Finite range simple effective interaction with tensor terms

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Simple Effective Interaction (SEI)

The finite-range simple effective interaction is given by

$$V_{eff}(r) = t_0(1 + x_0 P_\sigma)\delta(r) + \frac{t_3}{6}(1 + x_3 P_\sigma)\left(\frac{\rho}{1 + b\rho}\right)^\gamma \delta(r) + (W + BP_\sigma - HP_\tau - MP_\sigma P_\tau)\mathbf{f(r)}$$

f(r)= $\frac{e^{-r/\alpha}}{r/\alpha}$ (Yukawa), e^{-r^2/α^2} (Gaussian), $e^{-r/\alpha}$ (Exponential)

- SEI has 11 parameters:(b, t_0,x_0, t_3, x_3,γ, α, W, B , H , M) and (W_0 $\Rightarrow$ Enters in the description of finite nuclei.)

Protocol adopted for parameter fixation

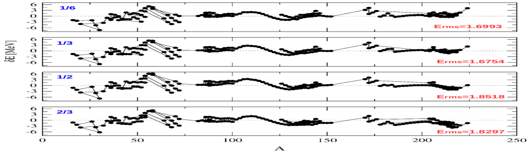
- mean field in SNM $\Rightarrow$ kinetic energy 300 MeV
- entropy in PNM ($\not>$) entropy in SNM
- $\frac{m*}{m}$ splitting in spin polarized PNM $\sim$ (DBHF) prediction
- b $\Rightarrow$ to prevent the NM to become supraluminous
- stiffness of the SNM $\Rightarrow$ γ $\Rightarrow$ pressure-density relation curve
- density dependence of the isospin asymmetric part $\Rightarrow$ saturation properties and β-stable charge neutral n + p + e + μ matter be maximum
- t_0 and W_0 $\Rightarrow$ BEs of the two doubly closed ^{40}Ca and ^{208}Pb nuclei

Behera et al 1998 JPG: Nucl. Part. Phys. **24** 2073, Behera et al, J. Phys. G: Nucl. Part. Phys. **38** (2011) 115104, Behera et al, J. Phys. G: Nucl. Part. Phys. **42** (2015) 045103, Behera et al, J. Phys. G: Nucl. Part. Phys. **40** (2013) 095105, Behera et al, Nuclear Physics A **794** (2007) 132148, Behera et al, J. Phys. G: Nucl. Part. Phys. **23** (1997) 445455

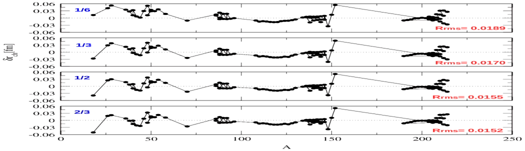
Quasilocal density functional theory(QLDFT)

$$\left[-\nabla\cdot\frac{\hbar^2}{2m_q^*}\nabla+U_q(\mathbf{R})-\mathbf{W}_q(\mathbf{R})(\nabla\times\boldsymbol{\sigma})\right]\phi_q=\epsilon_q\phi_q,$$

Deviation of BE for 161 even-even spherical nuclei for the four EOSs of SEI corresponding to γ=1/6, 1/3, 1/2 and 2/3



Deviation of Charge radii for 86 even-even spherical nuclei for the four EoS



B Behera et al, J. Phys. G: Nucl. Part. Phys. **42** (2015) 045103.

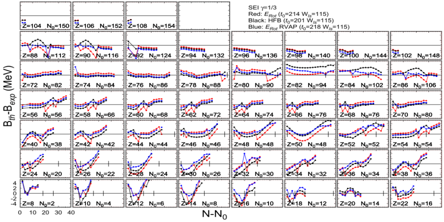
Table 4. Comparison of the quantal binding energies and radii obtained in doubly magic nuclei using the QLEDF of [1] and the HF approach.

Nucleus	E_{EDF}(MeV)	E_{HF}(MeV)	r_p^{EDF}(fm)	r_p^{HF}(fm)
^{16}O	−127.6240	−127.0907	2.6594	2.6646
^{28}O	−179.4020	−178.5816	2.7832	2.7922
^{40}Ca	−342.1981	−341.2690	3.3997	3.4053
^{48}Ca	−416.8068	−414.6866	3.4210	3.4326
^{56}Ni	−478.7936	−480.0659	3.6847	3.6907
^{78}Ni	−645.4516	−646.4948	3.8814	3.8928
^{100}Sn	−824.8094	−825.6940	4.4132	4.4244
^{132}Sn	−1105.0788	−1105.4751	4.6360	4.6456
^{208}Pb	−1636.6551	−1635.8961	5.4285	5.4344

Table 5. Comparison of quantal binding energies and radii obtained in the isotopes of *Sn* using the QLEDF plus improved BCS pairing of [1] and the HFB method.

