operator</u>

$$D_{1,+1}(\boldsymbol{P},\boldsymbol{q}) = \frac{\sqrt{2}}{2}\left(\nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}} - \pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}}\right),$$

$$D_{1,0}(\boldsymbol{P},\boldsymbol{q}) = \frac{1}{2}\left(\nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}} - \pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}} - \pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}} + \nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},\frac{1}{2}}\right),$$

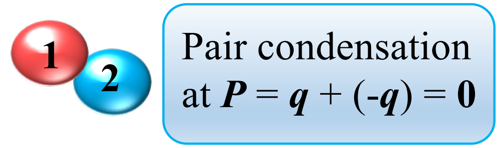
$$D_{1,-1}(\boldsymbol{P},\boldsymbol{q}) = \frac{\sqrt{2}}{2}\left(\nu_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\pi_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}} - \pi_{\frac{\boldsymbol{P}}{2}-\boldsymbol{q},-\frac{1}{2}}\nu_{\frac{\boldsymbol{P}}{2}+\boldsymbol{q},-\frac{1}{2}}\right)$$

$\boldsymbol{P}$: center-of-mass momentum of pair, $\boldsymbol{q},\boldsymbol{q}'$: relative momentum



Coherent BCS state (momentum space)

In the case of pairing, we have



$$|\Psi_{\mathrm{coh.}}\rangle = \exp\left(\sum_{\boldsymbol{q}} g_{\boldsymbol{q}} c^{\dagger}_{\boldsymbol{q},1} c^{\dagger}_{-\boldsymbol{q},2}\right)|0\rangle$$

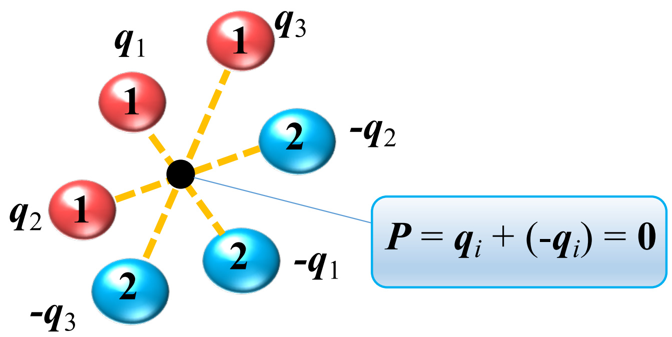
$$= \exp\left(g_{\boldsymbol{q}_1} c^{\dagger}_{\boldsymbol{q}_1,1} c^{\dagger}_{-\boldsymbol{q}_1,2} + g_{\boldsymbol{q}_2} c^{\dagger}_{\boldsymbol{q}_2,1} c^{\dagger}_{-\boldsymbol{q}_2,2} + \cdots\right)|0\rangle$$

$$= \left(1 + g_{\boldsymbol{q}_1} c^{\dagger}_{\boldsymbol{q}_1,1} c^{\dagger}_{-\boldsymbol{q}_1,2}\right)\left(1 + g_{\boldsymbol{q}_2} c^{\dagger}_{\boldsymbol{q}_2,1} c^{\dagger}_{-\boldsymbol{q}_2,2}\right)\times(\ldots)|0\rangle$$

$$= \prod_{\boldsymbol{q}}\left(1 + g_{\boldsymbol{q}} c^{\dagger}_{\boldsymbol{q},1} c^{\dagger}_{-\boldsymbol{q},2}\right)|0\rangle$$

$$\equiv \prod_{\boldsymbol{q}}\left(u_{\boldsymbol{q}} + v_{\boldsymbol{q}} c^{\dagger}_{\boldsymbol{q},1} c^{\dagger}_{-\boldsymbol{q},2}\right)|0\rangle$$

BCS ground state

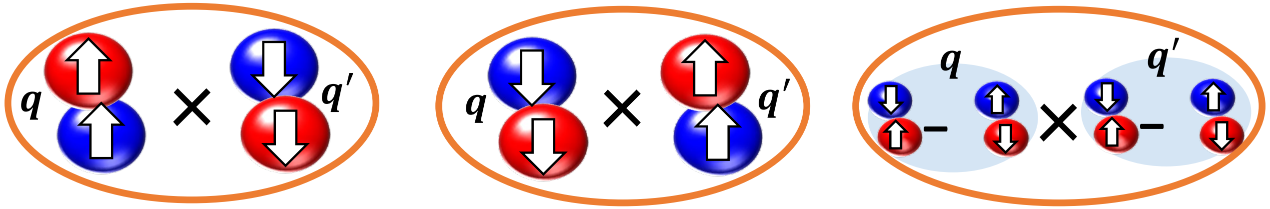


Superposition of pairs with q_i

Coherent BCS state (momentum space)

Alpha creation operator (S = T = 0, COM momentum of pairs $\boldsymbol{P}$ = 0)

$$\alpha^\dagger(\boldsymbol{q},\boldsymbol{q}') = \frac{\sqrt{3}}{3}\left[D^\dagger_{1,+1}(0,\boldsymbol{q})D^\dagger_{1,-1}(0,\boldsymbol{q}') + D^\dagger_{1,-1}(0,\boldsymbol{q})D^\dagger_{1,+1}(0,\boldsymbol{q}') - D^\dagger_{1,0}(0,\boldsymbol{q})D^\dagger_{1,0}(0,\boldsymbol{q}')\right]$$