Nucleus	E_{EDF}(MeV)	E_{HFB}(MeV)	r_p^{EDF}(fm)	r_p^{HFB}(fm)
^{102}Sn	−848.1597	−848.8267	4.4297	4.4389
^{104}Sn	−870.2672	−870.8571	4.4450	4.4535
^{106}Sn	−891.6288	−891.9747	4.4603	4.4685
^{108}Sn	−912.2156	−912.2815	4.4758	4.4839
^{110}Sn	−932.3108	−931.8436	4.4915	4.4995
^{112}Sn	−951.4225	−950.7060	4.5069	4.5150
^{114}Sn	−969.8919	−968.9010	4.5215	4.5301
^{116}Sn	−987.5958	−986.4534	4.5355	4.5447
^{118}Sn	−1004.5842	−1003.3858	4.5488	4.5586
^{120}Sn	−1020.8849	−1019.7195	4.5620	4.5721
^{122}Sn	−1036.5196	−1035.4725	4.5748	4.5850
^{124}Sn	−1051.5581	−1050.6561	4.5875	4.5977
^{126}Sn	−1065.9662	−1065.2716	4.6021	4.6100
^{128}Sn	−1079.7420	−1079.3066	4.6137	4.6221
^{130}Sn	−1092.7420	−1091.7298	4.6251	4.6341

RMS deviation of BE and charge radii of 620 even-even, both spherical and deformed,and 313 even-even nuclei computed in the Hartree-Fock-Bogoulibov formulation(HFB)



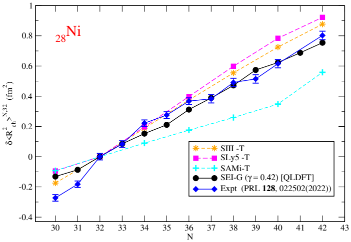
γ	t_0 MeV fm^3	x_0	W_0 MeV	σ(E) MeV	σ(R) fm	E_{rot}
1/3	201	3.1931	115	1.873	0.0253	no
1/2	438	1.4182	112	1.958	0.0252	no
1/3	214	3.0595	115	1.788	0.0253	yes: VAP
1/2	450	1.4071	112	1.843	0.0252	yes: VAP
1/3	218	3.0221	115	1.742	0.0255	yes: RVAP
1/2	455	1.4027	112	1.561	0.0255	yes: RVAP

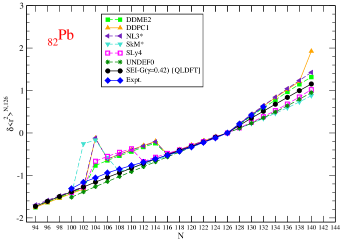
B Behera et al, J. Phys. G: Nucl. Part. Phys. **43** (2016) 045115

The twelve parameters for SEI-G($\gamma = 0.42$) along with the nuclear matter saturation properties (such as saturation density ρ_0, energy per nucleon $e(\rho_0)$, incompressibility for symmetric nuclear matter K, effective mass m^*/m, symmetry energy E_s, slope of symmetry energy L, and curvature of the symmetry energy K_{sym}

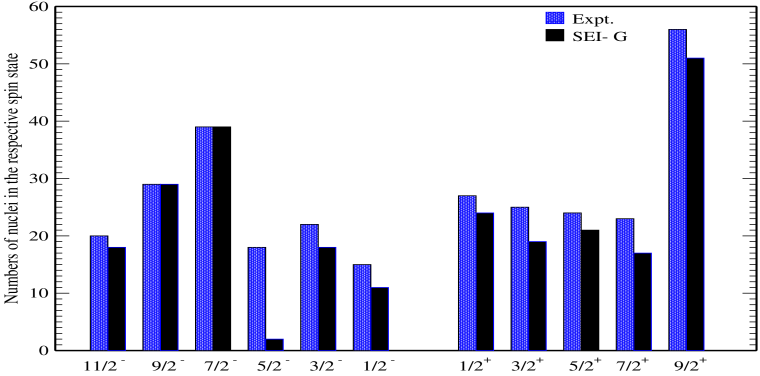
γ	b [fm^3]	α [fm]	ε_{ex} [MeV]
0.42	0.5050	0.7591	−95.0536
ε^l_{ex} [MeV]	ε_0 [MeV]	ε^l_0 [MeV]	ε_γ [MeV]
−63.3691	−91.6562	−53.1272	90.0035
ε^l_γ [MeV]	t_0 [MeV fm^3]	x_0	W_0 [MeV fm^5]
65.3966	341.2	1.7933	113.4
Nuclear matter saturation properties			
ρ_0 [fm^{-3}]	$e(\rho_0)$ [MeV]	K [MeV]	m^*/m
0.1584	−16.0	240	0.711
E_s [MeV]	L [MeV]	K_{sym} [MeV]	
35.5	76.71	−155.0	

Isotopic shift :

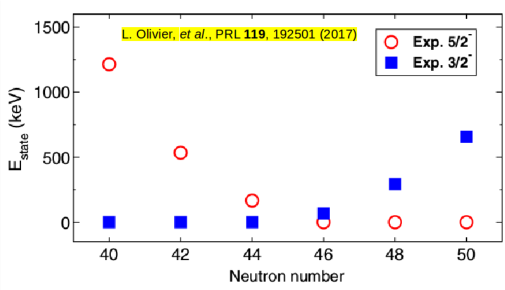


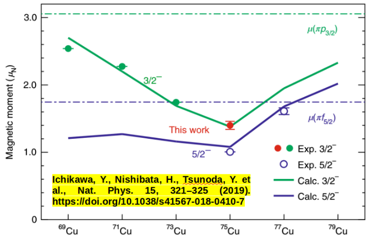


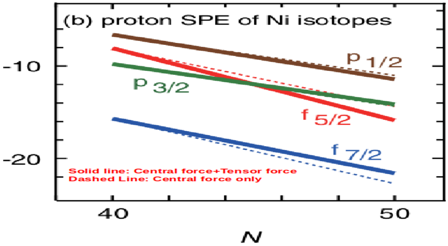
Comparison of experimental and SEI-G(γ=0.42)[at the QLDFT level and using the uniform blocking method] spins of 298 odd-nuclei in different spin states:



Similar results as : L. Bonneau, P. Quentin, and P. Mller, Phys. Rev. C **76**, 024320 (2007)

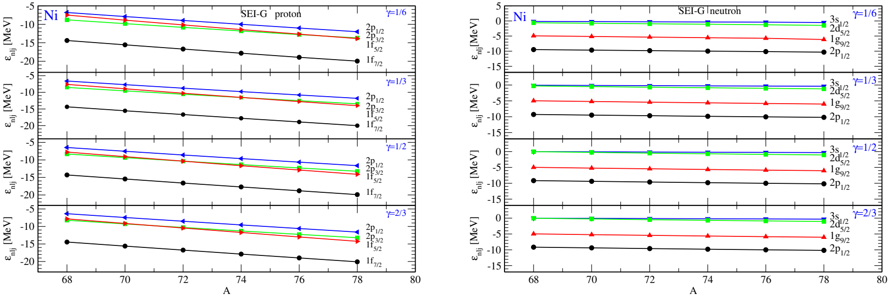






T. Otsuka, *et al.*, PRL **104**, 012501 (2010)

- Proton and neutron single-particle levels around the Fermi level for Ni isotopes from A = 68 to A = 78 computed with the SEI for four EoSs.

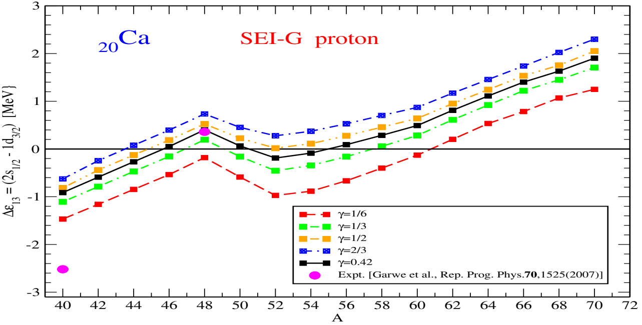


Nucleus	Spin-parity	SEI-G($\gamma = 0.42$) E[MeV]	Expt. E[MeV]	SEI-G E^*[keV]	Expt. E^*[keV]
^{69}Cu	3/2−	−599.40	−599.97	663	1215
^{71}Cu	3/2−	−613.73	−613.09	449	537
^{73}Cu	3/2−	−626.51	−625.51	156	263
^{75}Cu	5/2−	−638.25	−637.13	103	62
^{77}Cu	5/2−	−649.11	−647.42	292	295
^{79}Cu	5/2−	−658.94	−656.65	620	660