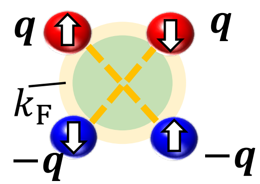


$$\begin{aligned}
|\Psi_{\rm coh}\rangle &= \exp\left(\sum_{\boldsymbol{q},\boldsymbol{q}'} g_{\boldsymbol{q},\boldsymbol{q}'}\alpha^\dagger(\boldsymbol{q},\boldsymbol{q}')\right)|0\rangle \\
&= \exp\left(g_{\boldsymbol{q}_1,\boldsymbol{q}_1}\alpha^\dagger(\boldsymbol{q}_1,\boldsymbol{q}_1) + g_{\boldsymbol{q}_1,\boldsymbol{q}_2}\alpha^\dagger(\boldsymbol{q}_1,\boldsymbol{q}_2)\right. \\
&\quad \left. + g_{\boldsymbol{q}_2,\boldsymbol{q}_1}\alpha^\dagger(\boldsymbol{q}_2,\boldsymbol{q}_1) + \cdots\right)|0\rangle \\
&= \left(1 + g_{\boldsymbol{q}_1,\boldsymbol{q}_1}\alpha^\dagger(\boldsymbol{q}_1,\boldsymbol{q}_1)\right)\left(1 + g_{\boldsymbol{q}_1,\boldsymbol{q}_2}\alpha^\dagger(\boldsymbol{q}_1,\boldsymbol{q}_2)\right) \\
&\quad \times \left(1 + g_{\boldsymbol{q}_2,\boldsymbol{q}_1}\alpha^\dagger(\boldsymbol{q}_2,\boldsymbol{q}_1)\right)\cdots|0\rangle \\
&= \prod_{\boldsymbol{q},\boldsymbol{q}'}\left[1 + g_{\boldsymbol{q},\boldsymbol{q}'}\alpha^\dagger(\boldsymbol{q},\boldsymbol{q}')\right]|0\rangle,
\end{aligned}$$

Quartet BCS ansatz

$$|\Phi\rangle = \prod_{\boldsymbol{q},\boldsymbol{q}'}\left[u_{\boldsymbol{q},\boldsymbol{q}'} + \sum_{S_z} v_{\boldsymbol{q},S_z}(\boldsymbol{q})D^\dagger_{1,S_z}(0,\boldsymbol{q}) + \sum_{T_3} x_{\boldsymbol{q},T_3}(\boldsymbol{q})P^\dagger_{1,T_3}(0,\boldsymbol{q}) + w_{\boldsymbol{q},\boldsymbol{q}'}\alpha^\dagger(\boldsymbol{q},\boldsymbol{q}')\right]|0\rangle$$

Difficult to be handled due to multiple infinite products…



- Assuming symmetric configuration ($\boldsymbol{q} \simeq \boldsymbol{q}'$)

Variational equations

$\boldsymbol{\lambda}$: Lagrange multiplier

Minimize the ground-state energy: $\partial\langle\Phi|H-\boldsymbol{\lambda}|\Phi\rangle=0$

$$v_{\boldsymbol{q},+1}=\frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},+1}+w_{\boldsymbol{q}}\Delta^{*(\mathrm{isp})}_{\boldsymbol{q},-1}}{B_{\boldsymbol{q}}+(\varepsilon_{\pi,\boldsymbol{q}}+\varepsilon_{\nu,-\boldsymbol{q}})},\quad v_{\boldsymbol{q},-1}=\frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},-1}+w_{\boldsymbol{q}}\Delta^{*(\mathrm{isp})}_{\boldsymbol{q},+1}}{B_{\boldsymbol{q}}+(\varepsilon_{\pi,\boldsymbol{q}}+\varepsilon_{\nu,-\boldsymbol{q}})},$$

$$v_{\boldsymbol{q},0}=\frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},0}-\frac{1}{2}(w_{\boldsymbol{q}}+w_{-\boldsymbol{q}})\Delta^{*(\mathrm{isp})}_{\boldsymbol{q},0}}{B_{\boldsymbol{q}}+(\varepsilon_{0,\boldsymbol{q}}+\varepsilon_{0,-\boldsymbol{q}})},$$

isoscalar

$$x_{\boldsymbol{q},+1}=\frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{ivp})}_{\boldsymbol{q},+1}}{B_{\boldsymbol{q}}+(\varepsilon_{\nu,\boldsymbol{q}}+\varepsilon_{\nu,-\boldsymbol{q}})},\quad x_{\boldsymbol{q},-1}=\frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{ivp})}_{\boldsymbol{q},-1}}{B_{\boldsymbol{q}}+(\varepsilon_{\pi,\boldsymbol{q}}+\varepsilon_{\pi,-\boldsymbol{q}})},$$