Nucleus	SEI($\gamma = \frac{1}{6}$) E [MeV]	SEI($\gamma = \frac{1}{3}$) E [MeV]	SEI($\gamma = \frac{1}{2}$) E [MeV]	SEI($\gamma = \frac{2}{3}$) E [MeV]	Expt. E [MeV]
^{68}Ni	−591.60	−591.08	−590.37	−590.46	−590.407
^{70}Ni	−604.76	−604.52	−603.80	−603.82	−602.300
^{72}Ni	−616.44	−616.32	−615.73	−615.83	−613.455
^{74}Ni	−627.04	−627.03	−626.49	−626.71	−623.820
^{76}Ni	−636.64	−636.75	−636.27	−636.53	−633.156
^{78}Ni	−645.81	−645.38	−644.96	−645.27	−641.550

P. Bano, X. Viñas, T. R. Routray, M. Centelles, M. Anguiano, and L. M. Robledo, PHYSICAL REVIEW C **106**, 024313 (2022)

sd- level splitting in Ca isotopic chain



Proton single-particle gaps between $1h_{11/2}$ and $1g_{7/2}$ level in Sn nuclei, Neutron single-particle gaps between $1i_{13/2}$ and $1h_{9/2}$ level in N=82 chain, $1h_{11/2}$ and $1g_{7/2}$ s.p.p levels in Sb nuclei, Reduction of shell gap in N =28(^{49}Ca and ^{47}Ar) nuclei and Evolution of the $1h_{11/2}$, $1g_{7/2}$, and $2d_{3/2}$ neutron s.p. levels in the N = 51 isotonic chain.

Simple effective interaction with a short-range tensor force

$$\begin{aligned} V_T &= \frac{T}{2}\Bigg\{\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}') - \frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}'^2\right]\delta(\mathbf{r_1}-\mathbf{r_2}) \\ &+ \delta(\mathbf{r_1}-\mathbf{r_2})\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) - \frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}^2\right]\Bigg\} \\ &+ U\Bigg\{(\boldsymbol{\sigma}_1\cdot\mathbf{k}')\delta(\mathbf{r_1}-\mathbf{r_2})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) \\ &- \frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\left[\mathbf{k}'\delta(\mathbf{r_1}-\mathbf{r_2})\mathbf{k}\right]\Bigg\}, \end{aligned}$$

T=triplet-even strength term and U=triplet-odd strength term.

The associate energy density: $\mathcal{H}_T = \frac{1}{2}\alpha_T\left[\mathbf{J}_n^2 + \mathbf{J}_p^2\right] + \beta_T \mathbf{J}_n\mathbf{J}_p,$

where, $\alpha_T = \frac{5}{12}U\,, \beta_T = \frac{5}{24}(T+U)$

The spin-orbit form factor is modified by the tensor force and reads:

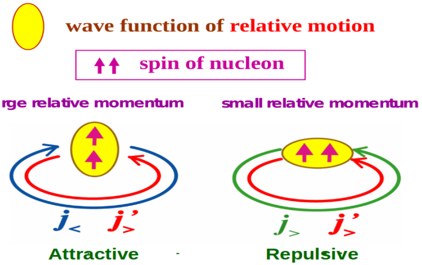
$$\mathbf{W}_q = \frac{W_0}{2}\left(2\nabla\rho_q + \nabla\rho_{q'}\right) + \alpha_T \mathbf{J}_q + \beta_T \mathbf{J}_{q'}\,.$$

Protocol adopted for T and U fixation:

- The crossing of $2p_{3/2}$ and $1f_{5/2}$ s.p. levels in Ni and Cu isotopes at neutron number N = 46 remains unchanged.
- For each pair of T and U values, the spin-orbit strength W_0 readjusted to reproduce the experimental BE of ^{208}Pb.