$$x_{\boldsymbol{q},0}=\frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{ivp})}_{\boldsymbol{q},0}+\frac{1}{2}(w_{\boldsymbol{q}}+w_{-\boldsymbol{q}})\Delta^{*(\mathrm{ivp})}_{\boldsymbol{q},0}}{B_{\boldsymbol{q}}+(\varepsilon_{0,\boldsymbol{q}}+\varepsilon_{0,-\boldsymbol{q}})},$$

isovector

$$w_{\boldsymbol{q}}=\frac{\frac{1}{2}(x_{\boldsymbol{q},0}+x_{-\boldsymbol{q},0})\Delta^{(\mathrm{ivp})}_{\boldsymbol{q},0}+v_{\boldsymbol{q},-1}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},+1}+v_{\boldsymbol{q},+1}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},-1}-\frac{1}{2}(v_{\boldsymbol{q},0}+v_{-\boldsymbol{q},0})\Delta^{(\mathrm{isp})}_{\boldsymbol{q},0}}{B_{\boldsymbol{q}}+2(\varepsilon_{\pi,\boldsymbol{q}}+\varepsilon_{\nu,-\boldsymbol{q}})}$$

quartet

$$B_{\boldsymbol{q}}=\frac{1}{2u_{\boldsymbol{q}}}\sum_{S_z,T_3}\left[v^*_{\boldsymbol{q},S_z}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},S_z}+v_{\boldsymbol{q},S_z}\Delta^{*(\mathrm{isp})}_{\boldsymbol{q},S_z}+x^*_{\boldsymbol{q},T_3}\Delta^{(\mathrm{ivp})}_{\boldsymbol{q},T_3}+x_{\boldsymbol{q},T_3}\Delta^{*(\mathrm{ivp})}_{\boldsymbol{q},T_3}\right]$$

Normalization condition

$$\sum_{S_z}\left|v_{\boldsymbol{q},S_z}\right|^2+\sum_{T_z}|x_{\boldsymbol{q},T_3}|^2+|u_{\boldsymbol{q}}|^2+|w_{\boldsymbol{q}}|^2=1$$

Pairing gaps

$$\Delta^{(\mathrm{isp})}_{\boldsymbol{q},S_z}=-\sum_{\boldsymbol{q}'}V_t(\boldsymbol{q},\boldsymbol{q}')\Big[u^*_{\boldsymbol{q}'}v_{\boldsymbol{q}',S_z}+\delta_{S_z,+1}v^*_{\boldsymbol{q}',-S_z}w_{\boldsymbol{q}'}+\delta_{S_z,-1}v^*_{\boldsymbol{q}',-S_z}w_{\boldsymbol{q}'}-\frac{1}{2}\delta_{S_z,0}\big(v^*_{\boldsymbol{q}',-S_z}w_{\boldsymbol{q}'}+v^*_{\boldsymbol{q}',-S_z}w_{-\boldsymbol{q}'}\big)\Big]$$

$$\Delta^{(\mathrm{ivp})}_{\boldsymbol{q},T_3}=-\sum_{\boldsymbol{q}'}V_s(\boldsymbol{q},\boldsymbol{q}')\Big[u^*_{\boldsymbol{q}'}x_{\boldsymbol{q}',T_3}+\frac{1}{2}\delta_{T_3,0}\big(x^*_{\boldsymbol{q}',T_3}w_{\boldsymbol{q}'}+x^*_{\boldsymbol{q}',T_3}w_{-\boldsymbol{q}'}\big)\Big].$$

Variational equations

$\boldsymbol{\lambda}$: Lagrange multiplier

Minimize the ground-state energy: $\partial\langle\Phi|H-\boldsymbol{\lambda}|\Phi\rangle = 0$

$$v_{\boldsymbol{q},+1} = \frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},+1} \text{[redacted]}}{B_{\boldsymbol{q}} + (\varepsilon_{\pi,\boldsymbol{q}} + \varepsilon_{\nu,-\boldsymbol{q}})}, \quad v_{\boldsymbol{q},-1} = \frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},-1} \text{[redacted]}}{B_{\boldsymbol{q}} + (\varepsilon_{\pi,\boldsymbol{q}} + \varepsilon_{\nu,-\boldsymbol{q}})},$$

$$v_{\boldsymbol{q},0} = \frac{u_{\boldsymbol{q}}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},0} \text{[redacted]}}{B_{\boldsymbol{q}} + (\varepsilon_{0,\boldsymbol{q}} + \varepsilon_{0,-\boldsymbol{q}})},$$

isoscalar

[redacted]

$$B_{\boldsymbol{q}} = \frac{1}{2u_{\boldsymbol{q}}}\sum_{S_z,T_3}\left[v^*_{\boldsymbol{q},S_z}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},S_z} + v_{\boldsymbol{q},S_z}\Delta^{*(\mathrm{isp})}_{\boldsymbol{q},S_z} \text{[redacted]}\right]$$

Pairing gaps

$$\Delta^{(\mathrm{isp})}_{\boldsymbol{q},S_z} = -\sum_{\boldsymbol{q}'} V_t(\boldsymbol{q},\boldsymbol{q}')\left[u^*_{\boldsymbol{q}'}v_{\boldsymbol{q}',S_z} \text{[redacted]}\right]$$

[redacted]

Normalization condition

$$\sum_{S_z}\left|v_{\boldsymbol{q},S_z}\right|^2 \text{[redacted]} + |u_{\boldsymbol{q}}|^2 \text{[redacted]} = 1$$

Well-known BCS theory has been recovered!

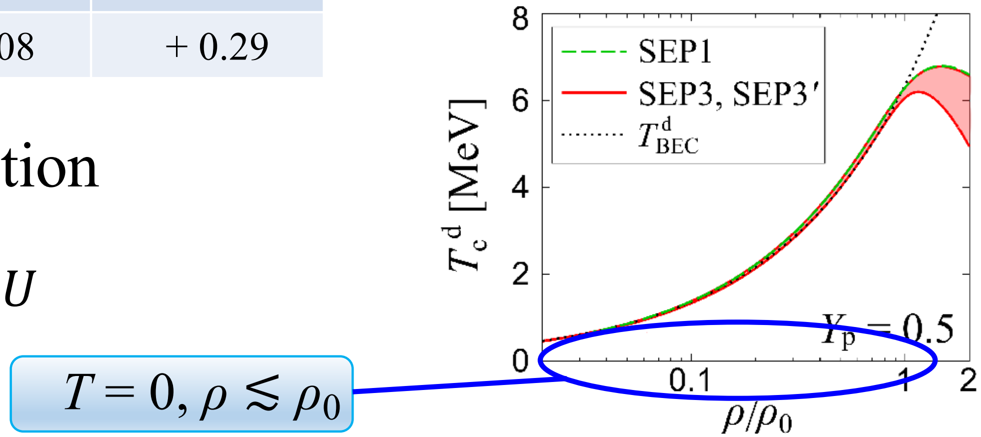
Numerical calculations

- We consider the infinite symmetric nuclear matter.
- Focus on the isoscalar interaction because isovector interaction is less relevant in the infinite symmetric nuclear matter.

	Isovector	Isoscalar
Scattering length a	−20 fm	+5.4 fm
$(k_F a)^{-1}$ at $\rho = 0.016\ \mathrm{fm}^{-3}$	- 0.08	+ 0.29

- Short-range contact interaction

$$V_t(\boldsymbol{q}, \boldsymbol{q}') \simeq -U$$



Tajima, Hatsuda, van Wyk, and Ohashi Sci. Rep. 9 (2019), 18477

Numerical calculations

Assumption: $x_q \simeq 0,\ V_t(\boldsymbol{q},\boldsymbol{q}') \simeq -U$

$k_F \simeq \sqrt{2M\mu} \simeq 0.457\ \mathrm{fm}^{-1}$

Input: $\Delta^{(\mathrm{isp})}_{q,S_z} \simeq 5\mathrm{MeV}, \mu = (1.16)^{-1}\Delta^{(\mathrm{isp})}_{q,S_z} = 4.31\mathrm{MeV}$

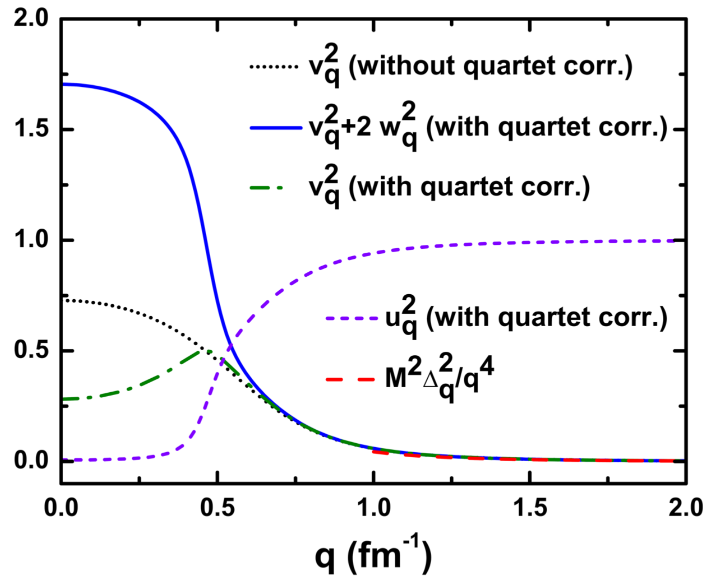
∗ 1.16 is borrowed from the BCS result at the unitarity limit

Self-consistently solve…

$$\frac{w_q}{u_q} = \frac{\left[2\Delta^{(\mathrm{isp})}_{\boldsymbol{q},+1}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},-1} - \left|\Delta^{(\mathrm{isp})}_{\boldsymbol{q},0}\right|^2\right]}{(B_q + 4\varepsilon_q)(B_q + 2\varepsilon_q) - \sum_{S_z}\left|\Delta^{(\mathrm{isp})}_{\boldsymbol{q},S_z}\right|^2}$$

$$B_q = -\varepsilon_q + \sqrt{\varepsilon_q^2 + \boldsymbol{\Delta}_q^2 + R_q}$$

$$R_q = \frac{w_q}{u_q}\left[2\Delta^{(\mathrm{isp})}_{\boldsymbol{q},+1}\Delta^{(\mathrm{isp})}_{\boldsymbol{q},-1} - \left|\Delta^{(\mathrm{isp})}_{\boldsymbol{q},0}\right|^2\right]$$