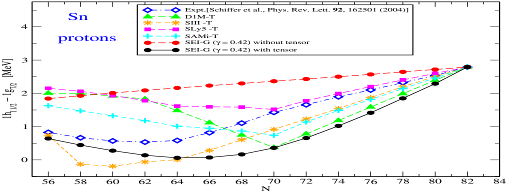
T=800MeV, U=-140MeV and $W_0 = 122 MeVfm^5$

- The tensor force provides an additional attraction between neutron and proton particle or hole states with spins $j_> = l + 1/2$ and $j'_< = l' - 1/2$ (or with $j_< = l - 1/2$ and $j'_> = l' + 1/2$) and repulsion with spins $j_> = l + 1/2$ and $j'_> = l' + 1/2$ (or with $j_< = l - 1/2$ and $j'_< = l' - 1/2$).
- These tensor interactions are stronger between states with similar radial wave functions, i.e., with the same principal quantum number and the same orbital angular momentum because in this case there is a large overlap along the radial directions.

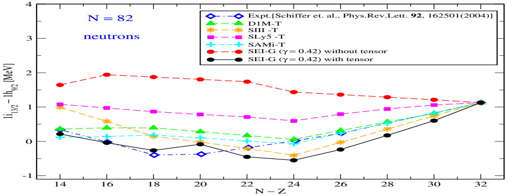


Takaharu Otsuka et al., PRL **95**, 232502 (2005)

Single-particle proton gaps[$1h_{11/2} - 1g_{7/2}$] in the Sn isotopic chains



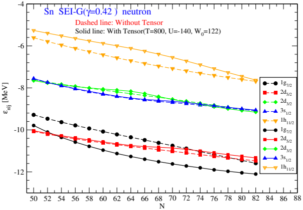
Neutron single-particle gaps[$1i_{13/2} - 1h_{9/2}$] in the N=82 isotonic chains

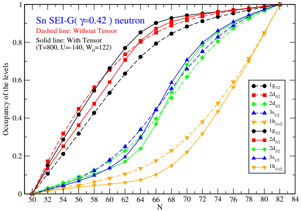


Shifts in MeV

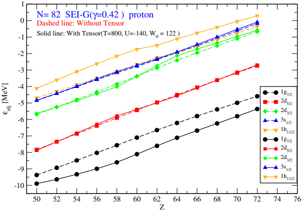
	SEI	SEI-T	SIII-T	SLy5-T	SAMi-T	D1M-T
Sn:($1h_{11/2}$−$1g_{7/2}$)	1.85	2.99	2.12	0.35	4.00	3.00
N=82:($1i_{13/2}$−$1h_{9/2}$)	0.90	1.16	0.57	−0.31	3.29	1.45

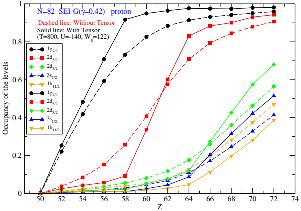
Energy levels and Occupation probability of the neutron levels of the Sn isotopes in the N = 50 to N = 82 major shell:



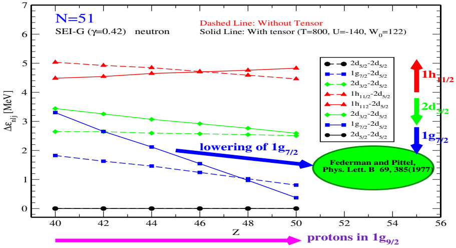


Energy levels and Occupation probability of the proton levels of the N =82 isotones in the Z = 50 to Z = 72 major shell:

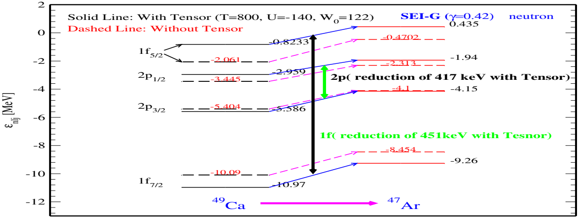




Neutron single particle levels in N = 51 isotones



Reduction of the N=28 gaps



Conclusion

- In this work we have seen that, by including a short-range tensor term to the standard spin-orbit interaction, one is able to explain in a qualitative way the experimentally observed behavior of some specific energy gaps in the Sn isotopes and in the $N = 82$ and $N = 51$ isotonic chains.
- But to have a more quantitative explanation, it appears that the tensor and the spin-orbit interactions should be modified, for example, by introducing a finite range in the tensor force and by exploring a more flexible spin-orbit part, which are tasks for future research.

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Thank you for your kind attention