Numerical calculations (Variational parameters)

Assumption: $x_{\boldsymbol{q}} \simeq 0,\ V_t(\boldsymbol{q}, \boldsymbol{q}') \simeq -U$

Input: $\Delta_{q,S_z}^{(\mathrm{isp})} \simeq 5\mathrm{MeV}, \mu = (1.16)^{-1}\Delta_{q,S_z}^{(\mathrm{isp})} = 4.31\mathrm{MeV}$

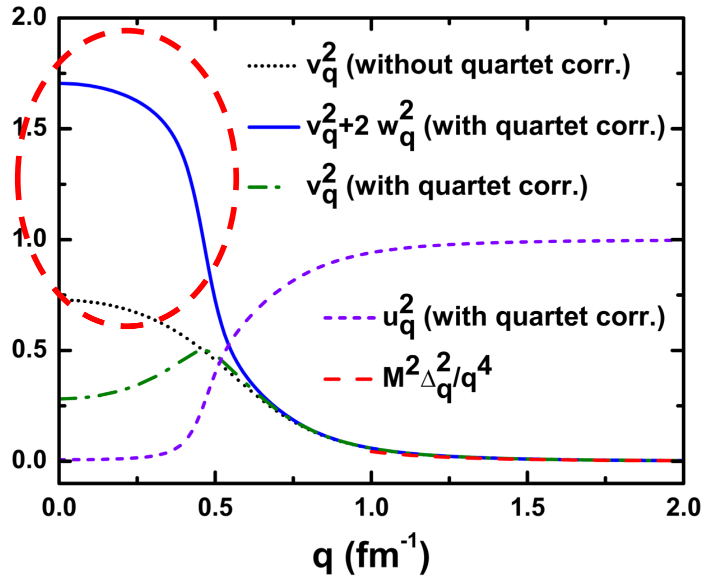
$k_{\mathrm{F}} \simeq \sqrt{2M\mu} \simeq 0.457\ \mathrm{fm}^{-1}$

$*$ 1.16 is borrowed from the BCS result at the unitarity limit

1. Quartet correlations mainly appear at low relative momentum (***q***)

2. An interplay between quartets (alpha) and pairs (deuteron) is found

3. A high-momentum tail is dominated by short-range pairs (deuteron)



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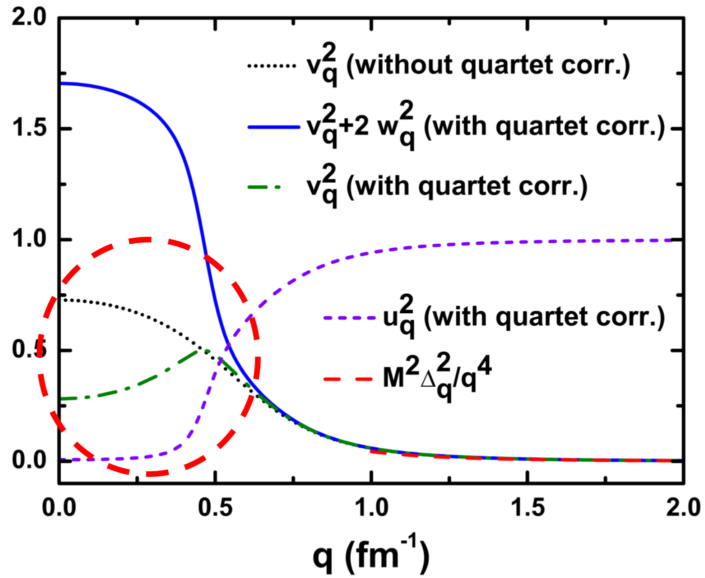


Numerical calculations (Variational parameters)

Assumption: $x_q \simeq 0,\ V_t(\boldsymbol{q},\boldsymbol{q}') \simeq -U$

$k_F \simeq \sqrt{2M\mu} \simeq 0.457\ \text{fm}^{-1}$

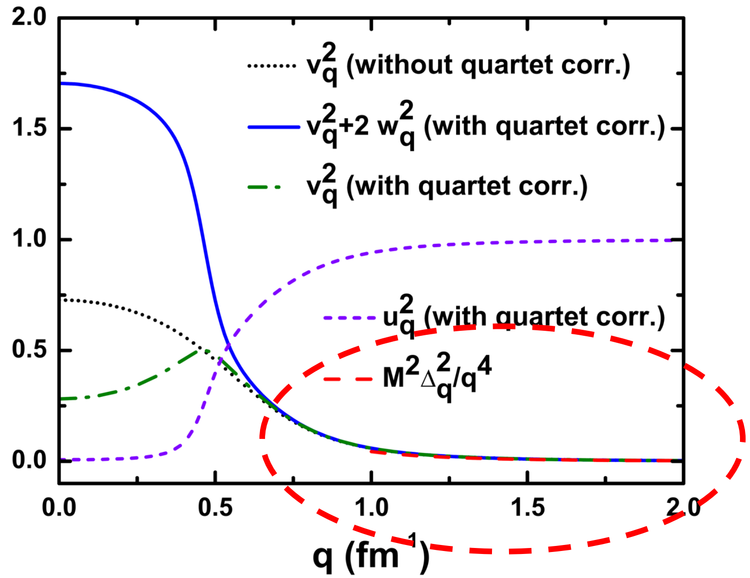
Input: $\Delta^{(\text{isp})}_{q,S_z} \simeq 5\text{MeV}, \mu = (1.16)^{-1}\Delta^{(\text{isp})}_{q,S_z} = 4.31\text{MeV}$

* 1.16 is borrowed from the BCS result at the unitarity limit

1. Quartet correlations mainly appear at low relative momentum (*q*)

2. An interplay between quartets (alpha) and pairs (deuteron) is found

3. A high-momentum tail is dominated by short-range pairs (deuteron)

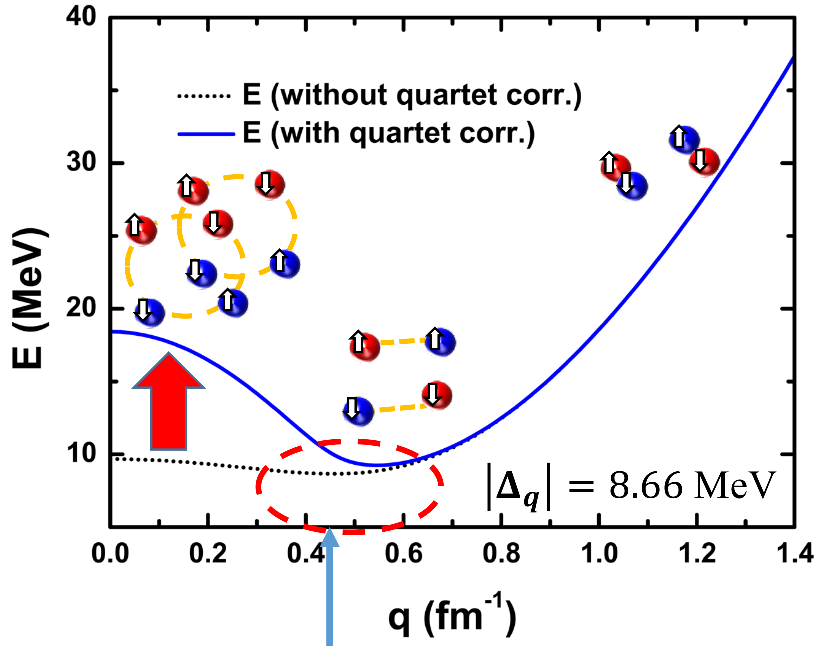


Numerical calculations (Quasiparticle excitation)

$$E = \sqrt{\varepsilon_{\boldsymbol{q}}^2 + \boldsymbol{\Delta}_{\boldsymbol{q}}^2 + R_{\boldsymbol{q}}}$$

$$R_{\boldsymbol{q}} = \frac{w_{\boldsymbol{q}}}{u_{\boldsymbol{q}}}\left[2\Delta_{\boldsymbol{q},+1}^{(\mathrm{isp})}\Delta_{\boldsymbol{q},-1}^{(\mathrm{isp})} - \left|\Delta_{\boldsymbol{q},0}^{(\mathrm{isp})}\right|^2\right]$$

1. E is enhanced at low relative momentum ($\boldsymbol{q}$)
2. Excitation gap (minima of E) is almost unchanged compared to the usual BCS one
3. Crossover from loosely bound quartets to short-range pairs



$\sqrt{2M\mu} \simeq 0.457\ \mathrm{fm}^{-1}$

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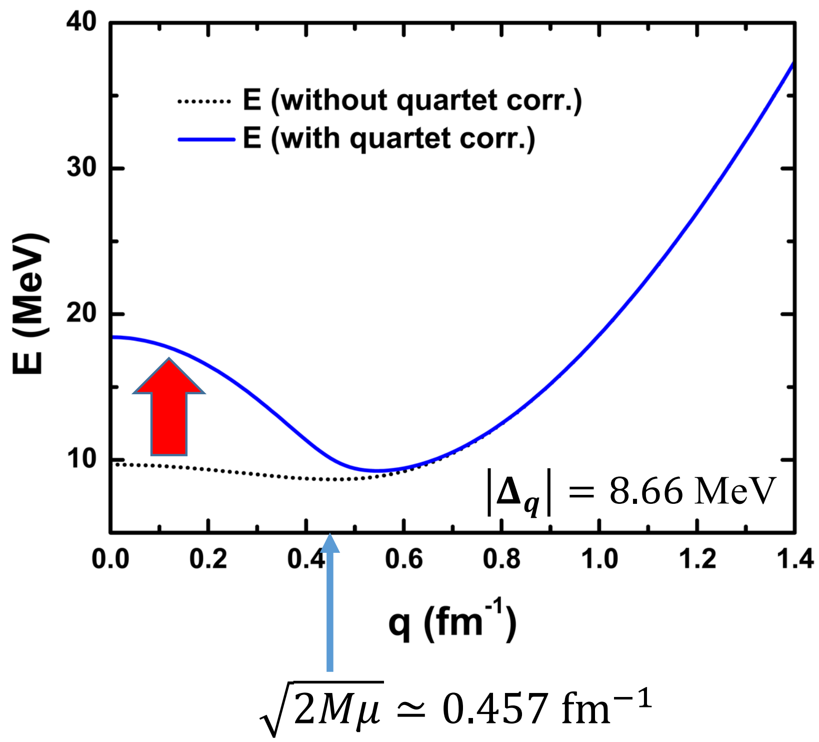

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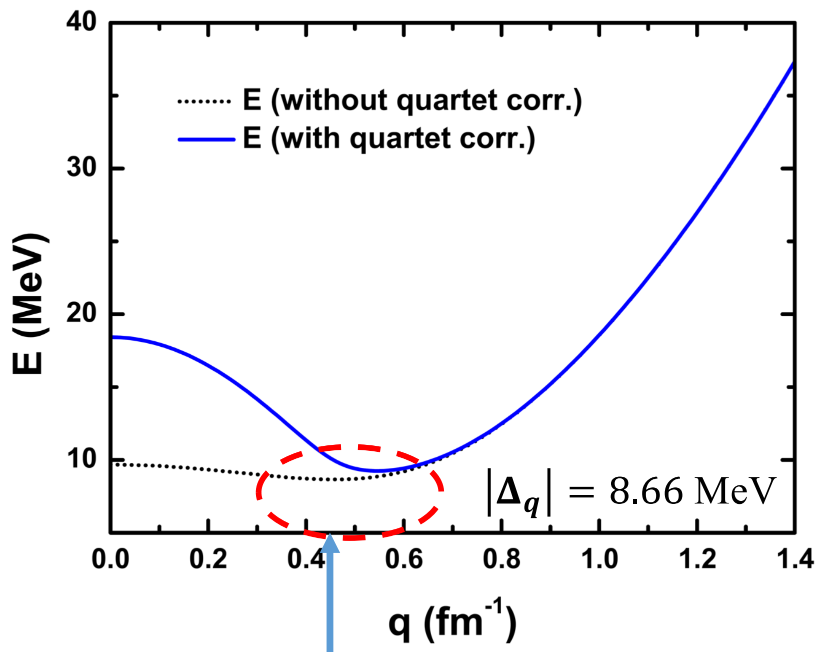

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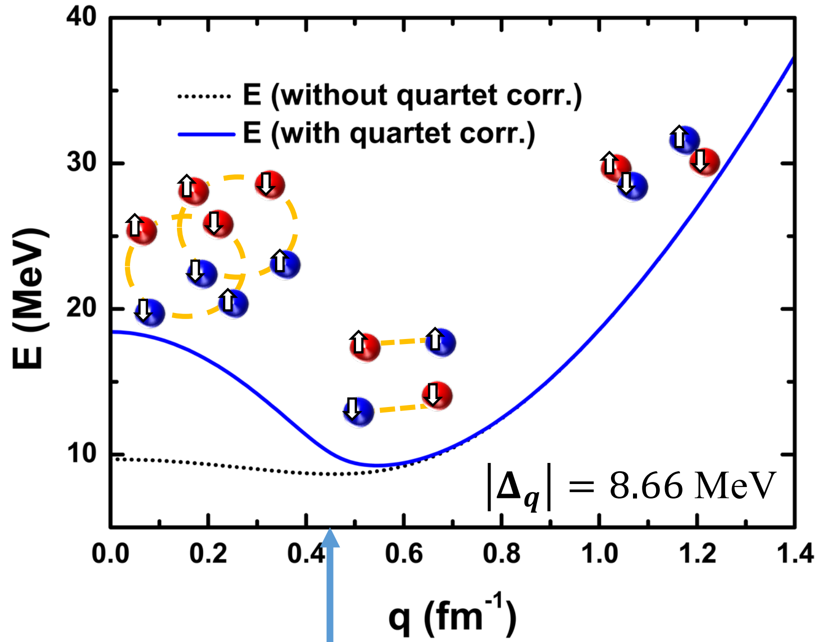
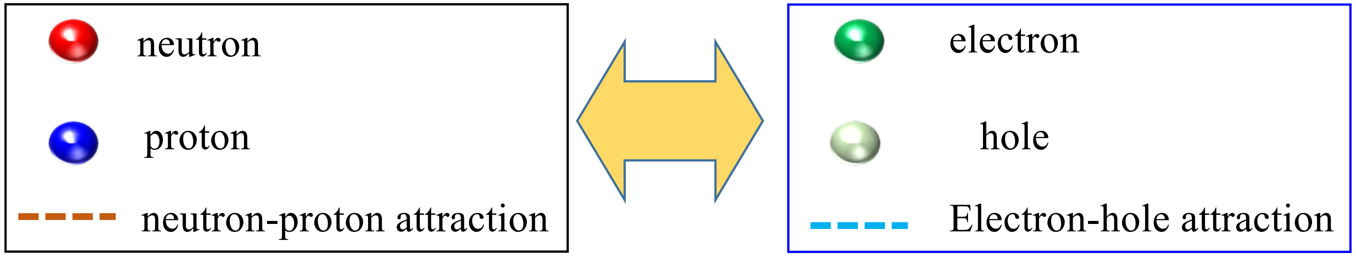


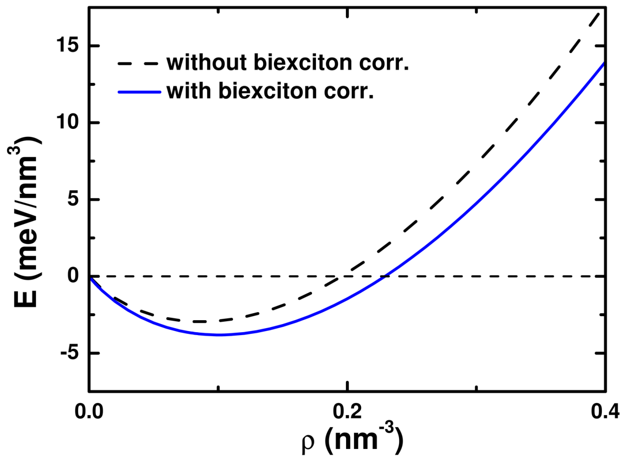
$\sqrt{2M\mu} \simeq 0.457\ \mathrm{fm}^{-1}$

Biexciton-like quartet correlation

Similar formalism can be applied to the electron-hole system



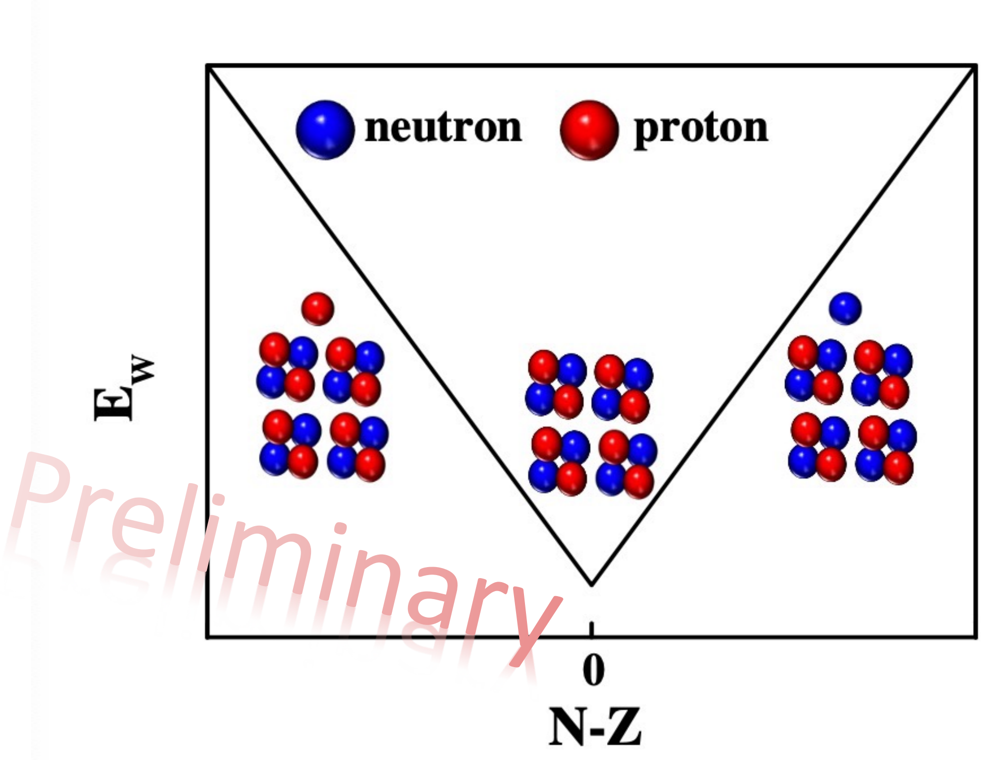
Ground-state energy density



Carrier density

YG, Tajima, and Liang, Phys. Rev. Res. **4** (2022) 023152

Wigner term arising from quartet-nucleon scattering



$$\frac{E}{\rho} = \frac{E_0}{\rho} + W\frac{|\rho_n - \rho_p|}{\rho} + \cdots$$

$$E_W = W\frac{|\rho_n - \rho_p|}{\rho}$$

$$W \propto f_{\mathrm{quartet}}(\rho = \rho_0)$$

Quartet condensation fraction

$$f_{\mathrm{quartet}} = \sum_{\boldsymbol{q}} |\langle \Psi | \alpha(\boldsymbol{q}) | \Psi \rangle|^2 = \sum_{\boldsymbol{q}} u_{\boldsymbol{q}}^2 w_{\boldsymbol{q}}^2$$

YG, Tajima, and Naito, in preparation

Summary

✧ **In this work**

- We have investigated Cooper quartet correlations in infinite symmetric nuclear matter at the thermodynamic limit, and discussed how physical properties would be affected.
- The BCS-type variational function is extended to the systems with the coexistence of pair and quartet correlations at zero temperature (quartet BCS theory).
- The present results may also contribute to the interdisciplinary understanding of fermionic condensations beyond the BCS paradigm.

✧ **For the NEXT step**

- Improvement of trial wave function (excited pair, Hartree-Fock term for the nonlocal interactions,……)
- Connection to nuclear experiments
- Calculations of various EOS, finite-temperature effect, transition temperature,……
- Wigner term arising from quartet-nucleon scattering

Thank you!

Q & A